EXPERIMENTAL CONSTRAINTS ON LEPTON MIXING ANGLES AND NEUTRINO MASS DIFFERENCES FOR THREE SIMULTANEOUSLY OSCILLATING NEUTRINO FLAVOURS

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Abstract

An analysis of experimental results in the framework of simultaneous oscillations of three neutrino flavours is used, in a combined fit, to constrain the elements of the 3 x 3 unitary lepton mixing matrix and to obtain limits on the three lepton mixing angles $\alpha_{e\mu}$, $\alpha_{e\tau}$ and $\alpha_{\mu\tau}$.

Lepton mixing can be described by a 3 x 3 unitary matrix. In fact, the weak eigenstates ν_e, ν_μ and ν_τ are related to the mass eigenstates ν_1, ν_2 and ν_3 by a unitary matrix U_{ik} [1]:

$$|\nu_\ell\rangle = \Sigma_k U_{\ell k}|\nu_k\rangle \qquad (\ell = e,\mu,\tau;\ k = 1,2,3) \tag{1}$$

and this matrix $U_{\ell k}$ can be parametrized in analogy to the quark mixing matrix. We use the parametrization of Maiani [2], given in table 1.

Table 1

Maiani parametrization of lepton mixing matrix

$$U = \begin{pmatrix} C_\beta C_\theta & C_\beta S_\theta & S_\beta \\ -S_\gamma C_\theta S_\beta e^{i\delta'} - S_\theta C_\gamma & C_\gamma C_\theta - S_\gamma S_\beta S_\theta e^{i\delta'} & S\gamma C\beta e^{i\delta'} \\ -S_\beta C_\gamma C_\theta + S_\gamma S_\theta e^{i\delta'} & -C_\gamma S_\beta S_\theta - S_\gamma C_\theta e^{-i\delta'} & C_\gamma C_\beta \end{pmatrix}$$

Up to now, the scarcity of data and the complexity of the problem have prevented a general analysis of experimental data without restrictive assumptions, i.e. with three free angles and neutrino masses. Instead, experimentalists searching for neutrino oscillations have usually made the ad hoc assumption that only one angle and one neutrino mass difference is different from zero, and have given their results in terms of these two quantities.

We present here a general analysis of available experimental data with five free parameters, i.e. three angles and two neutrino mass differences [3].

The time evolution of an initially pure ν_ℓ state can be expressed in terms of mass differences and the neutrino energy E. If this energy is large compared to the neutrino mass, the probability to find a $\nu_{\ell'}$ after a distance L(m) from the production point of an initially pure ν_ℓ state of energy E(MeV) is [4]:

$$P_{\ell\ell'} = \sum_{k,k'=1} U_{\ell k}\, U_{\ell k'}\, U_{\ell' k}\, U_{\ell' k'} \cos\,(2.54\ \Delta m^2_{k'k} L/E) \tag{2}$$

where $\Delta m^2_{k'k} = |m^2(\nu_{k'}) - m^2(\nu_k)|$ is measured in (eV^2).

As a simplification, the elements $U_{\ell k}$ are assumed to be real. This leaves us with five free parameters to be determined, because for thee neutrino masses only two Δm^2 values are independent.

For the Maiani parametrization, in the limiting case of only one angle being finite, this angle can be directly related to oscillations between two flavours of neutrinos. The Maiani angles θ, β and γ are thus uniquely related to the oscillations channels $\nu_e \leftrightarrow \nu_\mu$, $\nu_e \leftrightarrow \nu_\tau$ and $\nu_\mu \leftrightarrow \nu_\tau$ respectively. We call these angles $\theta = \alpha_{e\mu}$, $\beta = \alpha_{e\tau}$ and $\gamma = \alpha_{\mu\tau}$. We note that this decoupling of angles does not occur in the Kobayashi-Maskawa notation.

The experimental data on neutrino oscillations can be divided into two classes: 1. Disappearance experiments, where the flux of neutrinos of one flavour is measured at two distances from their production point. 2. Appearance experiments, where neutrinos of flavour ℓ are searched for in a beam of neutrinos of flavour $k \neq \ell$.

For the simultaneous fit of oscillation experiments of different kinds, we have used all data from reactor and accelerator experiments available at this time. They are listed in table 2 [4]. For each experiment, the original data were used, such that for a given neutrino energy each experiment gives a contribution to the global χ^2 in the fit. For each experiment we checked that, making the restrictive assumptions of the authors, the procedure used here gave back the quoted result of the authors in the two-parameter model with only one θ and one Δm^2.

Table 2

Oscillation experiments included in the fit [4]. The last column is the χ^2 contribution of the experiment for the hypothesis of no oscillation.

Experiment	Measured quantity	Δm^2 range [eV^2]	$\chi^2_{no\ osc.}$ / NDF
(1) CCFRR	$P_{\mu\mu}$ ratio	30. - 1000.	15.5/14
(2) CDHS	$P_{\mu\mu}$ ratio	0.24 - 90.	15.3/14
(3) CHARM	$P_{\mu\mu}$ ratio	0.60 - 20.	1.8/3
	$P_{e\mu}$ difference	1. - 10.	0/0
(4) GOESGEN	P_{ee} absolute	> 0.01	15.5/15
(5) GOESGEN	P_{ee} ratio	0.03 - 3.	7.4/15
(6) BUGEY	P_{ee} ratio	0.02 - 5.	20.9/8
(7) ν_τ Exp.'s	$P_{\mu\tau}/P_{\mu\mu}$	> 0.2	6.3/5
(8) BNL	$P_{\mu e}$ absolute	> 0.43	2.4/6

A total chisquare function was constructed from the measured quantities Q^m (ratios of oscillation probabilities at different L/E or absolute probabilities from a comparison to calculated initial fluxes) and the corrsponding computed values Q^c. The data points of each experiment i are allowed to vary together by a scale factor N_i within the quoted normalization uncertainty $\sigma(N_i)$. The expression

$$\chi^2 = \sum_{i=1} \left[\sum_j \left(\frac{N_i Q^m_{ij} - Q^c_{ij}}{\sigma(Q^m_{ij})} \right)^2 + \left(\frac{N_i - 1}{\sigma(N_i)} \right)^2 \right] \quad (3)$$

is minimized by fitting the mixing angles and mass differences, where the index i labels the experiments, and the index j gives the bin in L/E for a specific experiment.

The results of the best fit are listed in table 3. Limits on the oscillation parameters can be derived from (3) in three parameter planes ($\sin^2 2\alpha_{e\mu}$, Δm^2_{12}), ($\sin^2 2\alpha_{e\tau}$, Δm^2_{13}), and ($\sin^2 2\alpha_{\mu\tau}$, Δm^2_{23}). The curves in Fig.1 give the largest allowed $\sin^2 2\alpha$ values (for 90%C.L.) as a function of the corresponding Δm^2. Each such point has been obtained under the condition that in the other two planes all combinations ($0. < \sin^2 2\alpha < 1.$, $0.01 < \Delta m^2 < 1000\ eV^2$) are possible.

There are two best fit solutions with a $\chi^2_{min} = 78$ for 84 D.F. One of them is (solution a) reached for the parameters $\Delta m^2_{21} = 34\ eV^2$ and $\Delta m^2_{31} = 0.2\ eV^2$. For this solution, the angles are constrained to be $\sin^2 2\alpha_{e\mu} < 4 \times 10^{-3}$, $\sin^2 2\alpha_{e\tau} < 0.13$ and $\sin^2 2\alpha_{\mu\tau} < 0.02$, and the mixing matrix is given in Table 3a, with 90%C.L. limits. The values for Δm^2 are different from zero only at the 1.5 standard deviation level, i.e. the finite values are not significant.

Table 3a

90%C.L. limits on mixing matrix elements for best fit values $\Delta m^2_{12} = 34\ eV^2$ and $\Delta m^2_{13} = 0.2\ eV^2$

$$U = \begin{pmatrix} 1.00 - 0.98 & 0. - 0.03 & 0. - 0.18 \\ -0.04 - 0. & 1.00 - 0.99 & 0. - 0.07 \\ -0.18 - 0. & -0.07 - 0. & 1.00 - 0.98 \end{pmatrix}$$

$\sin^2 2\alpha_{e\mu} < 0.004$; $\sin^2 2\alpha_{e\tau} < 0.13$; $\sin^2 2\alpha_{\mu\tau} < 0.02$.

Table 3b

90%C.L. limits on mixing matrix elements for fit values $\Delta m^2_{12} = 0.2\ eV^2$ and $\Delta m^2_{13} = 38\ eV^2$

$$U = \begin{pmatrix} 1.00 - 0.97 & 0. - 0.19 & 0. - 0.15 \\ -0.21 - 0. & 1.00 - 0.98 & 0. - 0.07 \\ -0.14 - 0. & -0.10 - 0. & 1.00 - 0.98 \end{pmatrix}$$

$\sin^2 2\alpha_{e\mu} < 0.15$; $\sin^2 2\alpha_{e\tau} < 0.09$; $\sin^2 2\alpha_{\mu\tau} < 0.02$

A second **local** minimum of χ^2 is found at $\Delta m^2_{21} = 0.2\ eV^2$ and $\Delta m^2_{31} = 38\ eV^2$, i.e. interchanging the two values from solution a. Here also $\chi^2_{min} = 78$, the 90%C.L. limits on the angles are $\sin^2 2\alpha_{e\mu} < 0.15$, $\sin^2 2\alpha_{e\tau} < 0.09$ and $\sin^2 2\alpha_{\mu\tau} < 0.02$, and the range of matrix elements is given in Table 3b.

Since there is no way of dicriminating between the two solutions, global limits on angles for mass differences in the range $\Delta m^2_{12} > 0.06\ eV^2$, $\Delta m^2_{13} > 0.04\ eV^2$ and $\Delta m^2_{23} > 2\ eV^2$ are $\sin^2 2\alpha_{e\mu} < 0.15$, $\sin^2 2\alpha_{e\tau} < 0.13$ and $\sin^2 2\alpha_{\mu\tau} < 0.02$ while for any specific mass difference $\Delta m_{k'k}$. the 90%C.L. limit on the correponsing $\sin^2 2\alpha_{\ell\ell'}$ is given in Figs. 1a through 1c. For the hypothesis of no oscillation, i.e. all mixing angles fixed to zero, a χ^2 of 85 for 87 D.F. is obtained, i.e. the data are consistent with this hypothesis.

In conclusion, we have shown that an analysis of existing data on oscillations of three neutrino flavours yields limits on the mixing matrix and on the three mixing angles each corresponding to one channel of oscillations. While this method does not in general allow stringent constraints on the mixing parameters to be extracted from one experiment, the combination of the different experimental data gives limits on angles and the mixing matrix elements which are about as restrictive as earlier two-neutrino analyses in separate oscillation channels. The ensemble of data is consistent with no oscillation occuring.

If neutrino masses are finite [5], then mixing angles of the order of $\sqrt{(m_e/m_\mu)}$ or $\sqrt{(m_\mu/m_\tau)}$ are expected in some models invoking family symmetries [6]. Experiments have nearly reached the level of sensitivity needed for testing such models. As in the case of quark mixing, the problem of family mixing is still waiting for a solution.

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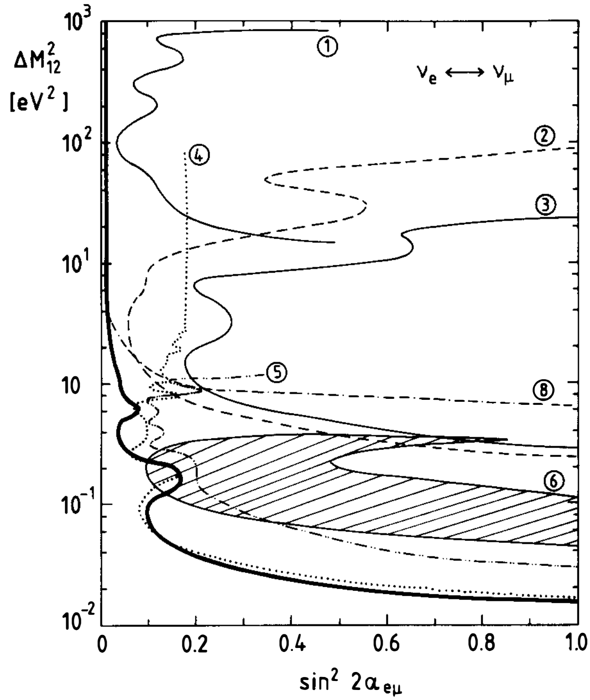

Fig.1a: Limits on mixing parameter $\sin^2 2\alpha_{e\mu}$ vs. neutrino mass difference ΔM^2_{12}. The thin lines are 90%C.L. upper limits on $\sin^2 2\alpha_{e\mu}$ from individual experiments 1) 2) 3) 4) 5) and 7) in ref. [4] assuming $\alpha_{e\tau} = \alpha_{\mu\tau} = 0$, the shaded arae is the allowed range from the Bugey experiment (6) in ref. [4]. The broad line is the 90%C.L. upper limit on $\sin^2 2\alpha_{e\mu}$ from the three-flavour oscillation analysis, allowing the other two mixing angles to vary over the whole range $0 \leq \sin^2 \alpha_{e\tau} \leq 1$ and $0 \leq \sin^2 2\alpha_{\mu\tau} \leq 1$.

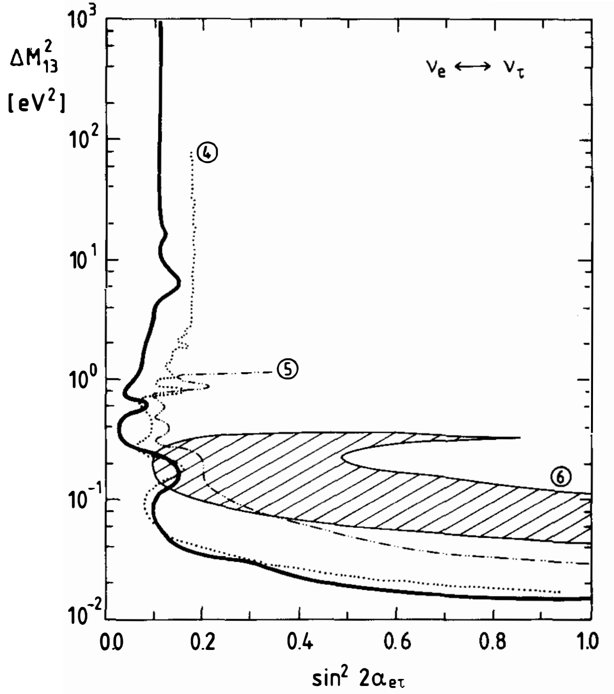

Fig.1 b:Limits on mixing parameter $\sin^2 2\alpha_{e\tau}$ vs. ΔM_{13}^2. Thin lines are 90% C.L. upper limits from individual experiments assuming $\alpha_{e\mu}=\alpha_{\mu\tau}=0$. Broad line from three-flavour analysis

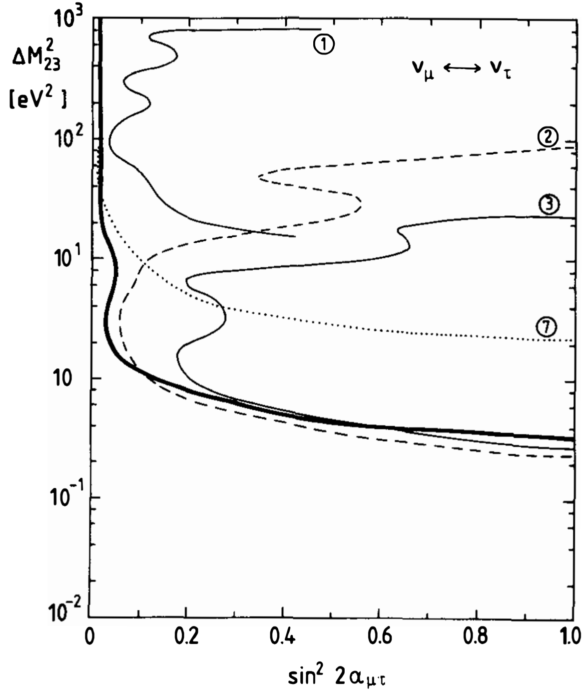

Fig.1c:Limits on mixing parameter $\sin^2 2\alpha_{\mu\tau}$ vs. ΔM_{23}^2.