

The Progress of Investigation on Neutrino Properties

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Abstract. Generally, particle physics plays an important role in the whole modern physics system. Thereinto, neutrino, as an elementary particle found from beta decay, is of great importance to discover its properties and mechanics. With the help of information retrieve and literature review, this paper aims at helping recollecting sorts of knowledge, e.g., the developing history of the principle, different kinds of matrix expressions, field theories etc. Additionally, some possible future developing directions have been proposed. Moreover, plenty of analytical models and experimental approaches for generating massive neutrinos are demonstrated, e.g., Lagrangian and beta decay. More experiments need to be carried out for the sake of verifying the theoretical predictions. Overall, these results offer a guideline for completing the Standard Model and hopefully the Grand Unified Theory, such as Majorana mass term.

1. Introduction

Neutrino is an elementary fermion (particle with a half spin), which can interact only via the weak interaction and gravity. The neutrino is electrically neutral and was thought to be zero mass since its rest mass is extremely small (much smaller than that of the other known elementary particles excluding massless particles). Since neutrinos have zero charge, they are unable to interact with electromagnetism force. The weak force has a very short range, i.e., it's not significant. Besides, the gravitational interaction is extremely weak due to its tiny mass, and neutrinos do not participate in the strong interaction neither. Thus, neutrinos typically pass through normal matter unimpeded and undetected [1].

This characteristic of neutrino enables it to be widely used in scientific research for many purposes. The low mass and neutral charge of neutrinos means that their interaction with other particles and fields is extremely weak. Therefore, neutrinos can be used to detect environments where other radiation sources cannot penetrate, e.g., the conditions of the solar core, astrophysical sources beyond the Solar System and the observation of supernovae.

The rest of the paper is organized as follows. The section 2 is mainly focused on basic descriptions and brief history neutrino. The section 3 introduces field theory and weak interaction related to neutrino. The section 4 discusses massive neutrino. Finally, a brief summary is given in section 5.



2. Basic Descriptions and History of Neutrino

2.1. History of neutrino

In the quantum world, the absorption and emission of energy are discontinuous, which is caused by the transition between different energy levels of the atoms. However, it is strange that the energy spectra of the beta rays released by the substance during the beta decay is continuous. In addition, it is also found that the electrons only take away part of the energy, and some of the energy is missing. On this basis, physicists like Niels Bohr believe that the law of conservation of energy fails in β decay [2].

In 1930, The proposal of the “neutrino” was first put forward by W. Pauli. He believes that in addition to electrons, a new particle with zero rest mass, electrically neutral, and different from photons is emitted in beta decay, taking away another part of the energy. The interaction between this particle and matter is so weak that it is difficult to detect it in experiments. Thus, the law of conservation of energy still holds, the energy distribution ratio between the (anti) neutrinos and the electrons can be varied in different cases. In 1933, the Italian physicist Fermi proposed a quantitative theory of beta decay. According to Fermi’s theory, in addition to the known gravitational and electromagnetic forces, there is a third kind of interaction called weak interaction. The beta decay is through the weak interactions, in which a neutron in the nucleus decays into an electron, a proton and an (anti) neutrino. His theory quantitatively described the law of β -ray energy spectrum continuity and β -decay half-life, and the reasons for β -ray energy spectrum continuity have been well explained [3].

In 1956, Reines and Cowan discovered the neutrino through inverse beta decay, which was using antineutrinos emitted by beta decay in nuclear reactors. The antineutrinos undergo anti-beta decay with hydrogen nuclei, forming fast and slow signals in the detector that are related to intensity and time. This is the first time that neutrinos have been detected in an experiment [4]. Madame Wu in 1956 demonstrated that parity conservation was violated in weak interactions following the idea of Lee and Yang [5, 6]. This leads to an awareness of the limitations of the standard model which only involves left-handed neutrino. This is the key issue why neutrino's mass cannot be explained at least in the minimum form of the standard model. It has to be extended, i.e., somehow right-handed neutrino is included.

Another type of neutrinos called Muon-neutrinos were discovered in 1962 by Lederman, M. Schwartz et al [7]. Neutrinos are produced correspondently with its charged leptons. For example, Muon neutrinos are produced in pion decay with Muons. When they performed the experiment, there were only muons but not electrons in the reactor. Therefore, they realized that there must be another type of neutrinos which is Muon neutrinos. Later, the third type of lepton tau was discovered, and it was also expected to have an associated neutrino. Nevertheless, tau neutrinos were announced to be detected until 2000, whose existence had already been inferred by both theoretical consistency and experimental data [8].

2.2. Neutrino oscillation

One of the main focuses of the neutrino research at early stage was the neutrino oscillation, which is a quantum mechanical phenomenon generated by different massive neutrinos. The first idea of neutrino oscillations was considered by B. Pontecorvo in 1957 [9], who developed the mathematical formalism and the modern formulation of neutrino oscillations in the vacuum.

Neutrinos oscillate between different flavours. For example, an electron neutrino produced in a beta decay reaction may interact in a distant detector as a muon or tau-neutrino. This oscillation occurs because the three mass state components of the produced flavour travel at slightly different speeds. Thus, there is a relative phase shift that change the way they combine to produce a varying superposition of three flavours. Each flavour component thereby oscillates as the neutrino travels, with the flavours varying in relative strengths. The relative flavour proportions when the neutrino interacts represent the relative probabilities for that flavour of interaction to produce the corresponding flavour of charged lepton [10, 11].

Neutrino oscillations always come along with the idea of mixing. Mixing was introduced at the beginning of the '60 by Maki et al. [12]. The relationship between flavour and mass eigenstates is encoded in the PMNS matrix, which is a unitary mixing matrix containing information on the mismatch of quantum states of neutrinos when they propagate freely and when they take part in weak interactions [13]:

$$U = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix} \begin{bmatrix} e^{\frac{i\alpha 1}{2}} & 0 & 0 \\ 0 & e^{\frac{i\alpha 2}{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad (1)$$

where s_{ij} and c_{ij} are used to denote $\sin \theta_{ij}$ and $\cos \theta_{ij}$ respectively, θ is the mixing angle and there are three of them ($\theta_{12}, \theta_{23}, \theta_{13}$).

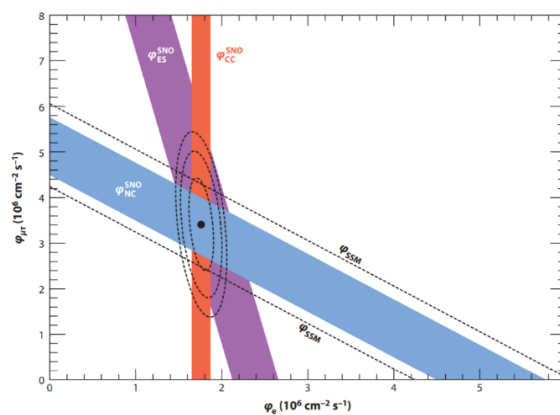


Figure 1. The flux of solar neutrinos of μ or τ flavor versus the flux of electron neutrinos deduced from the three neutrino reactions.

There were two well-known anomalies in the history of neutrinos, one was solar neutrino deficit. First indications of neutrino oscillations came from solar neutrino: R. Davis built the Homestake experiment to detect solar neutrino based on an experimental technique by Pontecorvo. Compared with the predicted solar neutrino fluxes, a significant deficit was first announced. This anomaly received further confirmation from various institutions (SAGE, GALLEX, Super-Kamiokande, SNO, etc.) and was finally interpreted as neutrino oscillations. The three bands intersect at the fit values, indicating that the combined flux results are consistent with neutrino flavour transformation with no distortion in the neutrino energy spectrum as shown in Fig. 1. Contemporarily, it is recognized that solar neutrinos do oscillate into other types of neutrinos like muon and tau neutrinos.

Another anomaly was found in atmospheric neutrinos. The experiments associated with this source of neutrino, including the confirmation of neutrino oscillation, was conducted at the Super-Kamiokande (Super-K) detector. Super-K is a large detector which located 1000 meters under a mountain in Japan, as schematically illustrated in Fig. 2. It contains a cylindrical stainless-steel tank, 39.3m in diameter and 41.4 m in height. The tank is closed during detection and filled with 50 kilotons of ultra-pure water. On each side of the wall, 11129 photo sensors called photo-multiplier-tubes (PMTs) are installed.

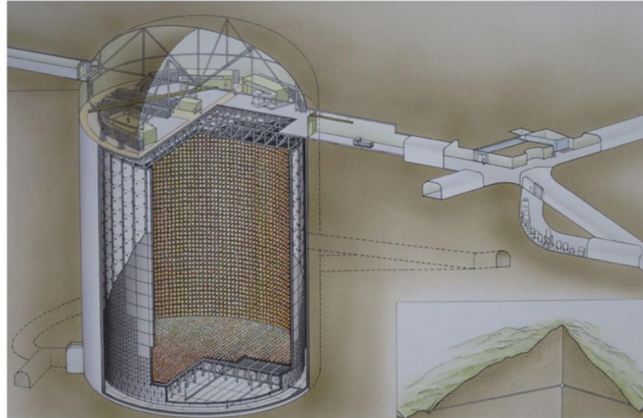


Figure 2. Overview structure of Super-K.

The detection of neutrino in Super-K is through the weak interaction between neutrino and water nuclei. After the interaction, a charged particle is produced and propagating through the water. If the speed of the particle is larger than the speed of light in the water, Cherenkov light will be emitted in a cone shape to the direction of a charged particle, eventually forming a Cherenkov ring on the PMTs sensor as illustrated in Fig. 3.

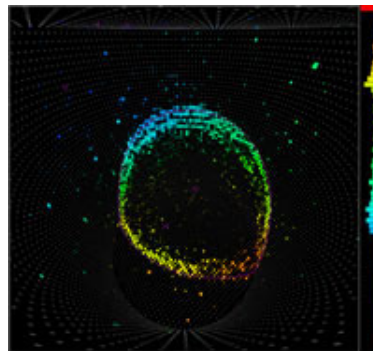


Figure 3. The Cherenkov ring emitted by a muon.

When cosmic rays (e.g., highly accelerated ions) hitting the atmosphere, pions and kaons are produced. Pions can decay into muons and muon neutrinos, and furthermore muons can decay into electrons, muon neutrinos and electron anti-neutrinos. The production of neutrino on the atmosphere is expected to be isotropic, and since neutrinos are hardly to interact with matter, the observed fluxes of up-going and down-going neutrinos in the SK should be equal. This statement had been proved to be true for electron neutrino. However, for the muon neutrino, measured results have shown that the flux of down-going neutrino is much larger up-going neutrino. The phenomenon can be explained by neutrino oscillation. Comparing to the down-going neutrino, the up-going neutrino has to travel longer to reach the detector. During such long distance, some muon neutrinos oscillate into the tau neutrino, decreasing the total flux of muon neutrino. Detailed explanation on experiment in Super-K will be given in the massive neutrino section. In general, the discovery in Super-K, along with solar neutrino deficit are recognized as the result and proof of neutrino oscillations.

3. Field theory and Weak interaction

Before talking in detail about the researches on neutrino, some basic analytical models will be introduced in terms of field theory and weak interaction.

3.1. Field Theory

In field theory, the Lagrangian of a system is different from the formal what we get used to in classical physics. Instead, one needs to rewrite it into the form of the sum of two term: free field and interaction field shown as followed.

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{int}. \quad (2)$$

For instance, in a charged fermion system with electromagnetism between photons, one can write down the exact expression of free field term within kinetic term and massive term.

$$\mathcal{L}_0 = \bar{\psi}(i\gamma^\mu\partial_\mu - m_0)\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}, \quad (3)$$

where ψ denotes the spinor field and the two F terms denote to electromagnetic field tensor. We can see that the first term of the R.H.S represents free fermions and the second term represents free photons. Based on Lagrangian function, one could easily gain Dirac function and Maxwell function.

Manifestly, the interactions between particles need three or more particles, e.g., the collision of two leads to a new particle. Therefore, it is evident that those two terms with productions only including up to two fields, which means they do not imply any interactions. An expression for the interaction field term is required, for instance:

$$\mathcal{L}_{int} = -eA_\mu\bar{\psi}\gamma^\mu\psi, \quad (4)$$

In which an electromagnetic field A_μ products with two fermion spinor fields ψ and $\bar{\psi}$. With the addition of \mathcal{L}_{int} , the Lagrangian function is able to describe the process of positive and negative electrons annihilation or the influence of virtual photons on two charged fermions.

3.2. Weak Interaction

As mentioned in the above sections, the principle of neutrino stemming from β decay. In β decay, it is known that a neutron decays and produces a proton, an electron and a neutrino according to the conservation of energy, momentum and leptons. In this process, we know that there are four kinds of fermions with spin of 1/2. As a result, the simplest way to derive the Lagrangian of interaction is to product these four terms directly, which origins from Fermi:

$$\mathcal{L}_{int} = \frac{G_F}{\sqrt{2}}\bar{p}\gamma_\mu n\bar{e}\gamma^\mu\nu_e. \quad (5)$$

which is so-called the weak interaction, involving neutrinos.

4. Massive Neutrino

The property of neutrino mass is the most crucial subject in neutrino physics. There are only left-handed components in neutrino fields to satisfy the local symmetry in Standard Model, which means neutrinos' mass-term cannot be derived under the Higgs mechanism. However, experiments about neutrino oscillation demonstrated that neutrinos were massive and mixed, which deviated from the standard model.

4.1. Neutrino oscillation in Super-K

As mentioned above, neutrino oscillation is a quantum mechanical phenomenon, which describes the probability for a certain neutrino to change its flavour spontaneously. As shown in Fig. 4, when a neutrino is moving in space, it can be thought of as superposition of three mass eigenfunctions, which also determines the flavour of this neutrino. Different eigenfunctions oscillate in different frequency, which may result in the change in phase of the superposition wave, hence changing the flavour of the neutrino.

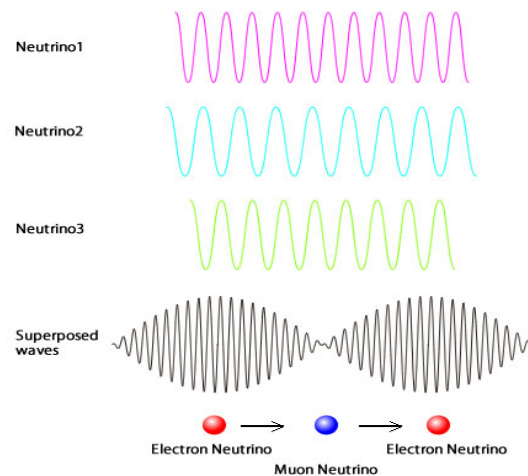


Figure 4. Demonstration of flavour oscillation.

Thus, the observation of neutrino oscillation between two types of neutrinos implies the fact that they have different mass eigenstates, which was the intrinsic behind neutrino oscillation experiments that always measure the squared mass-difference Δm^2 instead of an absolute mass. According to examination different values of Δm^2 , neutrinos with different flavors have different non-zero masses.

A typical result of Neutrino oscillation was taken by the Super-K experiment in 1998, in which an analysis of oscillation between atmospheric muon neutrino and tau neutrino was done. As exhibited in Fig. 5, there was inconsistency between the measured muon neutrino flux and expectation value, which meant neutrino oscillation happened when muon neutrino travelled through the Earth. The results showed that Δm^2 between the two oscillating states, under a zenith angle $\sin^2 2\theta > 0.82$, were between 5×10^{-4} and $6 \times 10^{-3} eV^2$, with 90% confident level.

It should be noted that the magnitudes of neutrino mass were much smaller than these of quarks and leptons. In general, neutrino masses are believed to be a low-energy manifestation of physics beyond the Standard Model. Their masses are suppressed by a new high-energy scale, which may give a clue to the unification of forces.

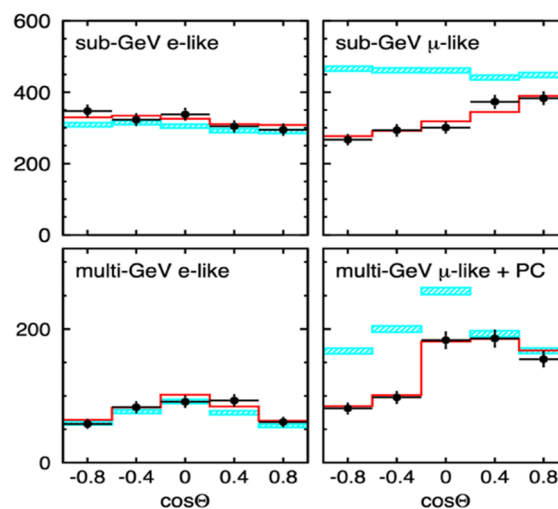


Figure 5. The plots were taken from a similar research of atmospheric neutrino carried out in Super-K detector, which shows the zenith angle distributions of e-like and μ -like events. The red line indicates the results of the best fit and the blue boxes indicates the expectations assuming no neutrino oscillations.

4.2. Dirac Mass term

One way to add a mass term for neutrino is to introduce three right-handed neutrino fields ($\nu_{\alpha R}$, $\alpha = e, \mu, \tau$) into the Standard Model, deriving the Minimally Extended Standard Model. Then, we can apply the Higgs mechanism to obtain Dirac neutrino mass. Unlike the left-handed component, the right-handed neutrino fields do not participate in any interactions (neither weak, strong, nor electromagnetic), which is why they are called sterile.

By applying the Yukawa interactions to the right-handed sterile neutrino fields, which is in analogy to the derivation of charged fermion masses, one derives the expression of the Dirac mass:

$$M_k = \frac{y_k^{\nu} v}{\sqrt{2}}, (k = 1, 2, 3), \quad (6)$$

where y_k^{ν} denotes an unknown parameter of the Standard Model, and v denotes the vacuum expectation value of the Higgs field. This expression shares the same form with the masses of the charged leptons.

However, the expression of M_k shows proportionality with v , as the masses of charged leptons and quarks, while it is known that the neutrinos are much lighter than those of quarks and leptons, which means y_k^{ν} must be small. However, the smallness of y_k^{ν} remain unknown under the Higgs-neutrino Yukawa coupling matrix. In fact, under the framework of the Standard Model, lepton and quark masses cannot be predicted and must be obtained from experiments, leading us to believe that Standard Model can be thought as a model that is under the low-energy limit of a complete theory. Thus, the investigation on masses of quarks and leptons may allow us to understand physics beyond the Standard Model.

4.3. Majorana Mass term

Another way of describing the neutrino mass is through the Majorana mass term. To derive this mass term, let's first consider the two-component theory proposed by H. Weil in 1929. For a generic fermion field ψ with left and right-handed component:

$$\psi = \psi_L + \psi_R, \quad (7)$$

its Dirac equation can be split into two different parts:

$$i\gamma^{\mu} \partial_{\mu} \psi_L = m\psi_R, i\gamma^{\mu} \partial_{\mu} \psi_R = m\psi_L. \quad (8)$$

For a massless neutrino we can decoupled the Eq. (7) into Weyl equations:

$$i\gamma^{\mu} \partial_{\mu} \psi_L = 0, i\gamma^{\mu} \partial_{\mu} \psi_R = 0. \quad (9)$$

Hence, one can describe the massless neutrino using a single chiral field (only left-handed) with only two independent components in the Standard Model.

E. Majorana, on the other hand, attempted to construct a massive neutrino using just a single left-handed field. He first tried to express ψ_R in terms of ψ_L by using the charge conjugation matrix C :

$$\psi_R = \xi C \overline{\psi_L}^T, \quad (10)$$

where T indicates the transpose of $\overline{\psi_L}$ and ξ is an arbitrary phase factor. Note that $C \overline{\psi_L}^T$ can be proved to be right-handed, indicating the rationality of this expression.

Thus, the Majorana field can be written as:

$$\psi = \psi_L + \psi_R = \psi_L + C \overline{\psi_L}^T = \psi_L + \psi_L^C, \quad (11)$$

Where the charge conjugate of this field is itself. Therefore, a Majorana particle described by Eq. (10) is its own anti-particle, which can only be a neutral fermion (neutrino). Majorana's statement have changed our understanding on neutrino and anti-neutrino, indicating the fact that we would only have one single particle, the neutrino.

Considering a single neutrino type ν with its right-handed component ν_L^C , and imposing this expression into the Dirac mass term (given by), we obtain:

$$\mathcal{L}_{mass}^D = -m\bar{\nu}\nu, \quad (12)$$

allowing us to derive the expression of Majorana mass term:

$$\mathcal{L}_{mass}^M = -\frac{1}{2}m\bar{\nu}_L^c\nu_L + H.c. \quad (13)$$

Unlike Dirac mass term, Majorana mass term is a single of the SM gauge group and, as such, it can appear as an essential mass term in Lagrangian. In addition, Majorana mass term differs from the Dirac mass term in that it violates the lepton number conservation. In Dirac field, neutrinos have a lepton number of +1, and antineutrinos have a lepton number of -1, which means during the reaction, the lepton numbers add up to zero and are conserved. Since neutrinos are the same as their antiparticle in the Majorana field, they have the same lepton number. Interactions involving Majorana neutrinos generally violate lepton number conservation by ± 2 .

4.4. Neutrino-less Double Beta Decay

One way to verify the Majorana neutrinos is through Neutrino-less Double Beta Decay. To understand this process, we first have to introduce the Two-Neutrino Double Beta Decay, in which the nuclear charge changes by two units while the atomic mass is left unchanged:

$$N(A, Z) = N(A, Z + 2) + 2e^- + 2\bar{\nu}_e, \quad (14)$$

where A denotes the mass number and D denotes the proton number. On the other hand, the Neutrino-less Double Beta Decay:

$$N(A, Z) = N(A, Z + 2) + 2e^-, \quad (15)$$

was proposed by W. H Furry in 1939. Clearly, this decay is forbidden under the Standard Model, as it violates the conservation of the total lepton number by two units. The mechanism of this process is given by Fig. 6.

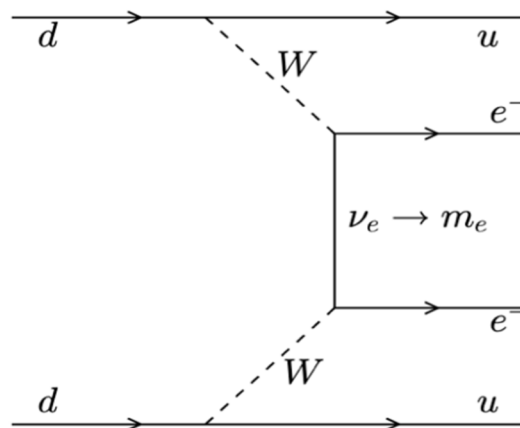


Figure 6. An illustration of the Tree-level quark diagram of Neutrino-less Double Beta Decay.

For this process, a neutrino decays and emits into a right-handed, which has to be absorbed by another neutron as a left-handed. Thus, the only way to fulfil these conditions is that the neutrino and the anti-neutrino are the same. On the other hand, the right-handed emitted by the first decay must evolve a left-handed chiral component as it propagates towards the second, which means the neutrino must have a mass to satisfy the helicity matching. Thus, a Majorana neutrino will fulfil all the conditions of the Neutrino-less Double Beta Decay, which means that proving the existence of Neutrino-less Double Beta Decay implies that neutrinos can be described in Majorana formation.

To sum up, experiments of neutrino oscillation indicate that neutrino indeed contain an absolute mass though really small comparing to other leptons. This result deviates the Standard Model and gives a necessity to derive a more complete theory. There are two ways of generating a neutrino mass term: Dirac and Majorana. Among these, the Majorana mass term suggests that neutrino is its anti-neutrino, which deviates the description from Standard Model. One way to prove that the Majorana mass term can describe neutrinos is by discovering Neutrino-less Double Beta Decay.

5. Conclusion

In summary, we investigate some basic knowledge about neutrino from its history to the recent research based on state-of-art facilities, e.g., massive neutrino. Due to the conservation laws of energy, momentum and lepton, people predict the existence of the new particle called neutrino theoretically. Then, as the development of science, in experiment three kinds of neutrinos have been discovered individually. Moreover, we give out basic knowledge of field theory in this paper and explore more focusing on the massive neutrino according to Yukawa theory and Standard Model. Manifestly, it is just a beginning which is written for introduction, which means more detailed theory and mathematical knowledge need to be given. Additionally, in this area particle physics still need more concrete experiments and evidence to support its theories instead of staying on papers. That is one important direction people have to work hard. Overall, these results pave a path for the aim of experiments need to be carried out and shed light on the possible research directions about future development of the theory.

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