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AN APPROACH TO HADRONIC SYMMETRY BREAKING IN GAUGE THEORIES *

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ABSTRACT

Evoking the spirit of (so-called) natural zeroth order symmetries of the quark mass matrix, an attempt is made to understand the origin and nature of approximate hadronic symmetries in the context of factorizable gauge theories of the weak and strong interactions. Within certain assumptions, dependent upon truly non-perturbative effects, formulas relating the parameters of $(3,\overline{3})+(\overline{3},3)$ breaking terms to mixing angles between weak gauge bosons and ratios of gauge bosons masses are obtained.

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The recent observation[1] that non-Abelian gauge theories may exhibit free-field asymptotic behavior has added support to the possibility that gauge principles underlie all of the dynamics, including strong as well as weak interactions. According to Weinberg[2,3], these interactions would correspond to the two factor groups, G_s and G_w, into which he would expect the total dynamical group to decompose. G_s and G_w are, respectively, non-chiral and chiral, and their associated coupling constants are of order 1 and e. While leptons are non-neutral only under G_w, quarks are charged with respect to both G_s and G_w, and can be represented in the form of a matrix with weak and strong interactions producing transitions along columns and rows, respectively. Sets of elementary canonical scalar fields non-neutral only under G_w are also introduced which, by their vacuum expectation values (VEV), break G_w, give masses to the weak gauge bosons and also produce or contribute to the fermion mass matrix. For the sake of asymptotic freedom, the use of elementary strongly interacting scalar representations is avoided and the option is (a priori) left open between having the semi-simple strong interaction group G_s not broken at all, or else broken dynamically. The interest in the former case is motivated by the hope[2] that the masslessness of the "colored gluons" would then provide a mechanism for preventing the production of colored particles, including quarks and gluons.

Weinberg's scheme, as outlined above, appears to fulfill basic requirements of a sensible theory, such as the vanishing to order α of parity and strangeness non-conservation effects.[2,4] Furthermore, thanks to the (possibly) asymptotically free nature of its strong interactions, this scheme would account for Bjorken's scaling in electroproduction and, also, enable one to perform exact calculations of order α corrections to the effective hadronic Lagrangian while ignoring all effects of the strong interactions.[2]

It is in this general framework[5] that here we wish to investigate certain aspects of the breaking of approximate hadronic symmetries such as isospin, chiral SU(2)xSU(2), SU(3) and chiral SU(3)xSU(3). Since these are the global symmetries which, in our approach, correspond to the weak interaction gauges, the weak interactions will be entirely responsible for their breakdown.[6] We shall argue that, if certain conditions are satisfied, it is possible to calculate the parameters $(\varepsilon_3, \varepsilon_8)$ of the $(3,\bar{3})+(\bar{3},3)$ symmetry breaking terms in the strong interaction Lagrangian $(u_0 + \varepsilon_3 u_3 + \varepsilon_8 u_8)$. In fact, we shall relate these quantities to mixing angles between weak gauge bosons and ratios of gauge bosons masses. From a technical viewpoint, the calculability of such symmetry breaking terms stems from the finiteness[8] of radiative corrections to natural zeroth order symmetries[8,9] of the fermion mass matrix. Although in an unconventional sense, chiral SU(3)xSU(3) will indeed, in our approach, be a natural zeroth order symmetry of the quark mass matrix. In particular, isospin will be such a natural zeroth order symmetry. This is especially desirable since, as emphasized by Weinberg[8,10], it is then possible to directly account for the order of magnitude of mass splittings within isospin multiplets.

More specifically, the parameters ε_3 and ε_8 associated with the $(3,\bar{3})+(\bar{3},3)$ symmetry breaking terms are (in a Lagrangian field theory with quarks as elementary fields) simply related to ratios of quark masses:

$$\varepsilon_3 = \frac{\sqrt{3}\,(m_p - m_n)}{\sqrt{2}\,(m_p+m_n+m_\lambda)} , \qquad \varepsilon_8 = \frac{(m_p+m_n-2m_\lambda)}{\sqrt{2}\,(m_p+m_n+m_\lambda)} , \tag{1}$$

in obvious notations. Thus, an attempt to evaluate these quantities may consist of setting up a scheme in which such ratios of masses are calculable. In the present context of "factorizable" gauge theories of the weak and strong interactions[2], this would be the case, e.g., if, corresponding to a natural zeroth order symmetry of the quark mass matrix, all these masses were forced to vanish when the weak interaction is turned off while the mass of a fourth, charmed quark (p') is kept different from zero. The uncharmed quarks would then acquire their masses by finite radiative corrections involving the charmed quark as a virtual state. The calculation of these radiative corrections, and of the symmetry breaking parameters in Eq. 1, would actually be carried out correctly ignoring the strong interactions of the quarks.[2]

This mechanism of quark mass generation is analogous to the one that has been invoked to generate the electron-muon mass ratio in the lepton sector.[11] The motivation is different: rather than the numerological observation of a ratio of order α, in the hadronic case we are attempting to understand the origin and the breakdown of symmetries as natural zeroth order symmetries of the quark mass matrix. Unfortunately, as in the case of the electron-muon mass ratio, serious difficulties appear to be in the way to the implementation of these ideas in all models that we have considered and that could be regarded as acceptable. These difficulties are of fundamental character and are connected with the generally accepted presence of elementary Higg's fields in the Lagrangian. As emphasized in a recent note by Goldman and the present author[12], we cannot generally afford to retain elementary scalar fields in the Lagrangian when it comes to calculating certain symmetry breaking effects, since these effects would then be artificially controlled by (arbitrary) parameters associated with Lagrangian terms involving the scalar fields in question. Of course, this is not true of the real world, at least if the latter is to satisfy our expectation of being described by a "minimal" theory, with a small number of independent parameters.

We shall therefore have to pay the price of giving up the simplicity of working all the way with elementary Higg's fields and persue our conjecture that in the real world the symmetry breaking role of elementary scalar fields in the Lagrangian is actually assumed by bound states formed as a result of the gauge interaction.[12] More specifically, we shall postulate that some representations of such scalar bound states are formed and that some elements in the representations acquire a non-vanishing VEV. These VEV's contribute tadpole terms to the physical gauge boson and quark mass matrices. A quark mass can be obtained by summing the tadpole expansion of Fig. 1. The first contribution in this expansion actually vanishes identically if one excludes, as we do, Schwinger's mechanism of mass generation or any other mechanism which does not involve explicit tadpoles. As for the one-tadpole contribution, our conjecture is that only one of the quarks acquires a non-negligeable mass from this term. We shall refer to this as the charmed quark (p'). Since the two-tadpole term is generally not expected to contribute by virtue of its group structure, the uncharmed quarks (q) will then acquire their masses essentially through the three-tadpoles diagram of Fig. 2.

It is easy to convince oneself that the conditions we are invoking lead to a theoretical structure which mimics the structure of a theory with elementary scalars in which the VEV of the Yukawa-coupled scalars is devised to make one of the fermions massive in zeroth order while keeping the others, in the same order, exactly massless. Thus, we appeal to dynamical symmetry breaking and define an analog to the natural zeroth order symmetries encountered in the conventional approach in which the symmetry breaking is manipulated with the help of elementary Higg's fields present at the Lagrangian level. The crucial difference between the two approaches is that, while in the approach which adopts elementary scalars the zeroth order conditions of interest to us are not generally "stable under radiative corrections"[14], in the approach that relies upon bound states it seems a priori possible to have the desired conditions realized. In fact, the analog of loop corrections which, in the former case, have a destabilizing effect are now rendered innocuous by the composite nature of the bound states (see Ref.12).

It should be remarked that the zeroth order condition which we want to simulate dynamically is the optimal condition one could hope to be satisfied since it incorporates the largest possible set of zeroth order symmetry relations and optimizes the predictive capability of simple perturbation theory. This reflects itself, in our case, in our ability to calculate ratios of quark masses by evaluating diagrams as in Fig. 2. These diagrams implicate bound state form factors where in the conventional approach one would have elementary vertices. However, the mass scale characterizing the fall-off in momentum space of such form factors is expected to be large and their presence should not appreciably affect the value of the loop integral. In reaching this conclusion, the role played by the effective cut-off of the loop integral, arising from the mutual cancellation of divergencies between diagrams corresponding to exchanges of different physical mixtures of left-handed ($W^{(L)}$) and right-handed ($W^{(R)}$) gauge fields (we have implied, e.g., the identification G_w=SU(4)xSU(4)), should be noted.

Thus we are led to the following approximate expression for the (order α) mass ratio of the charmed quark (p') to any uncharmed quark (q):

$$(m_{p'}/m_q)_1 \simeq (\alpha/\pi)\left|\sin 2\theta_{p'q} \ln\left(M^{(1)}_{p'q}/M^{(2)}_{p'q}\right)\right| \qquad (2)$$

where $M^{(1)}_{p'q}$ and $M^{(2)}_{p'q}$ are the masses of the physical (diagonal) gauge bosons connecting q to p', and $\theta_{p'q}$ is the mixing angle relating these bosons to $W^{(L)}_{p'q}$ an $W^{(R)}_{p'q}$. Let us remark, at this point, that it is possible to arrive at the same equation from a different approach, starting from a pole approximation to the Dyson-Schwinger equations for the quark self-energy parts, provided that a certain eigenvalue equation is satisfied by the gauge bosons masses[15]. This coincidence reinforces our confidence, if not in the assumptions, at least in the approximations we have used to arrive at Eq. 2.

Needless to say, all the quantities appearing on the r.h.s. of Eq. 2 are physical and measurable in principle. Through the use of current algebra, the symmetry breaking parameters ε_3 and ε_8, that we obtain by substituting Eq. 2 into Eq. 1, are also (indirectly) measurable. Consequently, our approach to the $(3,\overline{3})+(\overline{3},3)$ part of chiral SU(3)xSU(3) breaking can, in principle, be checked experimentally. However, since such a check would be based on the study of reactions involving (yet unobserved) charm-changing weak currents, it is clear that at this time we should concentrate on those implications of our approach which are more on the theoretical side.

Corresponding to favored values of ε_3 and ε_8[16] ($\varepsilon_3 \simeq -0.02, \varepsilon_8 \simeq -1.25$), the mass ratios $m_p/m_n \sim 1/1.5$ and $m_n/m_\lambda \sim 1/20$ are found. The latter ratio, whose deviation from 1 mirrors the medium-strong symmetry breaking, is of course especially large. In the scheme (with charm) under consideration, this would suggest a large suppression of the diagram (Fig. 2) giving n its mass or a large enhancement of the similar diagram giving λ its mass or, perhaps, both. Suppression can be achieved if the relevant left-handed and right-handed gauge fields are prevented from mixing significantly, i.e., if $\sin 2\theta_{p'n} \ll 1$ (see Eq. 2). Such suppression is, however, accompanied by a relative increase in the mass of the charmed (p') quarks and, with it, by a worsening of SU(4) as an approximate hadronic symmetry. If, on the other hand, SU(4) breaking is, by the GIM mechanism[17], tied to the effective strength of the coupling of strangeness changing neutral weak currents induced by radiative corrections, the p' mass cannot be arbitrarily large. Nonetheless, since what sets the scale here is a gauge boson mass, there may be yet some room in this direction. As to the possible enhancement of the diagram giving λ its mass, this would have to come from the gauge boson mass ratio ($M^{(1)}_{p'\lambda}/M^{(2)}_{p'\lambda}$) upon which the λ-mass depends only logarithmically.

Alternatively, the small values of m_n/m_λ and m_p/m_λ may be interpreted to imply that the p and n quarks acquire their masses in order α as compared to the λ quarks. With G_W=SU(4)xSU(4), this would be the case if $W^{(L)}_{p'n}$ and $W^{(R)}_{p'n}$ were not to mix at all (similarly, $W^{(L)}_{p'p}$ and $W^{(R)}_{p'p}$) and the pattern of quark mass generation were, consequently, $p' \xrightarrow{(\alpha)} \lambda \xrightarrow{(\alpha)} p, n$.

Clearly, this would also be the case in a model without charm and G_w=SU(3)xSU(3).

All of this physics is, in our scheme, controlled by the gauge boson sector, in which the symmetry breaking "first" manifests itself with the generation of gauge bosons masses and mixings. A mixing between W_{pn} and $W_{p\lambda}$ would also be responsible for the apparent Cabibbo rotation, if the observed weak interaction currents do indeed possess Cabibbo's universality[18]. These phenomena, because of their intrinsecally non-perturbative character, are beyond the domain of calculability of our approach.

Let us finally observe that, given a complete model of leptons and hadrons our approach could predict relationships between hadronic symmetry breaking parameters such as ε_3 and ε_8 and ratios of lepton masses. In this respect, we would be faced with serious difficulties if the neutrinos are indeed massless.

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14. In the sense that the renormalization of radiative corrections forces the introduction of Lagrangian counterterms of the "locking" type with a destabilizing effect on the desired configuration of VEV.

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FIGURE CAPTIONS

1. Tadpole expansion for fermion self-energies.

2. Diagram for the mass of uncharmed quarks.

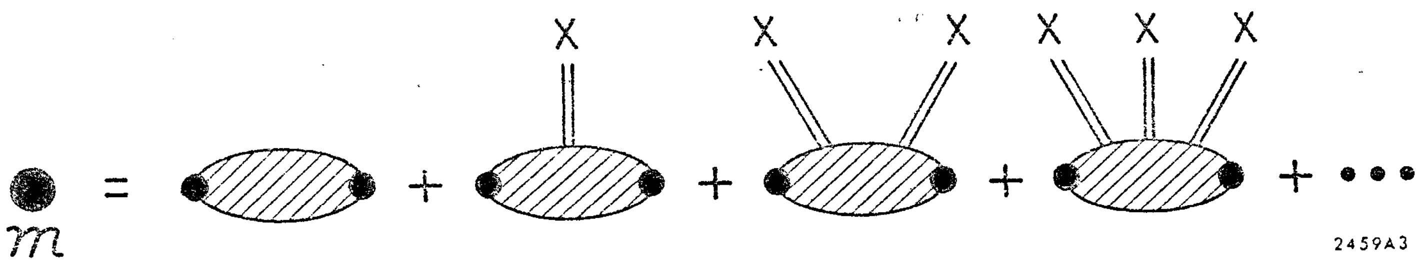

Fig. 1

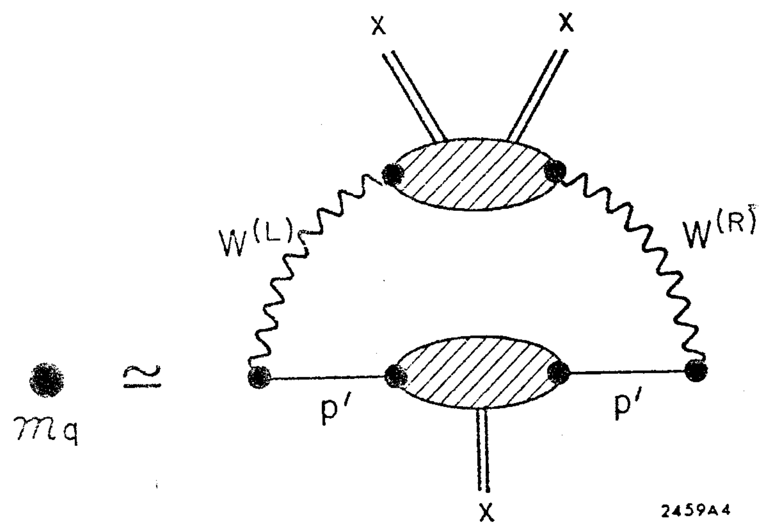

Fig. 2