Thermonuclear (n,γ) reaction rate equations for stellar nucleosynthesis

Vinay Singh,* Debasis Bhowmick, and D. N. Basu
Variable Energy Cyclotron Centre, 1/AF Bidhan Nagar, Kolkata 700064, INDIA.

Introduction

There is a high demand for nuclear data in multidisciplinary subject like nuclear astrophysics. The two areas of nuclear physics which are most clearly related to one another are stellar evolution and nucleosynthesis. The necessity for nuclear data for astrophysical applications puts experimental methods as well as reliability and predicative ability of current nuclear models to the test. Despite recent, considerable advances, there are still significant issues and mysteries. Only a few characteristics of nuclear astrophysics are covered in the current work which include ^{52}Fe(n,γ)^{53}Fe, ^{53}Fe(n,γ)^{54}Fe, ^{54}Fe(n,γ)^{55}Fe, ^{55}Fe(n,γ)^{56}Fe, ^{56}Fe(n,γ)^{57}Fe, ^{57}Fe(n,γ)^{58}Fe and ^{58}Fe(n,γ)^{59}Fe reactions which are important in stellar nucleosynthesis. The reaction rates are calculated using nuclear statistical model. These rates are subsequently fitted to polynomials of temperature T_9 in order to facilitate calculations for stellar nucleosynthesis.

Theoretical formalism

The astrophysical nuclear reaction rate can be calculated by folding the Maxwell-Boltzmann energy distribution for energies E at the given temperature T with the cross section $\sigma^{\mu}_{\alpha\alpha'}(E)$. The relative populations of various energy states of nuclei with excitation energies E^{μ}_{x} and spins I^{μ} in thermodynamic equilibrium follows the Maxwell-Boltzmann distribution. In order to distinguish between different excited states the superscript μ is used along with the incident α channel in the formulas that follow. Taking due account of various target nuclei excited state contributions, the effective nuclear reaction rate per mole in the entrance channel $\alpha \rightarrow \alpha'$ can be expressed as

$$N_A\langle\sigma v\rangle^{*}_{\alpha\alpha'} = \left(\frac{8}{\pi m}\right)^{1/2}\frac{N_A}{(kT)^{3/2}G(T)} \qquad (1)$$

$$\times\int_0^{\infty}\sum_{\mu}\frac{(2I^{\mu}+1)}{(2I^{0}+1)}\sigma^{\mu}_{\alpha\alpha'}(E)E\exp\left(-\frac{E+E^{\mu}_{x}}{kT}\right)dE$$

where N_A is the Avogadro number, k and m are the Boltzmann constant and the reduced mass in the α channel, respectively, and

$$G(T) = \sum_{\mu}(2I^{\mu}+1)/(2I^{0}+1)\exp(-E^{\mu}_{x}/kT)$$

is temperature dependent normalized partition function.

*Electronic address: vsingh@vecc.gov.in

Calculations and Results

The radiative neutron capture (n,γ) reaction cross sections have been calculated using Hauser-Feshbach statistical model calculations [1]. Latest level density based on temperature-dependent Hartree-Fock-Bogolyubov calculations utilizing the Gogny force and the Brink-Axel Lorentzian for the gamma-ray strength function [1] have been chosen for performing the present computations. These excitation functions highlight the variations of (n,γ) cross sections with energy and show a different energy dependence than $1/\mathrm{E}_{\mathrm{n}}^{1/2}$ behavior valid at very low energies in the thermal domain. The energy variation of radiative capture cross section is more expeditious in the astrophysical realm of energy range of 10 keV to 1.2 MeV than in the thermal domain. Thermonuclear reaction rates per mole ($N_A < \sigma v >$) have been fitted to the polynomials of temperature T_9 (10^9 K) of the form

$$N_A < \sigma v >= \sum_{n=0}^{7} a_n T_9^{n/3} + a_8 \log_{10} T_9 \qquad (2)$$

and have been presented in Figs.1-2 by continuous lines. The coefficients of fitting of thermonuclear reaction rate to the polynomial in temperature T_9 have been listed in Table-I. These calculations have been compared with the experimental data. The different structural properties of the Fe isotopes could be the possible reason for the deviation of the theoretical results from the experimental data. The JINA REACLIB reaction rates represented in the form of $\exp\left[c_0 + \sum_{n=1}^{5} c_n T_9^{(2n-5)/3} + c_6 \log_{10} T_9\right]$ [2, 3] using the values coefficients c_n listed in JINA REACLIB database have been presented by dotted lines in Figs.1-2. The solid circles in Figs.1-2 represent the experimental data. The Hauser-Feshbach statistical model estimates of thermonuclear (n,γ) reaction rates, however, could not be fitted with JINA REACLIB form of polynomial in temperature T_9. Moreover, from Figs.1-2 it is evident that JINA REACLIB form of polynomial fails poorly.

Summary and Conclusion

A few important thermonuclear reactions which may have significant impact on the chemical abundances in the region near ^{56}Fe have been explored. The ^{56}Fe nucleus formed at the endpoint of thermonuclear burning is the most energetically favorable one at the low densities. In order to facilitate the stellar nucleosynthesis computations, these rates are then fitted to polynomials of temperature T_9. The calculations of the (n,γ) reaction rates have been compared with the JINA REACLIB

TABLE I: The coefficients of fitting of thermonuclear reaction rate to the polynomial in temperature T_9.

Reaction	a_0	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
^{52}Fe(n,γ)^{53}Fe	-0.113774×10^{8}	-0.898350×10^{7}	0.109944×10^{9}	-0.212525×10^{9}	0.2117889×10^{9}	-0.117009×10^{9}	0.338290×10^{8}	-0.399156×10^{7}	-0.823436×10^{7}
^{53}Fe(n,γ)^{54}Fe	0.153882×10^{8}	-0.443333×10^{8}	0.751030×10^{8}	-0.722473×10^{8}	0.315010×10^{8}	0.129019×10^{7}	-0.552535×10^{7}	0.123653×10^{7}	0.185681×10^{7}
^{54}Fe(n,γ)^{55}Fe	-0.309640×10^{9}	0.101116×10^{10}	-0.200123×10^{10}	0.284575×10^{10}	-0.256504×10^{10}	0.137874×10^{10}	-0.399704×10^{9}	0.477580×10^{8}	-0.873182×10^{8}
^{55}Fe(n,γ)^{56}Fe	-0.843064×10^{9}	0.125863×10^{10}	0.798242×10^{9}	-0.357322×10^{10}	0.423271×10^{10}	-0.244802×10^{10}	0.697564×10^{9}	-0.779767×10^{8}	-0.359015×10^{9}
^{56}Fe(n,γ)^{57}Fe	-0.134373×10^{9}	0.426710×10^{9}	-0.794204×10^{9}	0.105426×10^{10}	-0.890778×10^{9}	0.452916×10^{9}	-0.125710×10^{9}	0.145606×10^{8}	-0.380546×10^{8}
^{57}Fe(n,γ)^{58}Fe	-0.177928×10^{9}	0.518892×10^{9}	-0.834466×10^{9}	0.936960×10^{9}	-0.667475×10^{9}	0.287460×10^{9}	-0.681916×10^{8}	0.683841×10^{7}	-0.527879×10^{8}
^{58}Fe(n,γ)^{59}Fe	-0.285497×10^{9}	0.843431×10^{9}	-0.141407×10^{10}	0.171328×10^{10}	-0.133758×10^{10}	0.637351×10^{9}	-0.168520×10^{9}	0.188948×10^{8}	-0.845940×10^{8}

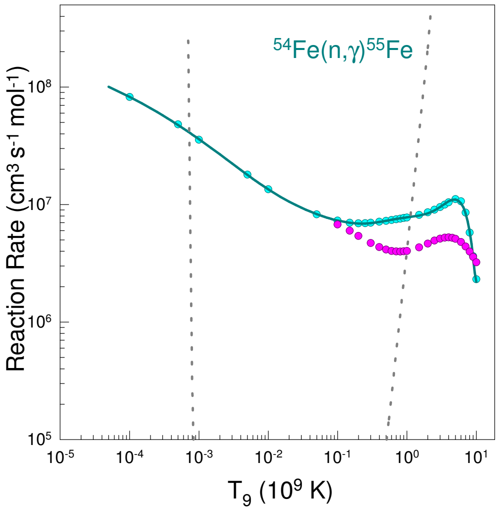

FIG. 1: The Hauser-Feshbach estimates of thermonuclear reaction rate per mole ($N_A < \sigma v >$) for ^{54}Fe(n,γ)^{55}Fe versus temperature T_9. The hollow circles (cyan) on the continuous line represent these estimates while the continuous line itself represents the polynomial fit. The solid circles (pink) represent the experimental data while the dotted line represents the JINA REACLIB reaction rate calculations [2, 3].

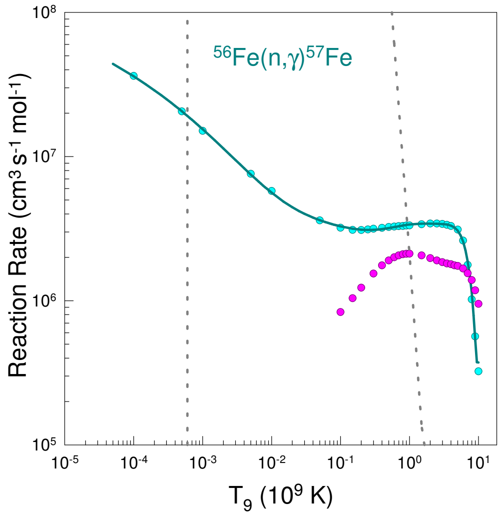

FIG. 2: Same as Fig.-1 but for ^{56}Fe(n,γ)^{57}Fe.

reaction rates. As in several cases there exist large deviations of JINA REACLIB fits to experimental data, estimates of the present calculations can be considered as good. Moreover, different structural properties of the Fe isotopes and uncertainties due to factors, such as level densities and mass models may have substantial effects on the rates while the low-energy upbend in the γ-strength function has a little (though non-negligible) effect on the rates. To conclude, the present investigation has important bearings on the relative abundance of the elements involved in the stellar nucleosynthesis. In order to study the consequence of the present results on the elemental abundances and isotopic ratios, comprehensive and detailed full reaction network calculations at evolving neutron densities needs to be performed in future. The lack of sufficient experimental data is one of the biggest impediments in constraining the (n,γ) reaction rates near the mass region fifty-six which can probably be acquired using new experimental techniques such as the surrogate method or the beta-Oslo method. This would also facilitate exclusion or establishing certain model inputs in future theoretical calculations.

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References

[1] Arjan Koning, Stephane Hilaire and Stephane Goriely, **TALYS-1.8** (2015).

[2] I. Dillman, M. Heil, R. Käppeler, R. Plag, T. Rauscher, F.-K. Thielemann, AIP Conf. Proc. **819**, 123 (2007).

[3] The third update of the 'Karlsruhe Astrophysical Database of Nucleosynthesis in Stars', I. Dillmann, R. Plag, F. Käppeler, T. Rauscher, Proceeding of the workshop "EFNUDAT Fast Neutrons - scientific workshop on neutron measurements, theory & applications" held on April 28-30, 2009 at Geel, Belgium.