Aspects of Quantum Black Holes

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Abstract. Black holes play a key role for any quantum theory of gravity. The main reasons are black hole radiation and evaporation as well as black hole entropy. I review these issues and address the problem of information loss. I then discuss the status of black holes in full quantum gravity. Particular attention is focused on quantum geometrodynamics, the direct quantization of general relativity. This allows scenarios in which the black-hole singularity is avoided, in particular the collapse of a wave packet towards a black-hole like state and its re-expansion as a white hole.

1. Hawking radiation and black-hole entropy

Black holes are mysterious objects. In Einstein's theory of general relativity (GR), they are described as regions of spacetime out of which nothing, not even light, can escape. There is strong evidence that such objects exist in Nature. One example is the Galactic Black Hole (Sgr A*) with a mass $M \approx 4.3 \times 10^6 M_\odot$ [1]. Another example is the supermassive black hole in the centre of the galaxy M87 (called M87*) observed by the Event Horizon Telescope (EHT) [2]; it has a mass of $M \approx 6.5 \times 10^9 M_\odot$. Stellar-mass black holes can be observed by X-ray emission (as in the case of V404 Cygni with $M \approx 9M_\odot$) or by the emission of gravitational waves (as in the case of the black-hole mergers observed by the Ligo Collaboration [3]). Observations of black holes give insight into the astrophysics of these objects and their surroundings, but can also provide the means to constrain fundamental physics (see e.g. [4]).

It has been known since the early 1970s that black holes obey laws that possess a striking analogy with the laws of thermodynamics. These analogies are presented in Table 1. In the presence of a negative cosmological constant Λ, one can derive a First Law of black-hole mechanics with an additional pressure term; the pressure derives from the cosmological constant as $p = -\Lambda/8\pi G$. This leads to an additional term $V\mathrm{d}p$ in the First Law and to the interpretation of M as enthalpy instead of energy [5]. We will not, however, consider this here.

Because Mc^2 plays the role of energy, we can compare the term $\mathrm{d}E = T\mathrm{d}S$ with $\mathrm{d}M = \frac{\kappa}{8\pi G}\mathrm{d}A$, where κ is the surface gravity. Since according to the Table κ stands in analogy to the temperature T, we can tentatively identify

$$T = \frac{\kappa c^2}{G\zeta}, \quad S = \frac{\zeta A}{8\pi}, \tag{1}$$

where ζ is so far undetermined. For dimensional reasons, k_B/ζ must have the dimension of a length squared. A universal length is not available in the classical theory, but if $\hbar$ is taken into account, one can use the *Planck length* [6]

$$l_\mathrm{P} = \sqrt{\frac{\hbar G}{c^3}} \approx 1.62 \times 10^{-35}\ \mathrm{m}. \tag{2}$$

This is a strong indication that quantum gravity is needed in order to give this analogy a precise meaning.

Table 1: Analogies between the laws of thermodynamics and the laws of black-hole mechanics

Law	Thermodynamics	Stationary black holes
Zeroth	T constant on a body in thermal equilibrium	κ constant on the horizon of a black hole
First	$\mathrm{d}E = T\mathrm{d}S - p\mathrm{d}V + \mu\mathrm{d}N$	$\mathrm{d}Mc^2 = \frac{\kappa c^2}{8\pi G}\mathrm{d}A + \Omega_\mathrm{H}\mathrm{d}J + \Phi\mathrm{d}q$
Second	$\mathrm{d}S \geq 0$	$\mathrm{d}A \geq 0$
Third	$T = 0$ cannot be reached	$\kappa = 0$ cannot be reached

Connected with the Planck length are the Planck time

$$t_\mathrm{P} = \frac{l_\mathrm{P}}{c} = \sqrt{\frac{\hbar G}{c^5}} \approx 5.39 \times 10^{-44}\ \mathrm{s}, \tag{3}$$

and the Planck mass

$$m_\mathrm{P} = \frac{\hbar}{l_\mathrm{P} c} = \sqrt{\frac{\hbar c}{G}} \approx 2.18 \times 10^{-8}\ \mathrm{kg} \approx 1.22 \times 10^{19}\ \mathrm{GeV}/c^2. \tag{4}$$

Classical black holes cannot radiate, so for them the analogy has at best mathematical value. But if quantum theory is taken into account, black holes radiate with a temperature proportional to $\hbar$, the famous Hawking temperature. It is given in the general case of stationary black holes by

$$T_\mathrm{BH} = \frac{\hbar\kappa}{2\pi k_\mathrm{B} c}; \tag{5}$$

in the particular case of a Schwarzschild black hole this can be written as

$$T_\mathrm{BH} = \frac{\hbar c^3}{8\pi G k_\mathrm{B} M} \approx 6.2 \times 10^{-8} \frac{M_\odot}{M}\ \mathrm{K}. \tag{6}$$

Because a black hole becomes hotter through the emission of Hawking radiation, it has a finite lifetime. Under the assumption that we deal with a Schwarzschild black hole and that only the emission of gravitons and photons is relevant, one obtains [8]

$$\tau_\mathrm{BH} \approx 8895 \left(\frac{M_0}{m_\mathrm{P}}\right)^3 t_\mathrm{P} \approx 1.159 \times 10^{67} \left(\frac{M_0}{M_\odot}\right)^3 \mathrm{yr}, \tag{7}$$

where M_0 denotes the initial mass of the black hole. For the stellar-mass black hole V404 Cygni this gives already a lifetime of 8.4×10^{69} years, which is about 6×10^{59} the age of the Universe. For M87*, this gives the even much higher lifetime of 3.2×10^{96} years.

From the expression of the Hawking temperature and the First Law of black-hole mechanics, one can derive the expression for the entropy of the black hole, the Bekenstein–Hawking entropy,

$$S_\mathrm{BH} = k_\mathrm{B}\frac{Ac^3}{4\hbar G} = k_\mathrm{B}\frac{A}{(2l_\mathrm{P})^2} \overset{\text{Schwarzschild}}{\approx} 1.07 \times 10^{77} k_\mathrm{B} \left(\frac{M}{M_\odot}\right)^2. \tag{8}$$

Inserting numerical values, one finds in the case of the black hole V404 Cygni a value of $8.7 \times 10^{78} k_{\rm B}$ for the entropy, more than 20 orders of magnitude bigger than the entropy of the star from which V404 Cygni has formed by gravitational collapse. For M87*, the entropy is about $4.5 \times 10^{96} k_{\rm B}$, which is more than seven orders of magnitude bigger than the whole non-gravitational entropy of the observable Universe.

The expressions for Hawking temperature and black-hole entropy are found at the level where gravity is treated classically and non-gravitational fields quantum. This approximation breaks down when the black hole has shrunk to Planck dimensions. One then needs a full theory of quantum gravity to describe the final black hole evaporation phase [6]. Quantum gravity should also shed led on the fate of the classical black-hole singularity. As for the entropy, the full theory should be able to provide a microscopic picture in the sense of statistical mechanics. Some of these issues are treated below. Hawking radiation and the final evaporation phase can only be observed, it seems, if primordial black holes can become astrophysically relevant. So far, however, there exist only upper limits on their existence.

A microscopic derivation of black-hole entropy has to start, for example, from the von Neumann formula

$$S_{\rm BH} = -k_{\rm B} {\rm tr}\left(\rho \ln \rho\right) , \tag{9}$$

using an appropriate density matrix from quantum gravity. Simple models can mimic this by assuming a certain counting procedure for discrete states. One example is the model provided by Bekenstein and Mukhanov in 1995 [9]. They assume a quantization condition for the allowed values A_N, $N \geq 1$, of the horizon area,

$$A_N = \alpha l_{\rm P}^2 N, \tag{10}$$

with some undetermined constant α. The energy level N is assumed to be degenerate with multiplicity $g(N)$, so one would expect

$$S = \frac{A}{4 l_{\rm P}^2} + \text{ constant } = \ln g(N). \tag{11}$$

With $g(1) = 1$ one gets

$$g(N) = {\rm e}^{\alpha(N-1)/4}. \tag{12}$$

Since the degeneracy must be an integer, one has the options

$$\alpha = 4 \ln k \ , \quad k = 2, 3, \dots \ . \tag{13}$$

For information-theoretic reasons ('it from bit'), $k = 2$ seems to be a preferred value. This case can be modelled by a chain of spin-1/2 particles [10]. Let us consider N such particles out of which n point up and $N - n$ point down:

$$\downarrow \uparrow \downarrow \downarrow \uparrow \downarrow \uparrow \downarrow \uparrow \uparrow \uparrow \downarrow \downarrow \uparrow$$

For the entropy of such a configuration one can take the logarithm of the number of states (setting Boltzmann's constant equal to one):

$$S = \ln \begin{pmatrix} N \\ N-n \end{pmatrix} = \ln \begin{pmatrix} N \\ n \end{pmatrix} . \tag{14}$$

Using Stirling's formula, one gets for the 'equilibrium case' $n = N/2$, neglecting terms of order $1/N$, the expression

$$S = N \ln 2 - \frac{1}{2} \ln N + \frac{1}{2} \ln \frac{2}{\pi}. \tag{15}$$

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With $S_0 := N \ln 2$, one can write this as

$$S \approx S_0 - \frac{1}{2} \ln S_0. \tag{16}$$

In the Bekenstein–Mukhanov model, we have

$$A_N = (4 \ln k) l_{\rm P}^2 N. \tag{17}$$

For $k = 2$ and using the spin model from above, one gets

$$S = \frac{A_N}{4l_{\rm P}^2} - \frac{1}{2} \ln \frac{A_N}{4l_{\rm P}^2} + \frac{1}{2} \ln \frac{2}{\pi} + \frac{1}{2} \ln(\ln 2). \tag{18}$$

(Loop quantum gravity predicts the same logarithmic correction term [6].)

Except for very small black holes, this yields almost the same result as the exact expression from (14) [10]:

$$S = \ln \frac{\left(\frac{A_N}{4l_{\rm P}^2 \ln 2}\right)!}{\left[\left(\frac{A_N}{8l_{\rm P}^2 \ln 2}\right)!\right]^2}, \tag{19}$$

where $A_N \geq 4(\ln 2) l_{\rm P}^2$. This spin-model is certainly oversimplified, but it demonstrates how *in principle* the Bekenstein-Hawking entropy (8) can be derived as an approximation from counting microscopic quantum states.

As one recognizes from (18), the first correction to (8) is a term that contains the logarithm of the area. It is clear that such a term arises from the next order of the Stirling formula. Numerically, it is very small and, except for small black holes, completely negligible compared with (8) [10]. In the case of V404 Cygni, for example, it gives the correction -90.9 to 8.7×10^{78}, and for M87* the correction -111.3 to 4.5×10^{96}.

2. Information-loss problem

The expressions for Hawking temperature and Bekenstein–Hawking entropy are calculated at the level of the semiclassical approximation in which gravity is treated classically. This approximation is expected to break down if the black hole approaches the Planck mass $m_{\rm P}$. In this limit, (8) is replaced by a formula such as (19). But what happens to the radiation?

If the black hole left only thermal radiation behind, a pure state for a closed system would evolve into a mixed system. This would stand in strong contradiction to standard quantum theory, where the von Neumann entropy (9) is conserved for a closed system. This conservation of entropy reflects the unitary evolution of quantum states for a closed system. If the black hole left only thermal radiation behind, this would lead to what one could call a *unitarity problem*. In the literature, this is better known as the *information-loss problem* because non-conservation of probability can be associated with a form of information loss.

The information-loss problem was discussed since the advent of Hawking radiation in the 1970s. That it is still considered as a major open issue is reflected by the continuous publication of papers on this subject and by contribution to conferences (see e.g. [11]). Hawking originally speculated that information is indeed lost in black-hole evaporation. He proposed a general evolution law of the full density matrix in the form

$$\rho \to \$\rho \neq S\rho S^\dagger, \tag{20}$$

for which he introduced a new operator $ ('dollar operator') which, in contrast to the standard scattering operator S, does not necessarily preserve unitarity. Motivated by developments in string-inspired approaches, he later changed his mind. The prevalent opinion today is, in fact, that the full evolution is unitary, but that this cannot be seen in the semiclassical approximation because of correlations (entanglement features) neglected there. (A third option, that the black hole leaves a remnant carrying all the information, comes with various problems and is not considered as a serious option by most experts.)

It is evident that the final answer to this question can be found only from a theory of quantum gravity.[1] Nevertheless, some important aspects can be discussed already at the semiclassical level. In the following, we briefly summarize some of them.

The first point is that in the calculations by Hawking and others, no exact mixed (canonical) state is used at any stage in the formalism. It is only expectation values of certain operators (e.g. of the particle number operator) in pure states, which exhibit a thermal spectrum. By using only such expectation values, the coherent superposition used by Hawking and others is indistinguishable from a local thermal mixture. They become distinguishable by using, for example, four-point functions. This can clearly be seen in calculations using the functional Schrödinger picture [12]. In the collapse of a star to a black hole, the ground state (vacuum) of a quantum field on this background is transformed into a two-modes squeezed state (see [13] and the references therein). The reduced state of each mode in a two-mode squeezed state is a thermal state (canonical ensemble). In general, the temperature associated with each mode depends on the mode number k. It is the particular feature of a black hole, due to the presence of a horizon, that the temperature is independent of k (universality feature). Two-mode squeezed states play also a role in the emergence of the CMB power spectrum from inflation, but there this universality is absent [13].

It is well known from quantum optics that squeezed states are very sensitive to decoherence. This applies, of course, also to situations where gravity is relevant. The quantum-to-classical transition for the primordial fluctuations during inflation in the early Universe can only be understood by decoherence [14]. Useful technical tools are the reduced density matrix and the Wigner function of the system. Decoherence is achieved by the arising entanglement with the ubiquitous degrees of freedom of the environment (where "environment" generally stands for irrelevant degrees of freedom). In the case of black holes, the thermal nature of Hawking radiation can be justified by this process of decoherence [13, 15, 16].

If black-hole evaporation is unitary, as we assume here, the information about the initial state from which the black hole originates can never disappear. For simplicity, one can envisage the initial state to be a pure quantum state. It is then clear that the entropy of Hawking radiation (more precisely, its von Neumann entropy (9)) cannot increase all the time until the black hole has evaporated (as it would do if it were exactly thermal). The global entropy for a pure state is zero and remains zero at all time. Non-zero entropies emerge for the subsystems of a globally entangled state (here, between black hole and Hawking radiation). Don Page has estimated the time when the radiation entropy has reached its peak value and then starts to decrease. This *Page time* is given by [8]

$$\tau_{\rm Page} \approx 0.53810\, \tau_{\rm BH} \approx 4786 \left(\frac{M_0}{m_{\rm P}}\right)^3 t_{\rm P} \approx 6.236 \times 10^{66} \left(\frac{M_0}{M_\odot}\right)^3 {\rm yr}. \tag{21}$$

After the time $\tau_{\rm BH}$ has elapsed, both the black-hole entropy and the radiation entropy have decreased to zero – the whole information of the initial state is present in the remaining radiation (which is, of course, non-thermal).

The microscopic picture of this process is subtle. It was claimed that unitary evolution can lead to the appearance of a "firewall" near the black-hole horizon [17]. Arguments for and against such a scenario can be found in the literature; see, for example, [18] for a review. Arguments in favour of a firewall come from a theorem in quantum information theory [19]: it is not possible to prepare three quibits in a way that any two of them are maximally entangled. In the black hole case this means that if there is a strong correlation between two Hawking quanta B and C at late times, there cannot be both a strong correlation between their earlier versions outside the horizon and a strong correlation between, say, the

[1] As Einstein emphasized in his famous remark to Heisenberg: "Only the theory decides about what can be observed."

earlier B and a quantum A which is inside the horizon. The authors of [17] claim that this leads to the breaking up of the A–B correlation resulting in a highly excited quantum state near the horizon (the "firewall"). Although there is evidence that such a firewall is harmless or even absent (see e.g. [20]), the investigation of [17] indicates that one should be careful when discussion a microscopic picture. Most likely, the classical notion of a horizon breaks down in quantum gravity (see next section). Whether features of a "quantum horizon" can be directly observed, is currently under investigation [22].

3. Quantum gravity and singularity avoidance

The final solution to the above problems can only come from a theory of quantum gravity. Although such a theory is not yet available in final form, there exist various approaches in which some of these issues can be addressed [6]. The main approaches are the direct quantization of general relativity (which actually is a whole class of approaches) and string theory. The latter is a candidate for a quantum theory of all interactions from which quantum gravity emerges as an effective theory.

Some progress has been achieved in deriving the formula (8) for the Bekenstein–Hawking entropy from the counting of microscopic states.[2] In loop quantum gravity, the microscopic degrees of freedom are the spin networks. Under certain assumptions, the expression for the entropy follows: the distinguishability of the microscopic states (a somewhat unnatural assumption from the perspective of ordinary quantum theory) and a specific, not very intuitive choice for the Barbero–Immirzi parameter, which is a free parameter in loop quantum gravity. In string theory, the microscopic degrees of freedom are the "D-branes". There, (8) can be derived without any ambiguity, but only for very special (extremal or near-extremal) stringy black holes – astrophysical black holes such as V404 Cygni do not fall into this class. In quantum geometrodynamics, one can derive $S \propto A$ in particular models, but no derivation of the exact expression (8) exists so far.

Quantum geometrodynamics is one of the most conservative approaches for the quantization of gravity. Its central equations are the Wheeler–DeWitt equation and the quantum diffeomorphism constraints [23, 6]. It is especially suitable for discussing conceptual issues, since it uses the language of wave functions familiar from quantum mechanics.

In quantum geometrodynamics, like in most other approaches to quantum gravity, spacetime is absent at the most fundamental level. The configuration space is the space of all *three*-geometries, so time has disappeared. Since the event horizon of a black hole is a *spacetime concept*, it cannot play a role in quantum gravity, only in its semiclassical approximation. It is for this reason that pictures of black-hole evaporation can be misleading; some of these pictures suggest the presence of a spacetime concept, the horizon, which in fact is absent.

A full and consistent description of black hole evaporation does not yet exist in quantum geometrodynamics (as well as other approaches). But one can use simplified models to investigate at least what can in principle happen. In [21], the combined system black hole – Hawking radiation – other degrees of freedom was mimicked by a system of oscillators with linear interaction terms. The corresponding Wheeler–DeWitt equations for this combined system in a semiclassical universe was solved exactly. It was found that an initial state evolves into a final state where the difference between the black hole and the Hawking radiation is blurred and one non-classical state remains.

Einstein's theory predicts that a singularity forms inside a black hole. What is the fate of this singularity in the underlying theory of quantum gravity? If black-hole evaporation is unitary, no such singularity should occur: if a collapsing object is described by a wave packet, this packet may disperse, but must not vanish in a singularity. This is, in fact, what concrete models exhibit.

The collapse of a spherically-symmetric null dust shell was discussed exactly in quantum geometrodynamics [24]. Its time evolution in the position (r-) representation was found to read:

$$\Psi_{\kappa\lambda}(t,r) = \frac{1}{\sqrt{2\pi}} \frac{\kappa!(2\lambda)^{\kappa+1/2}}{\sqrt{(2\kappa)!}} \left[\frac{\mathrm{i}}{(\lambda + \mathrm{i}t + \mathrm{i}r)^{\kappa+1}} - \frac{\mathrm{i}}{(\lambda + \mathrm{i}t - \mathrm{i}r)^{\kappa+1}} \right], \tag{22}$$

[2] For references, see e.g. [6].

where λ and κ are parameters describing the shape of the initial wave packet. As can be recognized from this solution, the wave function obeys the important condition

$$\lim_{r\to 0} \Psi_{\kappa\lambda}(t,r) = 0. \tag{23}$$

This means that the probability of finding the shell at vanishing radius is zero! In this sense, the singularity is avoided in the quantum theory. This singularity avoidance is a consequence of the unitary evolution that is imposed on the wave function. What happens is that the quantum shell describing the collapsing null shell bounces and re-expands, and no horizon forms. In a sense, the black hole becomes a while hole, although these names are here only metaphors: one has, in fact, a "gray" object without horizon and singularity. A similar behaviour also occurs in models of loop quantum gravity; see [25] for a review.

The case of a full dust cloud can no longer be treated exactly, but one can use models such as the Lemaître–Tolman–Bondi (LTB) model to describe the temporal evolution [26]. Also here, wave packets describing the various shells of the dust cloud do not collapse to a black hole but bounce and re-expand. In all of these models, the effect of Hawking radiation is not taken into account.

An important question in these scenarios is how long it takes from collapse to re-expansion. Since the re-expansion of collapsed stars is not observed in our Universe, the corresponding timescales should turn out to be at least as long as the age of the Universe; otherwise, these models cannot be considered as realistic. Various results can be found in the literature [25]. In the LTB model discussed in [26], the timescale turns out proportional to M_0^3, so the time is of the order of the evaporation time or the Page time discussed above, which by far exceed the age of the Universe. A similar timescale $\propto M_0^3$ arises for the spreading of a wave packet describing the evolution of a Reissner–Nordström black hole in quantum geometrodynamics [27]. It seems that this timescale plays an important role in the quantum evolution of black holes, although the connection between the various expressions is not yet clear.

Singularity avoidance is also extensively discussed in quantum cosmology, mostly in connection with singularities in classical models with dark energy; see, for example, the review [28]. There the "DeWitt criterion" of singularity avoidance, imposing the vanishing of the wave function in the region of the classical singularity [23], can be applied in many situations. Since time in quantum cosmology is absent, the DeWitt criterion is not the consequence of a unitary evolution.

Black holes are usually discussed in the framework of asymptotically flat spacetimes. A more realistic situation is the case of a black hole in quantum cosmology. It seems that our Universe has very peculiar boundary conditions: its initial state is very smooth (absence of condensed objects, in particular black holes), which from the gravitational point of view corresponds to a state of low entropy. Long ago, Roger Penrose has estimated the "probability" for our Universe using entropic arguments (see [7] for references and an update). Applying (8), the maximal entropy is achieved if all matter in our observable Universe is assembled into one black hole. Comparing this with the actual entropy of everything in the observable Universe (which includes, in particular, the entropy of all supermassive black holes such as M87*), one obtains for this probability the exceedingly tiny number

$$\frac{\exp\left(\frac{S}{k_{\rm B}}\right)}{\exp\left(\frac{S_{\rm max}}{k_{\rm B}}\right)} \sim \frac{\exp\left(3.1\times 10^{104}\right)}{\exp\left(1.8\times 10^{121}\right)} \approx \exp\left(-1.8\times 10^{121}\right). \tag{24}$$

This number is so small that one would expect a physical origin for its origin; anthropic arguments would fail to give an explanation. The Wheeler–DeWitt equation could in principle provide the means for such an explanation, because it allows the formulation of a simple boundary condition that corresponds to low gravitational (entanglement) entropy [7].

Cosmic boundary conditions also leave an imprint on the boundary conditions for black holes. This has particular consequences in the case of a classically recollapsing universe [29]. The black holes do not develop any horizon, nor any singularity, independent of the features discussed above for black holes in asymptotically flat spacetime. The combination of the above bounce scenarios with the quantum version of a recollapsing universe is an intriguing project that has not yet been pursued.

To summarize, there are strong indications that black holes are genuine quantum objects. The event horizon is a classical concept; it is expected to be absent in quantum gravity and to emerge only in the semiclassical limit. In this limit, the various macroscopic components of the black-hole wave function (which also include "no-hole states") can become dynamically independent by decoherence [12, 16]. Exact models of quantum gravitational collapse suggest the presence of a bounce and the transition into an expanding phase. No singularity forms. Many of these quantum features could only be studied from observations of primordial black holes. Whether this will ever be possible is not known.

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