SPIRAL-SECTOR FFAG MAGNETS

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In a spiral-sector FFAG machine, the magnetic field in the median plane may be represented approximately by

$$B_z = B_0\left(\frac{r}{r_0}\right)^k\left\{1 + f\cos\left[\left(\frac{1}{w}\ln\left(\frac{r}{r_0}\right) - N\theta\right)\right]\right\},$$

where the "flutter" f is usually of order one. Such a field may be produced by spiral magnets, as was done in MURA's first model.[1] These magnets would have a width of $\sim wr$ and a vertical gap of the same order. The lengths of a magnet would be

$$\sqrt{1 + \left(\frac{1}{wN}\right)^2}(r_{max} - r_{min})$$

where $(r_{max} - r_{min})$ is the radial aperture of the magnets. In order to have this quantity small one requires a large value of k. In this case $1/w$ must also be large to get reasonable vertical focusing. Typical values for a large machine are, $k \sim 95$, $1/w \sim 540$, $N \sim 48$. In this case the length of the spiral magnets, for a radial aperture of ~ 3 meters, would be ~ 30 meters, giving absurd lengths for the flux path.

For this reason, we have gone to magnets as shown in Fig. 1. This is a photograph of a pair of model magnets constructed to study methods of machining and coil winding, and also to investigate the actual field produced. The spiral poles are simply projections on the surface, and the flux is carried in a radial direction to the back leg. There are also "zero" poles, between the poles, in order to increase the flutter for a given gap. The poles are excited by two types of windings: pole windings, around each pole on the magnet block, whose excitation is adjusted to provide a field varying as r^k between the centers of each pole; and coils, differentially wound through each slot in the pole, to provide an r^k variation of the field across the face of each pole.

It would, in principle, be possible to construct the whole magnet without straight sections in this way. Assembly would be difficult and the coils would be hard to handle, but it might be possible. However, it has been found that, if a number of sufficiently short, radial straight sections are used, their effects on the orbits are tolerable. In addition, the problem of applying the rf is facilitated by the use of these short radial straight sections. The present idea is to make 13 straight sections every three magnets, the azimuthal length of each one being about one-third of the azimuthal length of the iron. Each magnet block, then, is about 15 feet in radius by 5 feet in azimuth, and weighs about 60 tons. The coils will be about 5 or 6 feet long and can be handled quite easily.

In designing a machine our usual procedure is to assume a pole shape, which we guess will give approximately the required flutter, and use the digital computer to calculate the field produced by this pole shape. This field is then fed back into the computer and the orbits of particles calculated. The frequencies of small-amplitude betatron oscillations, the effects of large amplitudes, and the effects of errors in the fields are all computed. The magnetostatic calculation generally proceeds in two steps: first a scaling machine without radial straight sections is calculated using the .coordinates,

$$\xi = \frac{1}{w}\ln\frac{r}{r_0} - N\theta \text{ and } \eta = \frac{z}{r}$$

n solving a Poisson-type equation; secondly, when the pole widths have been chosen in such

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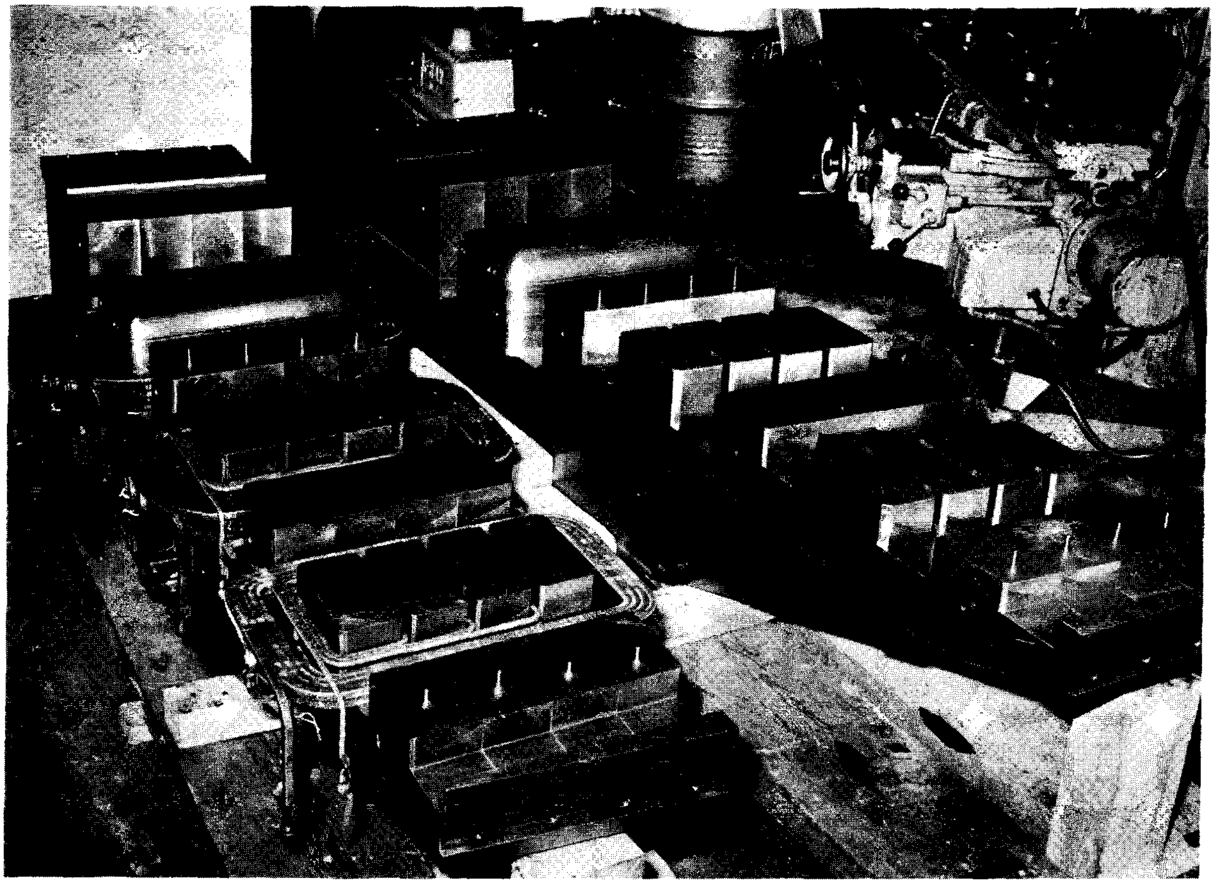

Fig. 1 Spiral sector FFAG model magnet.

a way as to give suitable orbit properties, the radial straight section is added by enhancing the coordinates to (ξ, η, φ) using $\varphi=\theta$ and accounting for the new type of boundary value problem. Orbit studies then are made to determine the effect of the radial straight sections.

At the large-radius (high-field) end of the magnets, it is desirable to reduce the gap to lessen the number of ampere-turns required. Moreover, there has been, hopefully, some damping of the betatron oscillations so that the gap need not be as large. To compute the shape of the pole, required to give the same field as the simple shape used in the low-field region, equipotential surfaces are calculated so that there need be no slots and the gap is gradually reduced as the radius is increased. In Fig. 1 the last pole shape was computed in this manner, using a three-dimensional calculation. The computer was asked to interpolate between the mesh points of potential values to find the contours of constant potential. From these, the computer calculated the path of a 1-inch radius ball-cutter, by calculating the normal to the surface at each point. Values were interpolated where the curvature was large. The magnetic tape was sent to Pratt Whitney Tool Company,[2] where it was converted to punched tape and inserted into a three-dimensional mill which machined the poles. On closer inspection, it can be seen that more contours might be desirable to give a smoother surface in the regions of high curvature. It is planned to measure the field first without doing anything to the surface and later, after filing down the cusps (produced by the 1-inch ball), to see what effect these have on the field.

The remainder of the magnet was constructed of narrow sections (0.21° wide) with the poles machined at the required angle [$\tan^{-1}(1/wN)$] with respect to the center line of each piece.

The four pieces, making up one block of each magnet, then were bolted and tack-welded together.

Another method for construction would be to do all of the final machining of slots, pole surfaces, and contoured poles on a large tape-controlled mill. Probably the large block would be built up of slabs which are rough machined, annealed, and fastened together, before going to the tape-controlled mill.

So far, no great effort has been made to include finite permeability in the actual computations, although the problem of how to include it has been studied. One difficulty is that the relation of B to H depends not only on the type of iron, but also on its previous history. For this reason, it is probably not of any real use in the low-field region of FFAG magnets. One example of this is MURA's experience with the 40-Mev model, where a new power supply, with the current turned on at a different rate, made a large change (1 or 2 percent) in the shape of the field in the region from 50 gauss to a few thousand gauss. With the contoured poles, where relatively high flux densities will be used, it may be valuable to investigate the effects of finite permeability with the computer.

It is expected that the measured field will differ from the computed field for several reasons. Among these are finite permeability, errors in machining and errors in the computer calculations. It is believed that the relaxation method, used by the computer, is only accurate to 0.1 or 0.2 percent. Such errors will affect the orbits and can be measured with our measuring equipment.

In order to make corrections, it is planned to have the computer calculate the magnetic field produced by a unit ampere-turn around individual sections of each pole. The magnetic field, without corrections, would then be measured and the computer asked to find the optimum number of ampere-turns around each section of each pole to minimize the field errors. A procedure, similar to this, was followed in trimming the 40-Mev model and it was found to work. It required several tries to get the best results. After the first computation, the required coils were installed and the fields remeasured. A new calculation gave further corrections. Small errors in measurements cause considerable trouble and the recent improvements in the measuring apparatus should improve this procedure.

REFERENCES

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