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Collective alternating-parity spectrum of the even-even nuclei with effective triaxiality

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Abstract. Model of the effective triaxiality of even-even nuclei with quadrupole and octupole deformation is developed in the present work. The Davidson potential was used to solve the radial part of the Schrödinger equation. The alternating parity energy spectrum and wave functions of the Schrödinger equation has been obtained. The contribution of quantity ϵ_ν–eigenvalues for the angular part of polar coordinate was taken into account. At the same time, in the expressions for the components of the moments of inertia of the nucleus, the variable of the angular part of the polar coordinates remained as a constant. The proposed model is used to describe the excited alternating-parity collective states yrast- and first-non-yrast-bands was proposed of even-even nuclei ^{146,148}Ba, ^{154}Sm, ^{158}Gd, ^{160}Dy, ^{170}Yb, ^{220,224}Ra and ^{224,226,230,232}Th.

Keywords: quadrupole and octupole deformation, effective triaxiality, alternating parity energy spectrum, triaxial rotor.

I. INTRODUCTION

It has been found that most of the atomic nuclei are quadrupole deformed in their ground states and have axially symmetric shapes [1]. Furthermore the octupole (reflection-asymmetric) mode appears in the region of heavy nuclei [1–8]. The nuclei with axial quadrupole and octupole deformations are determined by respective parameters β_2 and β_3, as well as with the projection K=0 of the total angular momentum $\hat{I}$ on the principal symmetry axis in the intrinsic reference frame. Yrast states of this an elongated rotating nucleus are well described by theoretical models [3–8].

On the other hand, a number of studies suggest the presence of non-axial (triaxial) deformations based on both quadrupole and quadrupole and octupole degrees of freedom [9–15]. The rotation of a triaxially deformed nucleus is an important problem. Note that this problem is solved sequentially. In Ref. [16] is considered the approach, where all deformation parameters were considered as constants whose values $\beta_{2\text{eff}}$, γ_{eff}, $\beta_{3\text{eff}}$ and η_{eff} effectively determine a rigid triaxial rotor. The alternating-parity spectra of even-even actinide nuclei: ^{228,230,232}Th, ^{230,232,234,236,238}U and ^{240}Pu has been good described.

A model formalism is developed in Ref. [17, 18] which considers the collective motion in the β_2 and β_3 dynamical variables by allowing non-zero effective values of the quadrupole and octupole axial asymmetry parameters γ=γ_{eff} and η=η_{eff}, respectively, with $K\neq$0. Obtained a good description the alternating-parity spectra of even-even nuclei: ^{154,160}Gd, ^{154}Sm, ^{156}Dy, ^{228,230,232}Th, ^{230,232,236,238}U and ^{240}Pu. But the energy contribution of the angular variable of polar coordinates is not taken into account in this work.

II. MODEL FORMALISM

General theory of quadrupole and octupole excitation of even-even nuclei is described by the Hamiltonian operator, which contains seven dynamical variables $\beta_2(\beta_2\geq 0)$, $\gamma(0\leq\gamma\leq\pi)$, $\beta_3(\beta_3\geq 0)$, $\eta(0\leq\eta\leq\pi)$, $\theta_1(0\leq\theta_1\leq 2\pi)$, $\theta_2(0\leq\theta_2\leq\pi)$, $\theta_3(0\leq\theta_3\leq 2\pi)$ [17]:

$$\hat{H}=\hat{T}_{\beta_2}+\hat{T}_{\beta_3}+\hat{T}_{\gamma}+\hat{T}_{\eta}+\hat{T}_{\text{rot}}+V(\beta_2,\beta_3,\gamma,\eta). \tag{1}$$

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Expressions for the kinetic energy operators $\hat{T}_{\beta_2}$ $\hat{T}_{\beta_3}$ and $\hat{T}_{\gamma}$ for β_2-, β_3-, γ-vibrations are given in the work [17]. And kinetic energy operator $\hat{T}_{\eta}$ for η-vibrations have the form [18]:

$$\hat{T}_{\eta} = -\frac{\hbar^2}{2B_3}\frac{1}{\beta_3^2}\left[\frac{\partial^2}{\partial\eta^2} + \frac{24\cos^2 2\eta - 6\cos 2\eta}{5 + 5\cos 2\eta + 8\cos^2 2\eta}\frac{\cos\eta}{\sin\eta}\frac{\partial}{\partial\eta}\right]. \tag{2}$$

Also $\hat{T}_{\text{rot}}$–rotational energy operator [17] and $V(\beta_2, \beta_3, \gamma, \eta)$–potential energy of β_2-, β_3-, γ- and η- vibrations.

General solution of Schrödinger equation with Hamiltonian (1) is a very complicated. Therefore different simplifications are used. Very simple simplification was performed in Ref. [17], where static approximation was used with β_2=$\beta_{2\text{eff}}$, β_3=$\beta_{3\text{eff}}$, γ=γ_{eff} and η=η_{eff}. A good description of the energy levels of excited collective states of heavy even-even nuclei in the actinide region has been obtained. And also in the work [3–7] the case with $\gamma \approx$0 and $\eta \approx$0 for heavy even-even nuclei of axial symmetry was examined.

Here we replace the variables γ and η with their effective values γ_{eff} and η_{eff} [10, 11, 17], and variables β_2 and β_3 are dynamic. Then, Hamiltonian (1) takes the following form [17]:

$$\hat{H} = \hat{T}_{\beta_2} + \hat{T}_{\beta_3} + \hat{T}_{\text{rot}} + V(\beta_2, \beta_3), \tag{3}$$

where

$$\hat{T}_{\beta_2} = -\frac{\hbar^2}{2B_2}\frac{1}{\beta_2^3}\frac{\partial}{\partial\beta_2}\left(\beta_2^3\frac{\partial}{\partial\beta_2}\right), \tag{4}$$

$$\hat{T}_{\beta_3} = -\frac{\hbar^2}{2B_3}\frac{1}{\beta_3^3}\frac{\partial}{\partial\beta_3}\left(\beta_3^3\frac{\partial}{\partial\beta_3}\right). \tag{5}$$

In this case, the rotational energy operator $\hat{T}_{\text{rot}}$ depends on the effective values of γ_{eff} and η_{eff} through projections of moments of inertia [17].

Schrödinger equation with Hamiltonian (3):

$$-\frac{\hbar^2}{2B_2}\left[\frac{3}{\beta_2}\frac{\partial}{\partial\beta_2} + \frac{\partial^2}{\partial\beta_2^2}\right]\Psi_I^{\pm}(\beta_2, \beta_3, \theta) - \frac{\hbar^2}{2B_3}\left[\frac{3}{\beta_3}\frac{\partial}{\partial\beta_3} + \frac{\partial^2}{\partial\beta_3^2}\right]\Psi_I^{\pm}(\beta_2, \beta_3, \theta) +$$

$$+\left[\hat{T}_{\text{rot}} + V(\beta_2, \beta_3)\right]\Psi_I^{\pm}(\beta_2, \beta_3, \theta) = E_I^{\pm}\Psi_I^{\pm}(\beta_2, \beta_3, \theta). \tag{6}$$

Suppose

$$\Psi_I^{\pm}(\beta_\lambda, \theta) = (\beta_2\beta_3)^{-3/2}\Phi_{I\tau}^{\pm}(\beta_2, \beta_3, \theta). \tag{7}$$

Then we obtain

$$\left[-\frac{\hbar^2}{2B_2}\frac{\partial^2}{\partial\beta_2} - \frac{\hbar^2}{2B_3}\frac{\partial^2}{\partial\beta_3} + \hat{T}_{\text{rot}} + W(\beta_2, \beta_3) - E_{I\tau}^{\pm}\right]\Phi_{I\tau}^{\pm}(\beta_2, \beta_3, \theta) = 0. \tag{8}$$

here

$$W(\beta_2, \beta_3) = V(\beta_2, \beta_3) - \frac{3\hbar^2}{8B_2\beta_2^2} - \frac{3\hbar^2}{8B_3\beta_3^2}. \tag{9}$$

We go to polar coordinates σ ($0\leq \sigma \leq \infty$) and ε ($-\frac{\pi}{2} \leq \varepsilon \leq \frac{\pi}{2}$) [3]:

$$\beta_2 = \sqrt{\frac{B}{B_2}}\sigma\cos\varepsilon,\ \beta_3 = \sqrt{\frac{B}{B_3}}\sigma\sin\varepsilon,\ B = \frac{B_2 + B_3}{2}, \tag{10}$$

then (8) we rewrite

$$\left\{-\frac{\hbar^2}{2B}\left[\frac{\partial^2}{\partial\sigma^2} + \frac{1}{\sigma\partial\sigma} + \frac{\partial^2}{\sigma^2\partial\varepsilon^2}\right] + \hat{T}_{\text{rot}} + W(\sigma, \varepsilon) - E_I^{\pm}\right\}\Phi_I^{\pm}(\sigma, \varepsilon) = 0. \tag{11}$$

A solution of the equation (11) we will find in the form

$$\Phi_I^{\pm}(\sigma,\varepsilon,\theta) = F_{I\tau}^{\pm}(\sigma,\varepsilon,)\varphi_{I\tau}^{\pm}(\theta). \tag{12}$$

Equation for $\varphi_{I\tau}^{\pm}(\theta)$ be present

$$\left(\hat{T}_{rot} - \epsilon_{I\tau}^{\pm}\right)\varphi_{I\tau}^{\pm}(\theta) = 0. \tag{13}$$

here $\epsilon_{I\tau}$-energy of triaxial rotor [16], the quantum number τ labels the different eigenvalues $\epsilon_{I\tau}^{\pm}$ of $\hat{T}_{\text{rot}}$ corresponding to a given angular momentum I. To solve equation (13) we assume, that the components of the moment of inertia $\mathcal{J}_i$ takes following form in polar coordinates:

$$\mathcal{J}_1 = 8B\sigma^2\left[\cos^2\varepsilon_0\sin^2\left(\gamma_{\text{eff}} - \frac{2\pi}{3}\right) + \sin^2\varepsilon_0\left(\frac{3}{2}\cos^2\eta_{\text{eff}} + \sin^2\eta_{\text{eff}} + \frac{\sqrt{15}}{2}\sin\eta_{\text{eff}}\cos\eta_{\text{eff}}\right)\right], \tag{14}$$

$$\mathcal{J}_2 = 8B\sigma^2\left[\cos^2\varepsilon_0\sin^2\left(\gamma_{\text{eff}} - \frac{4\pi}{3}\right) + \sin^2\varepsilon_0\left(\frac{3}{2}\cos^2\eta_{\text{eff}} + \sin^2\eta_{\text{eff}} - \frac{\sqrt{15}}{2}\sin\eta_{\text{eff}}\cos\eta_{\text{eff}}\right)\right], \tag{15}$$

$$\mathcal{J}_3 = 8B\sigma^2\left(\cos^2\varepsilon_0\sin^2\gamma_{\text{eff}} + \sin^2\varepsilon_0\sin^2\eta_{\text{eff}}\right). \tag{16}$$

The ability of the triaxial quadrupole-octupole rotor model for describing the structure of the lowest positive and negative parity levels in the spectra of heavy even-even nuclei is considered in Ref. [17], where $\varphi_{I\tau}^{\pm}(\theta)$ has the form

$$\varphi_{I\tau}^{\pm}(\theta) = \sum_{K\geq 0}^{I} A_{IK}^{\tau}|IMK\pm\rangle, \tag{17}$$

$$|IMK\pm\rangle = \frac{1}{\sqrt{2(1+\delta_{K,0})}}\left(|IMK\rangle \pm (-1)^{I-K}|IM-K\rangle\right), \tag{18}$$

with $|IMK\rangle = \sqrt{\frac{2I+1}{8\pi^2}}D_{MK}^{I}(\theta)$, here $D_{MK}^{I}(\theta)$-Wigner function. The expansion coefficients implicitly depend on the effective deformation parameters, $A_{IK}^{\tau} = A_{IK}^{\tau}(\gamma_{\text{eff}},\eta_{\text{eff}})$. M and K are the projections of the angular momentum $\hat{I}_i$ on the third axes of the laboratory and intrinsic frames, respectively.

If we take $W(\sigma,\varepsilon)$ in the vicinity of the minimum $\sigma_0, \pm\varepsilon_0$ accept as [3]

$$W(\sigma,\varepsilon) = V(\sigma) + \frac{C_\varepsilon}{2\sigma^2}(\varepsilon \mp \varepsilon_0)^2, \tag{19}$$

then the variables are (11) separated and $F_I^{\pm}(\sigma,\varepsilon)$ divided into factors:

$$F_I^{\pm}(\sigma,\varepsilon) = f_I^{\pm}(\sigma)\chi(\varepsilon \mp \varepsilon_0). \tag{20}$$

Equation for $\chi(\varepsilon \mp \varepsilon_0)$:

$$\frac{\partial^2\chi_\nu(\varepsilon \mp \varepsilon_0)}{\partial\varepsilon^2} + \frac{2B}{\hbar^2}\left[\pm\epsilon_\nu - \frac{C_\varepsilon}{2}(\varepsilon \mp \varepsilon_0)\right]\chi(\varepsilon \mp \varepsilon_0) = 0, \tag{21}$$

eigenvalue and eigenfunction of equation (21):

$$\epsilon_\nu = (\nu + \frac{1}{2})h\omega_\varepsilon, \qquad \chi_\nu(\xi^{\pm}) = N_\varepsilon H_\nu(\xi^{\pm})\exp(-\frac{1}{2}(\xi^{\pm})^2), \tag{22}$$

where N_ε –normalization factor, $H_\nu(\xi^{\pm})$ –Hermitian polynomial, ν=0,1,2,... quantum number ε-vibrations.

$$\omega_\varepsilon = \frac{C_\varepsilon}{B}, \qquad \xi^{\pm} = \sqrt{\frac{B\omega_\varepsilon(\varepsilon \mp \varepsilon_0)}{2}},$$

here C_ε - elasticity constant with respect to ε-vibrations. Now we will write an equation for $f_I^{\pm}(\sigma)$:

$$-\frac{\hbar^2}{2B}\left[\frac{\partial^2 F_I^{\pm}(\sigma)}{\partial\sigma^2} + \frac{\partial F_I^{\pm}(\sigma)}{\sigma\partial\sigma}\right] + \left[\frac{\hbar^2\epsilon_{I\tau}^{\pm}}{4B\sigma^2} + V(\sigma) \mp \frac{\hbar^2}{4B}\frac{\epsilon_\nu}{\sigma^2} - E_I^{\pm}\right]f_I^{\pm}(\sigma) = 0. \tag{23}$$

To solve the equation (23), the potential $V(\sigma)$ can offer

$$V(\sigma) = V_0 \left(\frac{\sigma}{\sigma_0} - \frac{\sigma_0}{\sigma} \right)^2. \tag{24}$$

Then the equation (23) will take the form:

$$\left[\frac{\partial^2}{\partial \sigma^2} + \frac{\partial}{\sigma \partial \sigma} - \frac{\epsilon_{I\tau}^{\pm}}{2\sigma^2} - 2g\frac{\sigma^2}{\sigma_0^4} + \frac{4g}{\sigma_0^2} - \frac{2g}{\sigma^2} \pm \frac{\epsilon_\nu}{2\sigma^2} + \frac{2BE_I^{\pm}}{\hbar^2} \right] f_I^{\pm}(\sigma) = 0,$$

where

$$g = \frac{BV_0\sigma_0^2}{\hbar^2}.$$

Enter new variable

$$x = \frac{\sqrt{2g}}{\sigma_0^2}\sigma^2.$$

Obtain following equation

$$\left[x\frac{\partial^2}{\partial x^2} + \frac{\partial}{\partial x} - \frac{\epsilon_{I\tau}^{\pm} + 4g \mp \epsilon_\nu}{8x} - \frac{x}{4} + \tilde{E} \right] f_I^{\pm}(x) = 0. \tag{25}$$

here

$$\tilde{E} = \frac{\sqrt{2g}}{2} + \sqrt{\frac{g}{2}}\frac{E_I^{\pm}}{2V_0}.$$

The wave functions of equation (25) are found in the following form:

$$f_{In\tau}^{\pm}(x) = e^{-\frac{x}{2}} x^{s^{\pm}} u_I^{\pm}(x), \tag{26}$$

here

$$s^{\pm} = \sqrt{\frac{\epsilon_{I\tau}^{\pm} + 4g \mp \epsilon_\nu}{8}}.$$

Then, we obtain following hypergeometric differential equation

$$\left[x\frac{\partial^2}{\partial x^2} + (2s^{\pm} + 1 - x)\frac{\partial}{\partial x} - \left(s^{\pm} + \frac{1}{2} - \tilde{E} \right) \right] u_I^{\pm}(x) = 0. \tag{27}$$

Wave functions of the equation (27) are confluent hypergeometric function $W_I^{\pm}(-n, 2s^{\pm}+1, x)$, $n = 0, 1, 2, ...$ – quantum number of σ-vibrations. Wave functions (26) we can write

$$f_{In\tau}^{\pm}(x) = N_\sigma x^{s^{\pm}} e^{-\frac{x}{2}} W_I^{\pm}(-n, 2s^{\pm}+1, x) \tag{28}$$

N_σ -normalization factor. Eigenvalues of the equation (27) are

$$E_{nI\tau}^{\pm} = \sqrt{\frac{2}{g}}V_0 \left(2n + \sqrt{\frac{\epsilon_{I\tau}^{\pm} + 4g \mp \epsilon_\nu}{2}} + 1 - \sqrt{2g} \right). \tag{29}$$

We introduce the energy factor

$$\hbar\omega = \sqrt{\frac{2}{g}}V_0.$$

Energy excited collective states:

$$\triangle E^{\pm}_{nI\tau} = \hbar\omega\left(2n + \sqrt{\frac{\epsilon^{\pm}_{I\tau} + 4g \mp \epsilon_{\nu}}{2}} - \sqrt{\frac{4g - \epsilon_0}{2}}\right).$$

We introduce the notation

$$\Delta^{\pm}_{\nu} = 4g \mp \epsilon_{\nu}, \qquad \Delta_0 = 4g - \epsilon_0.$$

And

$$\triangle E^{\pm}_{nI\tau} = \hbar\omega\left(2n + \sqrt{\frac{\epsilon^{\pm}_{I\tau} + \Delta^{\pm}_{\nu}}{2}} - \sqrt{\frac{\Delta_0}{2}}\right) \tag{30}$$

In presented approximation energy levels of the excited collective states are determined by parameters: $\hbar\omega$ (in keV), $\Delta^{\pm}_{\nu}$ (dimensionless), Δ^{+}_{0} (dimensionless). ε_0 (in degree), $\gamma_{\rm eff}$ (in degree) and $\eta_{\rm eff}$ (in degree).

The general solutions of the Schrödinger equation (23) are very complicated. This is because the quantities ϵ_{ν} are eigenvalues of equation (22). In this equation variable ε is dynamic. This makes the solution of the equation (22) more difficult because the variable ε is present in the expressions for the components of the moment of inertia (14)-(16). This difficulty was overcome in the work [17] as follows: here it is assumed that ε=ε_0. And the components of the moment of inertia are expressed by parameter ε_0. But there is a rather serious shortcoming here, since the contribution of the quantities ϵ_{ν} will not be taken into account. Here, to take into account the contribution of a quantity ϵ_{ν}, we assume that the quantity ε in the equation is dynamic. We also assume that the value of ε in terms of moments of inertia is constant, that is, a parameter ε_0. This assumption does not have a serious impact on our calculations, because the quantity ε_0 is present in the expressions for components of the moment of inertia (14)-(16) as the argument of the sine and cosine functions. Therefore, these functions are not very much affected by changes in the values the components of the moment of inertia of the even-even nuclei.

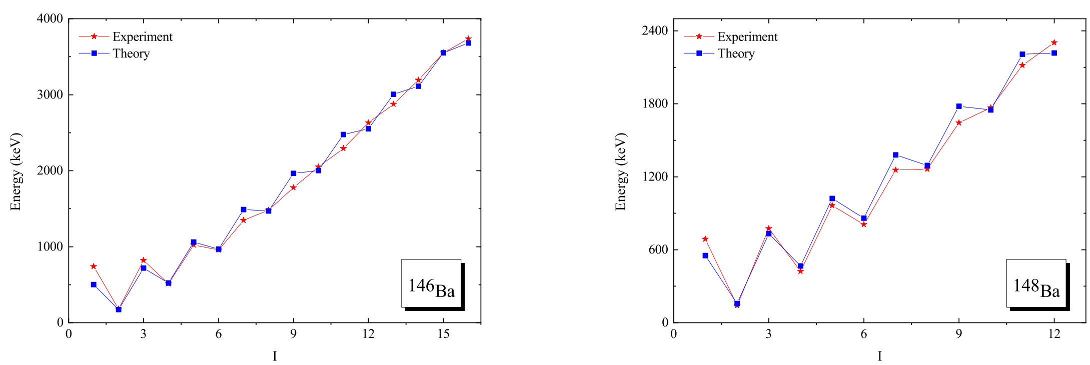

FIG. 1. Theoretical and experimental [20] energy levels of the yrast alternating-parity band for ^{146}Ba (left) and ^{148}Ba (right).

III. ENERGY LEVELS

The adjusted values of the model parameters are shown in the captions of Figs. (1)-(6). Even a quick glance shows that the parameter ε_0 takes on large values. These are very different from the values of this parameter in work [17]. In this work, the parameter ε_0 took small values or values close to zero (with the exception of nucleus ^{232}U). Then $\sin\varepsilon_0 \approx 0$ in the second terms of the expressions (14)-(16). This means that such a value excludes the contribution of the parameter $\eta_{\rm eff}$ to energy spectrum (30). Here this lack of is eliminated. Which makes it possible to describe the energy spectrum of variable parity from the point of view of taking into account vibrational-rotational motion and with K mixing, i.e. with $K\neq$0. In addition, if we take into account $\sin 45^0$=$\cos 45^0$, then the contribution of one of the triaxiality parameter increasesfor values of $\varepsilon_0 > 45^0$ or $\varepsilon_0 < 45^0$ with respect to the second, see (14)-(16).

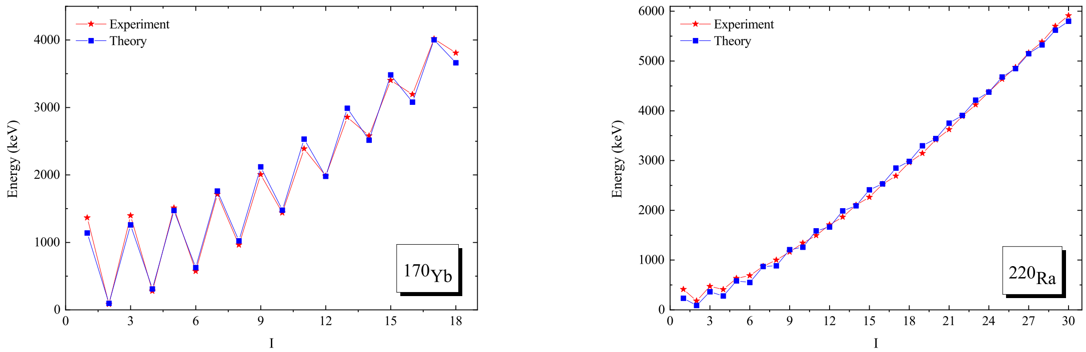

FIG. 2. The same as in Fig. 1, but for ^{170}Yb (left) and ^{220}Ra (right).

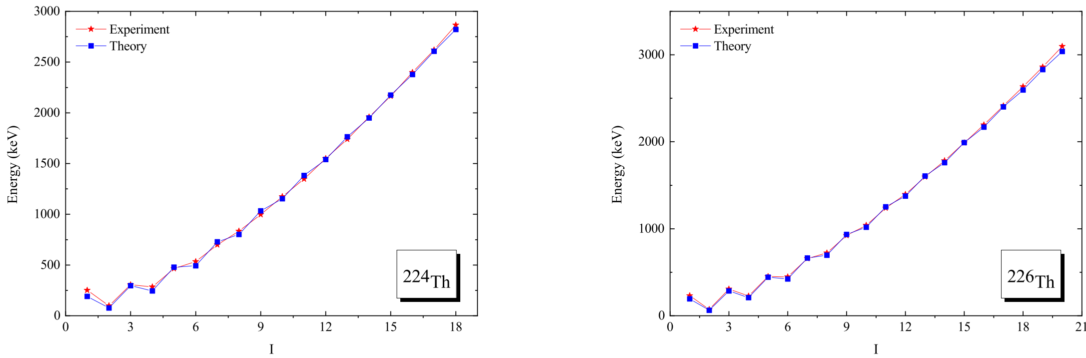

FIG. 3. The same as in Fig. 1, but for ^{224}Th (left) and ^{226}Th (right).

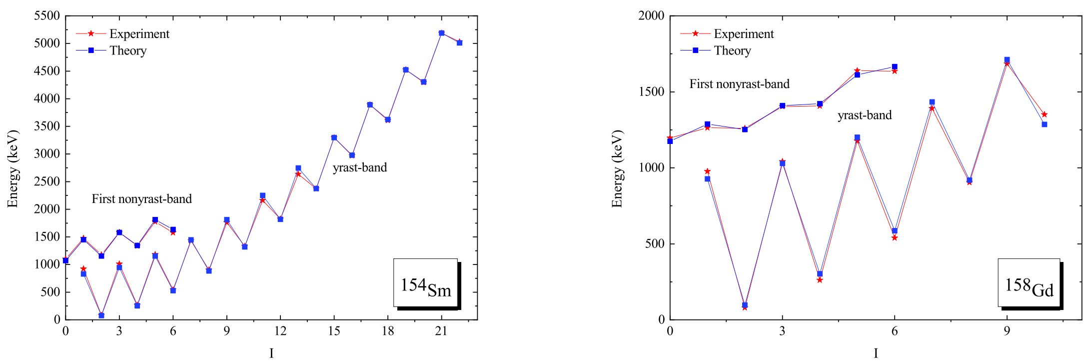

FIG. 4. Theoretical and experimental [20] energy levels of the yrast alternating-parity and first-non-bands for ^{154}Sm (left) and ^{158}Gd (right).

The values of the fitted model parameters and the RMS values (in keV) obtained for the alternating-parity spectra for even-even nuclei: ^{146,148}Ba, ^{154}Sm, ^{158}Gd, ^{160}Dy, ^{170}Yb, ^{220,224}Ra and ^{224,226,230,232}Th have been presented In the Table I. Note that the RMS value (at $\leq$ 100 keV) is a good criterion for the applicability of different models [19].

Theoretical and experimental [20] behaviour of the spectrum of energy levels of excited collective states of the yrast

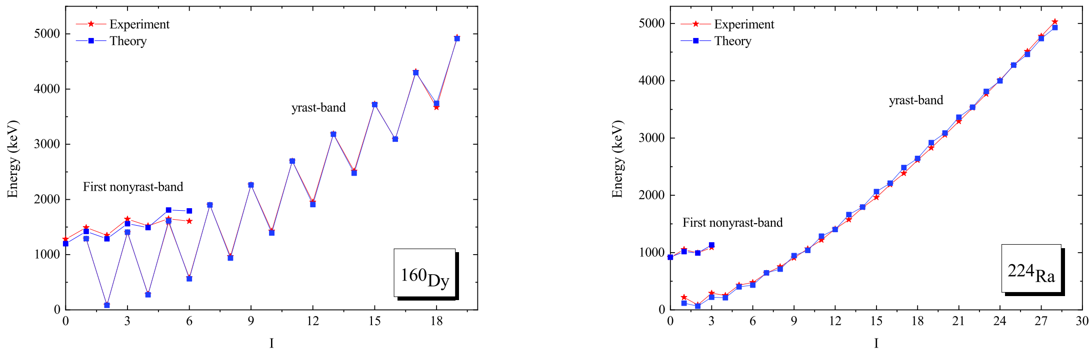

FIG. 5. The same as in Fig. 4, but for ^{160}Dy (left)and ^{224}Ra (right).

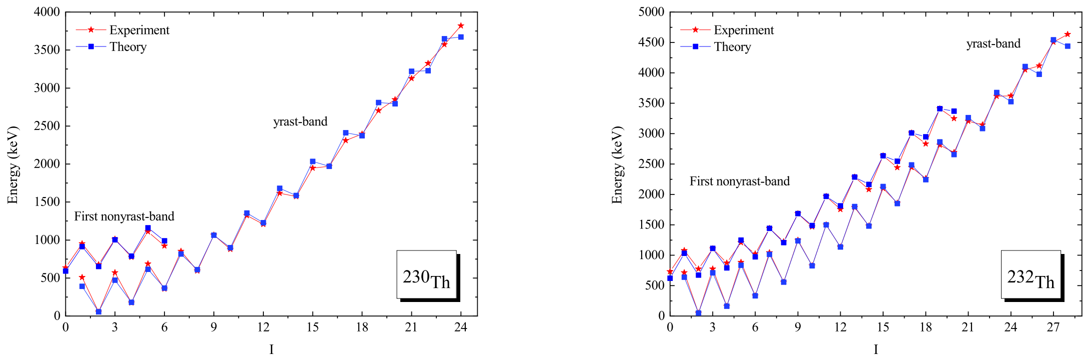

FIG. 6. The same as in Fig. 4, but for ^{230}Th (left) and ^{232}Th (right).

Nucleus	$\hbar\omega$	Δ_0^+	Δ_0^-	Δ_1^+	Δ_1^-	ε_0	γ_{eff}	η_{eff}	RMS
^{146}Ba	545.46	14.12	24.39	–	–	20.9	5.01	5.08	109.39
^{148}Ba	489.41	11.73	24.07	–	–	30.53	10.65	5.21	82.2
^{170}Yb	633.43	64.12	110.14	–	–	42.24	0.32	89.82	99.55
^{220}Ra	479.81	40.91	49.06	–	–	27.78	5.65	5.05	93.82
^{224}Th	516.17	107.71	107.81	–	–	4.55	48.14	87.5	29.9
^{226}Th	530.43	157.72	158.7	–	–	5.22	49.4	12.45	24.42
^{154}Sm	808.55	222.26	288.72	195.16	212.69	4.35	5.42	9.02	39.32
^{158}Gd	388.61	31.38	79.16	49.63	54.27	4.9	5.49	51.03	32.21
^{160}Dy	801.87	145.98	204.99	129.28	137.05	37.84	134.09	23.46	61.71
^{224}Ra	620.51	65.26	68.87	53.99	56.52	59.22	2.11	9.81	54.04
^{230}Th	456.67	130.9	158.69	109.12	129.59	10.49	2.88	13.66	63.65
^{232}Th	431.28	207.4	207.62	185.33	222.26	32.23	119.08	35.77	61.42

TABLE I. The values of adjusted model parameters and RMS values (in keV) obtained for the alternating-parity spectra of several even-even nuclei.

alternating-parity band heavy even-even nuclei: ^{146,148}Ba, ^{170}Yb, ^{220}Ra and ^{224,226}Th is presented in Fig. (1)-(3). All even-even nuclei under consideration are well described by the proposed model.

The theoretical and experimental behaviour of the spectrum of energy levels of excited collective states of the yrast and first non-yrast alternating-parity bands of heavy even-even nuclei: ^{154}Sm, ^{158}Gd, ^{160}Dy, ^{224}Ra and ^{230,232}Th is presented in Fig.

(4)-(6). All the considered even-even nuclei are well described by the proposed model.

IV. CONCLUSION

The effective triaxiality model of even-even nuclei with quadrupole and octupole deformation is developed in the present work. The Davidson potential was used to solve the Schrödinger equation for the radial part. In presented model energy levels of yrast-band of the alternating parity spectrum determined by six adjusted parameters. And energy levels of the yrast- and first-non-yrast-bands determined by eight adjusted parameters. Application of the model to the energy levels of the yrast and first non-yrast alternating parity bands in several rare earth and actinide nuclei shows good agreement with the corresponding experimental energy levels. A description of the alternating parity energy spectrum of even-even nuclei taking into account rotational and vibrational degrees of freedom is carried out for the first time.

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