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Electromagnetic influence on hyperbolically symmetric sources in $f(T)$ gravity

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Abstract The present study aims to see how gravitational modification, specifically, the $f(T)$ gravitational field, where T is torsion scalar, impacts static fluid content with hyperbolic symmetry and electromagnetic field. We enlarge Herrera's strategy (Herrera et al. in Phys Rev D 103:024037, 2021) to analyze the impact of electromagnetic field on $f(T)$ gravity. We distinguish the stress–energy tensor by considering the ingredients of the tetrad field in the Minkowski co-ordinate frame, commencing with modified field equations. With the advent of negative density, this sort of fluid is supposed to surpass extreme physical conditions, enabling quantum impacts to be detected. We calculate a viable formulation of mass utilizing the Tolman mass from the viewpoint of $f(T)$ gravity along with the electromagnetic field. The gravitational interference is repulsive, as made evident by the negative value of the Tolman mass. Also, we explored the structure scalars in $f(T)$ gravity and found significant solutions in presence of electric charge.

1 Introduction

The recognition of accelerated growth of the cosmos in the 1990s, experienced a major breakthrough. Although general relativity (GR) is a very effective paradigm, it incorporates substantial drawbacks, such as the failure to comprehend and interpret dark matter (DM) and dark energy (DE) [2,3]. The escalating conflict about indirect and direct estimations of the Hubble parameter [4–6] and the cosmic constant issue [7] have boosted interest in investigating feasible adaptations of GR and the λCDM model. The latest photograph of gigantic black hole's image in the core of the galaxy M87 [8] as well as the latest finding of gravitational radiation [9] have enhanced the ability of investigating what gravity works in a massive domain. This impact could also be employed to examine GR and look for feasible impacts in modified gravity. In this regard, many of the most efficient approaches to the foregoing difficulties are grounded on the expansion and adaptation of GR. It should be emphasized that certain approaches are highly effective not just in the perspective of the aforesaid cosmic growth, but also in other aspects of current cosmology. Furthermore, the conventional GR cannot be altered in terms of quantum theory, but this problem can be resolved with the aid of higher-curvature components in gravitational-action.

The need of the modification of GR has been well-drafted via some reviews on alternative gravities [10–14]. The idea we're addressing in this study is relevant to nominal teleparallel gravity (TG) theories. In this context, the torsion tensor is associated with the gravitational intensity but the curvature tensor has zero value [15]. In this paradigm, the teleparallel in equivalence of GR (TEGR) is a special framework with the analogous equations of GR. TEGR is the Einstein's theory that unites the electromagnetic and gravitational fields [16,17]. Using the energy–momentum tensor, the TEGR approach might be used to compute the conserved components, angular-momentum, and mass [18]. The fundamental justification for altering the TEGR framework was the recent appearance of problems in findings that TEGR could not answer [19]. This theory can then be adjusted, and many modified TG frameworks have been proposed and examined in recent years. The $f(T)$ gravity [19,20], in which T is the scalar torsion, is a prominent example, is the alternation of the TEGR framework. Not only inflation in the primordial universe [19], but late-time cosmological expansion [21–23] might be addressed using this idea.

We focus on $f(T)$ gravity for a variety of purposes, along with the notion that the Lagrangian of this framework is purely based on the torsion scalar, allowing it simple to accomplish than other modified approaches of gravity [12,24]. A further key aspect is that, apart from the numer-

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ous different modified approaches, modified field equations (MFEq's) are of second-order [25,26]. This notion has been the subject of numerous studies in a variety of contexts (as in cosmological and astrophysical domains). Following conformal transformation, Yang [27] observed that the $f(T)$ gravity is not dynamically analogous to the TEGR Lagrangian. Meng and Wang [28] discussed the validity of Birkhoff's theorem in $f(T)$ gravity. They elaborated the concept by studying its implementation on de-Sitter space-time. Bamba et al. [29] discussed $f(T)$ gravity conformal challenges. The de-Sitter solution is also revealed to emerge under conformal TG. Geng et al. [30] used observational data to restrict a newly schemed teleparallel DE strategy rooted on the TEGR, where a certain canonical scalar field is included, enabling for non-minimal gravity interaction. They depicted that the concept is acceptable with observed data using the approaches like exponential and power-law etc. Bhatti et al. [31–34] studied the dynamical instability ranges for self-gravitating objects in $f(T)$ gravity with planar and cylindrical geometry. The key findings of their analysis is that in $f(T)$ gravity, the stiffness parameter plays a vital impact on the stability of celestial object.

The Schwarzschild-solution (SS) of the Einstein-field equations (EFEq's) for null space is the hyperbolically-symmetric, static and asymptotically-flat solution. The term "static" refers to that for the time-like vector field, the metric-tensor please the Killing equations. And the SS please the time-like Killing vectors of the form $\chi_{(0)} = \partial_t$, $\chi_{(1)} = \partial_\phi$, $\chi_{(2)} = -cos\phi\partial_\theta + coth\theta sin\phi\partial_\phi$, $\chi_{(3)} = sin\phi\partial_\theta + coth\theta cos\phi\partial_\phi$. Harrison [35] examined the slightly different solutions of EFEq's from usual ones by considering it as an outcome of GR. He determined the solutions for null space by using the separation method. The expressions for 10 degenerate and 20 non-degenerate solutions are found explicitly. The geometrical and physical inspections of these thirty solutions are carried out. Gaudin et al. [36] inspected the SS of EFEq's in the appearance of without mass scalar field. They also investigated the physical properties of null hyperbolic spacetime in detail. Stephani et al. [37] studied certain exact SS for EFEq's in depth. Rizzi et al. [38] investigated the model (of empty space) which shows the emergence of homogenous and continuous classification of matter because of test particle movement. They further found that no matter existed as stress–energy tensor vanished. It indicates that the DM appears as dynamical impact because of space-time curvature. The probability of containing tunnels in hyperbolical-spacetime is studied by Lobo et al. [39]. They established a class of degenerate solutions by introducing exotic matter in hyperbolically symmetric static spacetime. The particular solutions and physical properties are studied by imposing specific limitations. The analytical assessment of inhomogeneous and spherical solutions of scalar-tensor gravities and FEq's is done by Faraoni et al. [40]. This gravity hub consists of dynamical and static solutions. They presented a connection between numerous solutions already discussed in the literature.

During the evolution of stellar objects, the static solutions of FEq's have become the topic of vast study, not only in GR but also in $f(T)$ gravity. Wang [41] explored static solutions for $f(T)$ models including a Maxwell term. He adopted a particular frame as selection of frames has a great influence on results in the $f(T)$ theory. In the chosen frame, he furnished the conditions of several solutions and determined only a delimited group of $f(T)$ models. While using the Weyl coordinates, Houndjo et al. [42] inspected the cylindrically symmetric static vacuum solutions in $f(T)$ background. They established the set of MFEq's and found the general solution as the outcome of constant torsion scalar. Moreover, they set up the cosmological constant and discussed the Linet Tian solution in the background of GR. Li et al. [43] checked the stability of static Einstein universe in both cases, i.e., open and closed with couple of exponential $f(T)$ models. They studied the existence of stable solutions in the context of $f(T)$ theory. Atazadeh and Mousavi [44] imposed new conditions on components of spherically symmetric metric and found new vacuum solutions in the context of $f(T)$ theory. They settled the analytical formation of $f(T)$ theory by eliminating coefficients of metric. Using anisotropic fluid configuration, Sharif and Rani [45] explored the static spherically-symmetric wormhole solutions in scenario of $f(T)$ theory. They explored MFEq's and determined the matter constituent expressions of transverse and radial pressure, and energy density. With the help of shape and particular $f(T)$ functions, they determined the nature of energy conditions in terms of wormhole solutions. It results in the existence of phenomenal admissible wormhole solutions in both cases.

An electromagnetic field (EMF) is the feature of space induced by the movement of an electric charge. The simultaneous connection of Maxwell and Einstein (or modified) fields, which is viewed as the dispersion of electromagnetic-waves because of curvature of space-time, has a rich background in curved space-time analysis. By introducing EMF in Lagrangian, Moffat [46] acquired field equations for new gravity and attained static spherically symmetric solutions. Ivanov [47] found the solutions for the Reissner–Nordström metric by using a new categorized strategy in the presence of EMF. Dehghani [48] furnished a new group of solutions for a modified gravity along with negative cosmological constant. He treated these solutions possibly as black brane solutions. Zhang et al. [49] explored the gravitational collapse in the presence of charge in de-Sitter space-time. They studied the impact of two factors on the charge and found that features of gravitational collapse are independent of the dimension of space-time. Bhatti and Yousaf [50] studied the inhomogeneity components for plane symmetry in $f(R)$ gravity with the effect of the charge on it. They used dissipative and anisotropic fluid for this analysis. The main findings indicate that the inhomogeneity components are influenced by electric charge. Yousaf et al. [51,52] explored the impact of modification and charge on self-gravitating objects. They studied the role of zero complexity factor and quasi-homologous constraint in considered structure. They constructed various solutions for MFEq's and deduced that few of them accept the Darmois constraints. Recently, in different modified theories, the influence of EMF on gravastars is studied [14,34,53] where the researchers found the stable regions for gravastars in this context. It is deduced that the entropy, length of the thin shell, and the energy content of the gravastar are all impacted by the EMF. In this article, we will communicate with issues listed below :

1. We inspect the structure scalars for static fluid content in $f(T)$ gravity within the presence of EMF. We will find effective analytical answers in turn of generating functions.
2. The contribution of $f(T)$ dark source components and impact of EMF will be scrutinized.

The following scheme is used to line up our article. We explore the physical characteristics of the static fluids equipped with hyperbolical symmetry in the framework of $f(T)$ gravity and the presence of EMF. In Sect. 2, we developed the MFEq's in terms of EMF and work out the expression for stress–energy tensor. The $f(T)$-Maxwell field equations along with mass function will be explored in Sect. 3. We also find relations of kinematical quantities in terms of the mass function. The relationship between the conformal tensor and the intrinsic curvature is built up in Sect. 4. The total energy budget formula given by the Tolman–Whittaker is utilized to evaluate its value in our analysis. In Sect. 5, the approach of an orthogonal division of the intrinsic curvature in the framework of $f(T)$ gravity and presence of EMF is carried out to evaluate structure scalars. Section 6 includes few effective analytical answers to concern fluid content. In Sect. 7, the closing comments are presented.

2 Formulation of $f(T)$ gravity; metric and matter distribution

In this section, first of all, we will give a formal idea of $f(T)$ gravity, where T is referred to as torsion scalar. Later on, the complete description of line elements and matter content will be given. This theory has great influence in investigating inflation and late-time accelerated expansion of the universe. The EHA for $f(T)$ gravity is defined as [29,54]

$$S_{f(T)} = \int d^4x \left(\mathcal{L}_M + \mathcal{L}_{EMF} + \frac{f(T)}{2\kappa^2}\right)|\mathfrak{h}|, \tag{1}$$

where $|\mathfrak{h}| = det(\mathfrak{h}^\tau_\phi)$, while $\mathfrak{h}^\tau_\phi$ behaves as the dynamical field of $f(T)$ theory. Here, coupling constant is represented by κ, $\mathcal{L}_M$ is the representation for matter field Lagrangian and differential function of the torsion scalar is given by $f(T)$. This set of orthonormal vector fields is associated with metric tensor by following relation $g_{\phi\tau} = \vartheta_{ij}\mathfrak{h}^i_\phi\mathfrak{h}^j_\tau$ with $\vartheta_{ij} = diag(1,-1,-1,-1)$. The coordinates for the manifold is given by the indices $(i, j, \ldots)$ and coordinates for tangent space is given by $(\tau, \phi, \ldots)$. The relation for torsion scalar is defined as

$$T = S_\rho^{\ \tau\phi} T^\rho_{\ \tau\phi}, \tag{2}$$

where the tensor $T^\rho_{\ \tau\phi}$ satisfy $T^\rho_{\ \tau\phi} = -T^\rho_{\ \phi\tau}$. The Weitzenböck connection ($\bar{\Gamma}^\rho_{\ \phi\tau} = \mathfrak{h}_i^{\ \rho}\partial_\phi\mathfrak{h}^i_{\ \tau}$) has vital role in defining the above tensor as

$$T^\rho_{\ \tau\phi} = \bar{\Gamma}^\rho_{\ \phi\tau} - \bar{\Gamma}^\rho_{\ \tau\phi} = \mathfrak{h}_i^{\ \rho}(\partial_\phi\mathfrak{h}^i_{\ \tau} - \partial_\tau\mathfrak{h}^i_{\ \phi}), \tag{3}$$

where

$$S_\rho^{\ \tau\phi} = \frac{\phi^\tau_\rho}{2}T^{\beta\phi}_{\ \ \beta} - \frac{\phi^\phi_\rho}{2}T^{\beta\tau}_{\ \ \beta} + \frac{1}{4}\left(T^{\tau\phi}_\rho + T^{\phi\tau}_\rho - T^{\tau\phi}_\rho\right). \tag{4}$$

The following result is attained by pursuing variation on the action of Eq. (1) with reference to tetrad field

$$\mathfrak{h}_i^{\ \rho}S_\rho^{\ \tau\phi}\partial_\tau T f_{TT} + \frac{f}{4}\mathfrak{h}_i^{\ \phi} + \frac{f_T}{\mathfrak{h}}\partial_\tau(\mathfrak{h}\mathfrak{h}_i^{\ \rho}S_\rho^{\ \tau\phi}) + \mathfrak{h}_i^{\ \rho}T^\alpha_{\tau\rho}S_\alpha^{\ \phi\tau}f_T = \frac{\kappa^2}{2}\mathfrak{h}_i^{\ \rho}(T^{\phi(m)}_\rho + E^{\phi(m)}_\rho), \tag{5}$$

whereas, $f_T \equiv \frac{\partial f}{\partial T}$, $f_{TT} \equiv \frac{\partial^2 f}{\partial T^2}$, while $T^{\phi(m)}_\rho$ is the fluid stress–energy tensor and $E^{\phi(m)}_\rho$ is EMF stress tensor. To carry our analysis in $f(T)$ gravity, initially a general form of fluid as an amalgam of anisotropic tensor $\Pi_{\tau\phi}$, isotropic stress P and energy density μ is taken and then hyperbolic symmetry will be applied later on. The stress–energy tensor is given as

$$T^{(m)}_{\tau\phi} = (\mu + P)V_\tau V_\phi - Pg_{\tau\phi} + \Pi_{\tau\phi}, \tag{6}$$

whereas V^τ is four-velocity vector and fluid satisfies the following relations

$$\mu = T_{\tau\phi}V^\tau V^\phi, \quad P = -\frac{1}{3}\hbar^{\tau\phi}T_{\tau\phi},$$
$$\Pi_{\tau\phi} = \hbar^\beta_\tau\hbar^\alpha_\phi(T_{\beta\alpha} + P\hbar_{\beta\alpha}),$$

with $\hbar_{\beta\alpha} = g_{\beta\alpha} - V_\alpha V_\beta$ represents the projection tensor. Since, TG is equivalent to GR, so the influence of covariant formulation of its extension, i.e., $f(T)$ gravity has equivalence with the subtraction of torsion scalar from the Ricci scalar. The $f(T)$ field equations by using covariant formulation is attainted as follows

$$\Upsilon_{\tau\phi} f_{TT} - \frac{T}{2}\left(f_T - \frac{f}{T}\right) g_{\tau\phi} + G_{\tau\phi} f_T = \kappa^2 \left(T^{(m)}_{\tau\phi} + E^{(m)}_{\tau\phi}\right), \tag{7}$$

where $\Upsilon_{\tau\phi} = S^{\rho}_{\tau\phi}\nabla_\rho T$, while the Einstein tensor is given by $G_{\tau\phi}$. After reshaping Eq. (5), we get

$$G_{\tau\phi} = \frac{\kappa^2}{f_T}\left(T^{(T)}_{\tau\phi} + T^{(m)}_{\tau\phi} + E_{\tau\phi}\right), \tag{8}$$

where $f(T)$ based corrections are as follows

$$T^{(T)}_{\tau\phi} = -\frac{1}{\kappa^2}\left\{\Upsilon_{\tau\phi} f_{TT} + \frac{1}{4}\left(R f_T - \Upsilon f_{TT} + T\right) g_{\tau\phi}\right\}. \tag{9}$$

Applying the usual limit, i.e., $f(T) = T$, TEGR equations can be attained. Mathematically, EMF stress tensor is expressed as

$$E_{\tau\phi} = \frac{1}{4}\left(-F^{\mu}_{\tau} F_{\phi\mu} + \frac{1}{4} F^{\mu\delta} F_{\mu\delta} g_{\tau\phi}\right),$$

where $F_{\phi\mu}$ is EMF tensor and mathematically it is given by $F_{\phi\mu} = \varphi_{\mu,\phi} - \varphi_{\phi,\mu}$ along with four-current density given by $J_\phi = \sigma(r) V_\phi$ and four-potential expressed by $\varphi_\phi = \varphi(r)\delta^0_\phi$. In compact form, the Einstein–Maxwell field equations (E-MFEq's) can be read as

$$F^{\tau\phi}_{\ \ ;\phi} = \mu_0 J^\tau; \quad F_{[\tau\phi;\gamma]},$$

where the magnetic permeability is described by the constant $\mu_0 = 4\pi$. The non-vanishing constituents of the E-MFEq's furnish the subsequent second-order equation of the form

$$\varphi'' - \left(\frac{\lambda'}{2} + \frac{\nu'}{2} - \frac{2}{r}\right)\varphi' = 4\pi\sigma e^{\lambda + \frac{\nu}{2}}.$$

By solving the above equation using method of integrating factor, we attain $r^2\varphi' e^{\frac{-\lambda-\nu}{2}} = \check{q}$, where $\check{q} = \int 4\pi r^2 \sigma e^{\frac{\lambda}{2}}$. The components of EMF tensor will summed up as

$$E_{00} = \frac{e^\nu \check{q}^2}{8\pi r^4}; \quad E_{11} = -\frac{e^\lambda \check{q}^2}{8\pi r^4}; \quad E_{22} = \frac{\check{q}^2}{8\pi r^2}; \quad E_{33} = \frac{sinh^2\theta \check{q}^2}{8\pi r^2}.$$

To study the effect of staticity and EMF within $f(T)$ gravity, we take the line element of hyperbolically static symmetric geometry (for the interior region) as

$$ds^2 = e^\nu dt^2 - e^\lambda dr^2 - r^2(d\theta^2 + sinh^2\theta d\phi^2), \tag{10}$$

where $\lambda(r)$ and $\nu(r)$ are only functions of r. The anisotropic fluid [55,56] has great influence in investigating behavior of self-gravitating objects. We consider our fluid in co-moving frame, where constituents of four velocity vectors in terms of orthonormal tetrad are as follows

$$e^{(0)}_\tau \equiv V_\tau = (e^{\frac{\nu}{2}}, 0, 0, 0); \quad e^{(1)}_\tau \equiv K_\tau = (0, -e^{\frac{\lambda}{2}}, 0, 0);$$
$$e^{(2)}_\tau \equiv L_\tau = (0, 0, -r, 0); \quad e^{(3)}_\tau \equiv S_\tau = (0, 0, 0, -r sinh\theta).$$

With aid of the Bondi idea [57] carried by Herrera [1], the stress–energy tensor is given as

$$T_{\tau\phi} = (\mu + P_{zz}) V_\tau V_\phi - P_{zz} g_{\tau\phi} + (P_{xx} - P_{zz}) K_\tau K_\phi. \tag{11}$$

To analyze the bounded-ness of anisotropic matter from the exterior region, the idea of smooth combining of the interior region given by Eq. (10) is used. In this scenario, the Darmois conditions as computed by Bhatti et al. [58] are executed.

3 $f(T)$-Maxwell equations

The $f(T)$ field equations including non-zero constituents of EMF and the stress–energy tensor are as follows

$$\frac{e^{-\lambda}\acute{\lambda}}{r} - \frac{(1+e^{-\lambda})}{r^2} = \frac{8\pi e^\nu}{f_T}\left[\mu + \frac{\check{q}}{8\pi r^4} + \frac{1}{16\pi} \times \left\{(T f_T - f) + e^{-\lambda} f_{TT}\left(\frac{2}{r}\acute{T} - \frac{\acute{\nu}}{4}\right)\right\}\right], \tag{12}$$

$$\frac{e^{-\lambda}\acute{\nu}}{r} + \frac{(1+e^{-\lambda})}{r^2} = \frac{8\pi e^\lambda}{f_T}\left[P_r - \frac{\check{q}}{8\pi r^4} - \frac{1}{16\pi}(T f_T - f)\right], \tag{13}$$

$$\left(\nu'' + \frac{\nu'}{r} + \frac{\nu'^2}{2} - \frac{\lambda'\nu'}{2} - \frac{\lambda'}{r}\right)\frac{e^{-\lambda}}{2} = \frac{8\pi r^2}{f_T}\left[P_\perp + \frac{\check{q}}{8\pi r^4} - \frac{1}{16\pi}\left\{(T f_T - f) - \frac{f_{TT} \times e^{-\lambda}}{2}(\nu' - \frac{3}{r}) T'\right\}\right], \tag{14}$$

where f_T is used for differentiable function of torsion and prime for derivative with regard to r. We made following assumptions in computing constituents of stress–energy tensor, i.e., $P_{xx} = P_r$ and $P_\perp = P_{yy} = P_{zz}$. The engagement of $f(T)$-Maxwell equations and $T^{\tau\phi(eff)}_{;\phi} = 0$ emerge in hydrostatic equilibrium form like

$$P_r' + (\mu + P_r)\frac{\nu'}{2} + \frac{2\Pi}{r} - \frac{\check{q}\check{q}'}{4\pi r^4} + \frac{h_1}{8\pi} = 0, \tag{15}$$

where presence of dark source terms are indicated by h_1 listed in Appendix I. The mass formula which basically provide information about change of energy of spherical fluid [59] corresponding to interior region is given as

$$m(r) = \frac{(1+e^{-\lambda})r}{2} - \frac{\check{q}^2}{2r}. \tag{16}$$

By making use of Eq. (16) into (12), we find

$$m(r) = -4\pi \int_0^r \frac{r^2}{f_T}\left[\mu e^\nu + \frac{\check{q}\check{q}'}{4\pi r} + \Theta_{00}^{(T)}\right] dr, \tag{17}$$

where $\Theta_{00}^{(T)}$ is stated in Appendix I. The above equation stipulate that the mass has certainly positive amount. For this sake, energy density should owe negative value which is encroachment of weak energy conditions (WEC). For more physical relevance, the detail analysis is examined in [1]. The above equation can be rewritten as

$$m = 4\pi \int_{r_{min}}^r \frac{r^2}{f_T}\left[|\mu| e^\nu - \frac{\check{q}\check{q}'}{4\pi r} - \Theta_{00}^{(T)}\right] dr, \tag{18}$$

where by making use of the fact that energy density owe negative value, $|\mu|$ is used in place of μ. The amalgam of Eqs. (13) and (16) yield

$$\nu' = \left(2\frac{\left\{\frac{4\pi}{f_T}(r^3 P_r - (2m - r)r^2\Theta_{11}^{(T)}) - \frac{\check{q}^2}{r}\right\}}{(2m - r + \frac{\check{q}^2}{r})^2} - m\right), \tag{19}$$

where $\Theta_{11}^{(T)}$ is stated in Appendix I. Substituting the above in Eq. (15), it leads to

$$P_r' + (P_r - |\mu|) \times \frac{\left(2\frac{\left\{\frac{4\pi}{f_T}(r^3 P_r - (2m-r)r^2\Theta_{11}^{(T)}) + \frac{\check{q}^2}{r}\right\}}{(2m - r - \frac{\check{q}^2}{r})^2} - m\right)}{2} + \frac{2\Pi}{r} + \frac{h_1}{8\pi} = 0. \tag{20}$$

This equation is in full engagement with pressure, anisotropic factor, and gravitational power in course of fluid composition in hydrostatic form. The out-turn in this scenario is well narrated in [1].

4 Conformal tensor in association with total energy budget

The relation between the conformal tensor and the intrinsic curvature is given by [60]

$$R^\lambda_{\tau\phi\sigma} = C^\lambda_{\tau\phi\sigma} + \frac{1}{2}R^\lambda_\phi g_{\tau\sigma} - \frac{1}{2}R_{\tau\phi}\delta^\lambda_\sigma + \frac{1}{2}R_{\tau\sigma}\delta^\lambda_\phi - \frac{1}{2}R^\lambda_\sigma g_{\tau\phi} - \frac{1}{6}\Re(\delta^\lambda_\phi g_{\tau\sigma} - g_{\tau\phi}\delta^\lambda_\sigma). \tag{21}$$

The conformal tensor infers knowledge about the tidal force pertain by a body when an object moves across a geodesic. It is in a relationship with the intrinsic curvature and both have contrast on the idea that the conformal tensor only allows information about the geometry of the object deformed because of tidal force. The conformal tensor contains electric as well as magnetic parts. But in the case of the exact Einstein solution, the magnetic part diminishes. The idea behind this fact is that the nearest flow lines spread unrelated to each one, in other words, the progression is merely dependent locally on fluid quantities. In this context, the electric component of the conformal tensor together with the conformal scalar is addressed as

$$C_{\sigma\kappa\nu\lambda} = (g_{\sigma\kappa\tau\phi}g_{\nu\lambda\mu\xi} - \eta_{\sigma\kappa\tau\phi}\eta_{\nu\lambda\mu\xi})V^\tau V^\mu E^{\phi\xi},$$

where $g_{\sigma\kappa\tau\phi} = g_{\sigma\tau}g_{\kappa\phi} - g_{\sigma\phi}g_{\kappa\tau}$ and the representation for the Levi-Civita tensor is observed as $\eta_{\sigma\kappa\tau\phi}$. The use of above argument in Eq. (10) provide the electric component as $E_{\tau\phi} = \varepsilon(K_\tau K_\phi + \frac{1}{3}h_{\tau\phi})$. The conformal tensor has no dependence on the four-velocity vector. But it's electric and magnetic components are linked to four-velocity vector orthogonally as interpreted by subsequent relation $E_{\tau\sigma}V^\sigma$ with trace-free $E^\tau_\tau = 0$. The value of conformal scalar observe as

$$\varepsilon = \frac{e^{-\lambda}}{4}\left(\frac{\lambda'\nu'}{2} - \nu'' - \frac{\nu'^2}{2}\right) + \frac{e^{-\lambda}}{2r}\left(\frac{\nu'}{2} - \frac{1}{r}\right) - \frac{1}{2r}\left(\frac{e^{-\lambda}\lambda'}{2} + \frac{1}{r}\right). \tag{22}$$

Employing field equations (12)–(14), Eq. (16) and the value of ε from Eq. (22), it provides

$$\frac{3m}{r^3} = \frac{4\pi}{f_T}\left[\left(\mu e^\nu - r^2 P_\perp + e^\lambda P_r\right) + \left(\Theta_{11}^{(T)} - \Theta_{00}^{(T)} - \Theta_{22}^{(T)}\right)\right] - \varepsilon - \frac{5\check{q}^2}{2r^4}. \tag{23}$$

Here, the value of $\Theta_{22}^{(T)}$ is written down in Appendix I. This expression reveals the effects of the conformal scalar, fluid components, extra curvature terms of $f(T)$ gravity and terms arise because of EMF on the mass function. The differentiation of the above equation with regard to radial constituent in presence of EMF and utilizing mass value from Eq. (17), bring out the equation of the form

$$\begin{aligned}\varepsilon = &\frac{4\pi}{f_T}\left[\left(\mu e^{\nu} - r^2 P_\perp + e^{\lambda} P_r\right)\right.\\ &\left.+\int_0^r \left(\Theta_{11}^{(T)} - \Theta_{22}^{(T)} - 2\Theta_{00}^{(T)}\right) dr\right]\\ &+\frac{4\pi}{r^3}\left[\int_0^r \left(\frac{\Theta_{00}^{(T)}}{f_T}\right)' + \left(\frac{e^{\nu}|\mu|}{f_T}\right)'\right]\tilde{r}^3 d\tilde{r}\\ &+\frac{3}{2r^3}\int_0^r \frac{\check{q}^2}{r^2} dr - \frac{\check{q}^2}{r^4}. \qquad (24)\end{aligned}$$

This manifest the conformal scalar rest on the fluid constituents (in other words, local anisotropy), density inhomogeneity, and expressions because of $f(T)$ curvature and EMF. Using value of the conformal scalar into Eq. (23), we obtain

$$\begin{aligned}m = &\frac{4\pi r^3}{3 f_T}\left[\int_0^r \left(\Theta_{22}^{(T)} - \Theta_{11}^{(T)} + 2\Theta_{00}^{(T)}\right) dr\right.\\ &\left.+\left(\Theta_{11}^{(T)} - \Theta_{22}^{(T)} - \Theta_{00}^{(T)}\right)\right]\\ &+\frac{4\pi}{r^3}\left[\int_0^r \left(\frac{\Theta_{00}^{(T)}}{f_T}\right)' - \left(\frac{e^{\nu}|\mu|}{f_T}\right)'\right]\tilde{r}^3 d\tilde{r}\\ &-\frac{1}{2}\int_0^r \frac{\check{q}^2}{r^2} dr - \frac{\check{q}^2}{2r}. \qquad (25)\end{aligned}$$

From the above equation, it is indicated that only density inhomogeneity, curvature terms of $f(T)$ gravity, and expression because of EMF affects the mass function in this scenario. Equations (24) and (25) is in accordance with [1] with added terms because of EMF. The Tolman–Whittaker mass serves as active gravitational mass for static as well as slow progression cases. It provides a better explanation of mass in the study of anisotropic fluids. This was established as an estimate of the entire energy budget of the system, without the faithfulness of its localization. The Tolman–Whittaker mass formula in our case with EMF is illustrated as

$$\begin{aligned}m_T = &\int_0^r \int_0^{2\pi} \int_0^{\pi} Sinh\theta r^2 e^{\frac{\lambda+\nu}{2}} \left(T_0^0 + E_0^0 - T_1^1\right.\\ &\left.-E_1^1 - 2T_2^2 - 2E_2^2\right) d\theta d\phi d\tilde{r}. \qquad (26)\end{aligned}$$

Here, T_i^i and E_i^i for $i = 1, 2, 3$ denote the constituents of stress–energy tensor and EMF, respectively. The solution of Eq. (26) turns out as

$$\begin{aligned}m_T = &(cosh\pi - 1)2\pi \int_0^r e^{\frac{(\nu+\lambda)}{2}} \tilde{r}^2\\ &\times\left(-|\mu| + 2P_\perp + P_r + \frac{\check{q}^2}{4\pi r^4}\right) d\tilde{r}. \qquad (27)\end{aligned}$$

The execution of Eqs. (12)–(14) and integration with regard to radial component becomes

$$\begin{aligned}m_T = &\frac{(cosh\pi - 1) f_T}{4} \times \nu' e^{\frac{(\nu-\lambda)}{2}} r^2\\ &-\frac{(cosh\pi - 1)}{24\pi}\left[\int_0^r \{k_1(r) d\tilde{r}\}_{,1} + k_1(r)\right], \qquad (28)\end{aligned}$$

where $k_1(r)$ is stated in Appendix I. The amalgam of the above equation with Eq. (19) is observed as

$$\begin{aligned}m_T = &\frac{cosh\pi - 1) f_T (}{2} e^{\frac{(3\lambda+\nu)}{2}}\\ &\times\left\{\frac{4\pi}{f_T}\left(P_r r^3 + r^2(r - 2m)\Theta_{11}^{(T)}\right) - m - \frac{\check{q}^2}{r}\right\}\\ &-\frac{(cosh\pi - 1)}{24\pi}\left[\int_0^r \{k_1(r) d\tilde{r}\}_{,1} + k_1(r)\right]. \qquad (29)\end{aligned}$$

The Eq. (27) is estimate of total energy budget and manifest that it is negative in nature. From Eq. (29), this argument only possible in presence of EMF if $\frac{4\pi}{f_T}\left(P_r r^3 + r^2(r - 2m)\Theta_{11}^{(T)}\right) - \frac{\check{q}^2}{r} < m$. The illustration of four-acceleration with terminology a_τ and relation $a_\tau = V_{\tau;\,\phi} V^{\phi}$ is yielded as $a_\tau = a K_\tau$ with $a = \frac{\nu' e^{\frac{-\lambda}{2}}}{2}$. Using this value in Eq. (28) gives

$$\begin{aligned}a = &\frac{2e^{\frac{-\nu}{2}}}{r^2(cosh\pi - 1) f_T}\\ &\times\left[m_T + \frac{(cosh\pi - 1)}{24\pi}\left\{\int_0^r \{k_1(r) d\tilde{r}\}_{,1} + k_1(r)\right\}\right].\end{aligned}$$

This equation manifests the inward-pointing of four-acceleration, i.e., negative value. This way exhibit the repulsive nature of gravitational force. Now, differentiating Eq. (27) and implement of Eq. (29) make as

$$\begin{aligned}\acute{m}_T - \frac{3}{r} m_T = &-\frac{(cosh\pi - 1)}{2}\\ &\times f_T\, r^2\left\{\varepsilon + 4\pi\Pi - \frac{\check{q}^2}{4\pi r^4} - 4\pi\Theta_{22}^{(T)} - 4\pi\Theta_{00}^{(T)}\right\}\\ &-\frac{(cosh\pi - 1)}{8\pi r}\left\{\int_0^r \{k_1(r) d\tilde{r}\}_{,1} + k_1(r)\right\}.\end{aligned}$$

The integration of equation acquire

$$m_T = (m_T)_\Psi(\frac{r}{r_\Psi})^3 + \frac{(cosh\pi - 1)}{2}r^3\int_r^{r_{\Psi e}}\left[\frac{f_T \times e^{\frac{\nu+\lambda}{2}}}{\tilde{r}}\left(\varepsilon + 4\pi\Pi - \frac{\check{q}^2}{4\pi\tilde{r}^4} - 4\pi\Theta_{22}^{(T)} - 4\pi\Theta_{00}^{(T)}\right)\right.$$
$$\left. - \frac{1}{4\pi r^4}\left\{\int_0^r \{k_1(r)d\tilde{r}\}_{,1} + k_1(r)\right\}\right]. \quad (30)$$

Now, making substitution of value from Eq. (24), we acquire

$$m_T = (m_T)_\Psi(\frac{r}{r_\Psi})^3 + \frac{r^3 \times (cosh\pi - 1)}{2}\int_r^{r_{\Psi e}}\left[\frac{f_T \times e^{\frac{\nu+\lambda}{2}}}{\tilde{r}}\left\{\frac{4\pi}{r^3}\int_0^s\left((\frac{\Theta_{00}^{(T)}}{f_T})' + (\frac{e^\nu|\mu|}{f_T})'\right)\tilde{s}^3 d\tilde{s}\right\}\right.$$
$$\left. + 4\pi\Pi + \frac{4\pi}{f_T}(\mu e^\nu - r^2 P_\perp + P_r e^\lambda - \frac{\check{q}^2}{\tilde{r}^4}) + \varsigma_1\right]dr, \quad (31)$$

where ς_1 value is stated in Appendix I. The above-mentioned equations are well-matched within [1] and its physical relevance can be studied in presence of EMF and $f(T)$ gravity.

5 Evaluation of structure parameters in $f(T)$ gravity

The work is about the review of anisotropic fluids concerning scalars which are acquired by the orthogonal division of the intrinsic curvature because of $f(T)$ gravity and EMF. The concept of orthogonal splitting of the Riemann tensor to obtain the structure scalars is the pioneering work of Herrera et al. [61]. They used spherically symmetric geometry linked with anisotropic dissipative fluid content. They performed the orthogonal splitting for the Riemann tensor and attained five structure scalars in the framework of GR. These are trace and trace-free components of tensors $X_{\alpha\beta}$, $Y_{\alpha\beta}$ and $Z_{\alpha\beta}$. These scalars have a direct influence on the physical properties of the fluid content. Few realistic solutions in terms of these scalars are also evaluated by these authors. To understand the concept of stability without shear-free constraint, Herrera et al. [62] evaluated the shear evolution equation in terms of scalar Y_{TF}. They found that the scalar Y_{TF} may be defined through the pressure anisotropy and the Weyl tensor or as in form of density inhomogeneity, pressure anisotropy, and dissipative parameters.

Herrera et al. [63] further extended their idea to acquire structure scalars from the orthogonal splitting of the Riemann tensor in the presence of charged dissipative fluid. The different scalars are labeled as X_T, X_{TF}, Y_T, Y_{TF} and Z in this case. They studied their physical features and deduced that each of them has different characteristics. The fluid energy density is controlled by the factor X_T while Z is responsible for all feasible dissipative fluxes. To understand inhomogeneities in the energy density of the fluid, the factor X_{TF} plays a significant role without contributing to dissipation. While the effect of pressure anisotropy and density inhomogeneity on the value of the Tolman mass is studied with the help of factor Y_{TF}. The factor Y_T depicts proportional relation with Tolman mass density. They determined that only factors Y_T and Y_{TF} are true candidates to understand the evolution of shear and expansion. Each scalar plays a significant role in describing fluid properties. With the help of these structure scalars, the evolution and structure of self-gravitating fluids can be studied [64]. One of the scalars Y_{TF} is also referred to as the complexity factor and is the subject of vast study. Herrera et al. [65] extended his strategy to study self-gravitating systems in fulfillment of quasi-homologous conditions and vanishing complexity factor. They calculated some models under these constraints and found that the evolution of spherical fluids is explained with help of few models. The contribution of the scalars in the inspection of fluid content is concerned deeply by [66–68]. In this framework, we employ three tensors with subsequent expressions as

$$Y_{\tau\phi} = R_{\tau\sigma\phi\rho}V^\sigma V^\rho,$$
$$Z_{\tau\phi} =^\star R_{\tau\sigma\phi\rho}V^\sigma V^\rho = \frac{1}{2}\eta_{\tau\sigma\pi\kappa}R^{\pi\kappa}_{\ \ \phi\rho}V^\sigma V^\rho,$$
$$X_{\tau\phi} =^\star R^\star_{\tau\sigma\phi\rho}V^\sigma V^\rho = \frac{1}{2}\eta_{\tau\sigma}^{\ \ \pi\kappa}R^\star_{\pi\kappa\phi\rho}V^\sigma V^\rho,$$

alongside $R^\star_{\tau\phi\sigma\rho} = \frac{1}{2}\eta_{\pi\kappa\sigma\rho}R_{\tau\phi}^{\ \ \pi\kappa}$. Turning into account $f(T)$-Maxwell equations, we observe

$$R^{\tau\sigma}_{\ \ \phi\rho} = C^{\tau\sigma}_{\ \ \phi\rho} + 16\pi(T^{(T)} + T^{(m)} + E^{(m)})^{[\tau}_{[\phi}\delta^{\sigma]}_{\rho]} + 8\pi T(\frac{1}{3}\delta^\tau_{[\phi}\delta^\sigma_{\rho]} - \delta^{[\tau}_{\ [\phi}\delta^{\sigma]}_{\rho]}). \quad (32)$$

Using EMF along side stress–energy tensor components, we observe

$$R^{\tau\sigma}{}_{\phi\rho}=R^{\tau\sigma}_{(I)\phi\rho}+R^{\tau\sigma}_{(II)\phi\rho}+R^{\tau\sigma}_{(III)\phi\rho},$$

here

$$\begin{aligned}R^{\tau\sigma}_{(I)\phi\rho}&=16\pi\left(\mu+\frac{\check{q}^2}{8\pi r^4}\right)V^{[\tau}V_{[\phi}\delta^{\sigma]}_{\rho]}\\&-16\pi\left(P+\frac{\check{q}^2}{8\pi r^4}\right)h^{[\tau}_{[\phi}\delta^{\sigma]}_{\rho]}\\&+8\pi\left(\mu-3P+\frac{(Tf_T-f)}{16\pi}\delta^{\tau\phi}-\frac{f_{TT}}{8\pi}S^{\tau}\nabla T\right)\\&+8\pi\left(\frac{1}{3}\delta^{\tau}_{[\phi}\delta^{\sigma}_{\rho]}-\delta^{[\tau}_{[\phi}\delta^{\sigma]}_{\rho]}\right),\\R^{\tau\sigma}_{(II)\phi\rho}&=16\Big(\Pi-\frac{\check{q}^2}{4\pi r^4}{}^{[\tau}_{[\phi}\delta^{\sigma]}_{\rho]}+\frac{1}{8\pi}(\frac{(Tf_T-f)}{2}\delta^{[\tau}_{[\phi}\delta^{\sigma]}_{\rho]}\\&-f_{TT}S^{[\tau}_{[\phi}\delta^{\sigma]}_{\rho]}\nabla_T\Big),\\R^{\tau\sigma}_{(III)\phi\rho}&=4V^{[\tau}V_{[\phi}E^{\sigma]}_{\rho]}-\epsilon^{\tau\sigma}_{\kappa}\epsilon_{\phi\rho\psi}E^{\kappa\psi}.\end{aligned}$$

The overhead equations provide the pave to calculate the expressions of three tensors in the way spotted as

$$X_{\tau\phi}=-\frac{8\pi}{3}|\mu|h_{\tau\phi}+4\pi\left(\Pi_{\tau\phi}-\frac{\check{q}^2}{r^4}\right)-E_{\tau\phi}+\varsigma_2, \tag{33}$$

$$Y_{\tau\phi}=\frac{4\pi}{3}(-|\mu|+3P)h_{\tau\phi}+4\pi\left(\Pi_{\tau\phi}-\frac{\check{q}^2}{r^4}\right)+E_{\tau\phi}+\varsigma_3, \tag{34}$$

$$Z_{\tau\phi}=0, \tag{35}$$

where, the expression for ς_2 and ς_3 is stated in Appendix I. In their trace and trace-free components, the overhead Eqs. (33) and (34) can be turned down as

$$X_{\tau\phi}=X_T\frac{h_{\tau\phi}}{3}+X_{TF}\left(K_\tau K_\phi+\frac{h_{\tau\phi}}{3}\right);$$
$$Y_{\tau\phi}=Y_T\frac{h_{\tau\phi}}{3}+Y_{TF}\left(K_\tau K_\phi+\frac{h_{\tau\phi}}{3}\right).$$

The value for scalars is given as

$$X_T=-8|\mu|\pi+\frac{\check{q}^2}{r^4}+\varsigma_4;\quad X_{TF}=4\pi\Pi-\frac{\check{q}^2}{r^4}-\varepsilon+\varsigma_5;$$
$$Y_T=4\pi(-|\mu|+3P)+\frac{\check{q}^2}{r^4}+\varsigma_6;$$
$$Y_{TF}=4\pi\Pi-\frac{\check{q}^2}{r^4}+\varepsilon+\varsigma_7, \tag{36}$$

where, the expression for ς_4, ς_5, ς_6 and ς_7 is stated in Appendix I. The values of trace-free and trace scalars presented in Eq. (36) with components of EMF and $f(T)$ dark source factors is well-matched in [1]. Turning into account the value of ε from Eq. (24) into the trace free scalar Y_{TF} from Eq. (36) consequences as

$$\begin{aligned}Y_{TF}&=4\pi\Pi+\varsigma_7+\frac{4\pi}{f_T}\left[\left(\mu e^{\nu}-r^2P_\perp+P_re^{\lambda}\right)\right.\\&\left.+\int_0^r\left(\Theta^{(T)}_{11}-2\Theta^{(T)}_{00}-\Theta^{(T)}_{22}\right)dr\right]\\&-\frac{4\pi}{r^3}\left[\int_0^r\left(\frac{\Theta^{(T)}_{00}}{f_T}\right)'+\left(\frac{e^{\nu}|\mu|}{f_T}\right)'\right]\tilde{r}^3d\tilde{r}\\&+\frac{3}{2r^3}\int_0^r\frac{\check{q}^2}{r^2}dr-\frac{\check{q}^2}{r^4}.\end{aligned} \tag{37}$$

We observe from Eq. (36) that

$$Y_{TF}+X_{TF}=8\pi\Pi+\varsigma_5+\varsigma_7-2\frac{\check{q}^2}{r^4}.$$

To reveal the physical implication of Y_{TF} and Y_T, we employ Eq. (31) with (36) and observe

$$\begin{aligned}m_T&=(m_T)_{\Psi e}\left(\frac{r}{r_\Psi}\right)^3+\frac{cosh\pi-1}{2}\\&\times\int_r^{r_{\Psi e}}f_T\times\frac{e^{\nu+\lambda}}{\tilde{r}}\left(Y_{TF}+\frac{\check{q}^2}{\tilde{r}^4}+\varsigma_8\right)d\tilde{r},\\m_T&=\frac{cosh\pi-1}{2}\int_0^r\tilde{r}^2\times e^{\frac{\nu+\lambda}{2}}\left(Y_T+\varsigma_6\right)d\tilde{r},\end{aligned} \tag{38}$$

where $\varsigma_8=-\varsigma_7+\varsigma_{7I}$, and value of ς_{7I} is stated in Appendix I. We interpose the impact of Y_{TF} on the total energy budget in terms of anisotropic matter, dark source terms of $f(T)$, components of EMF, and density inhomogeneity. It also narrates the proportional association of Y_T with dark source terms of $f(T)$ and total energy budget.

6 Charged static form of solutions

This portion of article is allocate to deduce results for the static fluid content encompass with hyperbolic symmetry in occupancy of EMF and $f(T)$ gravity. These solutions are evaluated in frame of two generating functions which further utilized to explore more explicit solutions with different conditions imposed on them. This approach is well-matched in [69]. From Eqs. (13) and (14), we observe

$$\begin{aligned}\frac{8\pi}{f_T}(P_r-P_\perp)-2\frac{\check{q}^2}{r^4}&=\left(\frac{e^{-\lambda}+1}{r^2}\right)\\&+\frac{e^{-\lambda}}{2}\left(\frac{\lambda'}{r}+\frac{\lambda'\nu'}{2}-\frac{\nu'^2}{2}-\nu''+\frac{\nu'}{r}\right)\end{aligned}$$

$$+\frac{8\pi}{f_T}\left(\Theta_{22}^{(T)}-\Theta_{11}^{(T)}\right).$$

By taking advantage of novel functions described in [1] as $e^{-\lambda}=y;\quad \frac{\nu'}{2}=-(\frac{1}{r}-z)$, the above equation is observed as

$$\frac{2}{z}\left(-\frac{8\pi\Pi}{f_T}+\frac{1}{r^2}+\frac{2\check{q}^2}{r^4}\right)=y'-y\left[\frac{6}{r}-2z-\frac{2z'}{z}-\frac{4}{zr^2}\right]$$
$$-\frac{2}{z}\times\frac{8\pi}{f_T}\left(\Theta_{22}^{(T)}(y)-\Theta_{11}^{(T)}(y)\right). \quad (39)$$

Integration of above yields

$$e^{\lambda}=\frac{z^2e^{\int\frac{4}{r^2z}+2z}dr}{r^6\left(2\int(\frac{1+\frac{\check{q}}{r^2}-8\pi\Pi r^2}{r^2f_T}\right)+\frac{8\pi}{f_T}\left(\Theta_{22}^{(T)}(y)-\Theta_{11}^{(T)}(y)\right)e^{\int\frac{4}{r^2z}+2z}dr+A_1}.$$

Here, constant of integration is labeled with A_1. From this it is evident that in case of static anisotropic matter content, novel generating functions can be picked up to describe solutions in form of them. In our scenario, the role of dark source components of $f(T)$ gravity and EMF components has significance. The physical variables in this argument observe as

$$\frac{m'(r)}{r^2}+\frac{\check{q}\check{q}'}{r^3}=\frac{4\pi}{f_T}\left[|\mu|e^{\nu}-\Theta_{00}^{(T)}\right],$$
$$\frac{4\pi}{f_T}\times\frac{r}{(2m-r)}\times\left[P_r-\frac{1}{16\pi}(Tf_T-f)\right]$$
$$=\frac{z(2m-r+\frac{\check{q}^2}{r})-m+r}{r^3},$$
$$8\pi P_{\perp}=\frac{f_T}{r^2}\left[\left(\frac{2m}{r}+\frac{\check{q}^2}{r^2}-1\right)\left(z^2+z'+\frac{1}{r^2-\frac{z}{r}}\right)\right.$$
$$\left.+z\left(\frac{m'}{r}+\frac{\check{q}\check{q}'}{r^2}-\frac{m}{r^2}-\frac{\check{q}^2}{r^3}\right)-\frac{\check{q}^2}{r^4}+\frac{r^2}{2f_T}\zeta_1\right],$$

where value of ζ_1 is stated in Appendix I.

6.1 Conformally flat static fluid

Here, we utilize one generating function along with an auxiliary ansatz to evaluate the solutions. As, we explained in preceding section that the conformal tensor has only one component, i.e., electric and magnetic component diminish in spherically symmetric form. The reduction of conformal scalar, i.e., ε=0 yield an equation of the form

$$\left(\frac{e^{-\lambda}+1}{r^2}\right)'-\left(\frac{e^{-\lambda}\nu'}{2r}\right)'e^{-(\nu+\lambda)}-\left(\frac{e^{-\lambda}\nu'}{2r}\right)'=0. \quad (40)$$

Now, introducing novel function as used in [1] of form $\mathbf{w}\frac{\nu'}{2}=\mathbf{w}';\quad y=e^{-\lambda}$, Eq. (40) turn up as

$$y'+\frac{2y\left(\mathbf{w}''-\frac{\mathbf{w}'}{r}+\frac{\mathbf{w}}{r^2}\right)}{\left(\mathbf{w}'-\frac{\mathbf{w}}{r}\right)}+\frac{2\mathbf{w}}{r^2\left(\mathbf{w}'-\frac{\mathbf{w}}{r}\right)}=0,$$

and the solution of above equation is attained after integration in subsequent form

$$y=e^{-\int n_1(r)dr}\left[\int e^{\int n_1(r)dr}n_2(r)dr+A_2\right], \quad (41)$$

here, constant of integration is labeled as A_2, and n_1, n_2 defined as

$$n_1=2\frac{d}{dr}\left[ln\left(\mathbf{w}'-\frac{\mathbf{w}}{r}\right)\right];\quad n_2=-\frac{2\mathbf{w}}{r^2\left(\mathbf{w}'-\frac{\mathbf{w}}{r}\right)}.$$

In terms of native variables, Eq. (41) is reconstructed as

$$-r\left(\frac{1}{r}-\frac{\nu'}{2}\right)=e^{\frac{\lambda}{2}}\sqrt{e^{-\nu}\beta_1r^2-1}, \quad (42)$$

here, β_1 serves as constant of integration and whose value is attained with the aid of matching conditions [58], giving

$$\beta_1=\frac{(3M+\frac{2Q^2}{r_{\Psi e}}-r_{\Psi e})^2+r_{\Psi e}(2M+\frac{Q^2}{r_{\Psi e}}-r_{\Psi e})}{r_{\Psi e}^4}.$$

After integration of Eq. (42), we observe

$$e^{\nu}=r^2\beta_1Sin^2\left(\int e^{\frac{\lambda}{2}}r^{-1}dr+\zeta_2\right),$$

here, ζ_2 is used as constant of integration, which with the aid of matching conditions in [58], gives

$$\zeta_2=-\left\{\left(\int e^{\frac{\lambda}{2}}r^{-1}dr\right)-\arcsin\right.$$
$$\left.\times\left[r_{\Psi e}\sqrt{\frac{(\frac{2M}{r_{\Psi e}}+\frac{Q^2}{r_{\Psi e}^2}-1)}{(3M+\frac{2Q^2}{r_{\Psi e}}-r_{\Psi e})^2+r_{\Psi e}(2M+\frac{Q^2}{r_{\Psi e}}-r_{\Psi e})}}\right]\right\}.$$

In the scenario of conformally flat static fluid, only one generating function works out. So, in this context, a subsidiary condition is mandatory. For this , we place $P_r=0$.

Using this condition in Eq. (13), we observe

$$\nu' = -\left(\frac{Tf_T - f}{2}\right) \times \frac{e^{2\lambda}}{f_T} - \left(\frac{1 + \frac{\check{q}^2}{r^2} + e^{\lambda}}{r}\right). \quad (43)$$

Replacing above value in expression of the conformal tensor and setting up $\varepsilon = 0$, we acquire equation of the form

$$8(e^{\lambda} + 1) + (1 + \frac{\check{q}^2}{r^2} + e^{\lambda})^2 - re^{\lambda}\lambda' + 3\lambda' r + \left(1 - \frac{3r\lambda'}{2} - r\right) \times \frac{Tf_T - f}{f_T} r^2 e^{2\lambda} - r^3 e^{2\lambda}\left(\frac{Tf_T - f}{f_T}\right)' - \frac{r^4 e^{4\lambda}}{2f_T^2}(Tf_T - f)^2 - \frac{8\check{q}^2}{r^2} - \frac{4\check{q}\check{q}'}{r} + \frac{\lambda'\check{q}^2}{r} = 0.$$

Setting $2g - 1 = e^{\lambda}$ in above equation, we obtain the subsequent form

$$-rg'(3g - 2) + g(9g - 4) + \zeta_3 = 0. \quad (44)$$

Where value of ζ_3 is stated in Appendix I. The integration of Eq. (44) yield

$$A_3 r^6 = \frac{4g^3}{(9g - 4)} + r^6 \int \zeta_4 dr,$$

here A_3 serve as constant of integration and ζ_4 is stated in Appendix I. The combination of both Eqs. (42) and (43) resulted as

$$e^{\nu} = \frac{r^2\beta_1(2g - 1)}{g(9g - 4) + (2g - 1)^2\frac{\check{q}^2}{4r^2} - \frac{\check{q}^2(2g-1)(1-3g)}{r^2} + \zeta_5},$$

where ζ_5 has value stated in Appendix I. The subsequent relations for physical quantities are attained in this case

$$|\mu| = \frac{3f_T}{2\pi r^2}\left(\frac{g(9g - 4) + \zeta_5 + H_1}{(3g - 2)}\right) + \zeta_6 - \frac{\check{q}^2}{2r^4};$$

$$P_{\perp} = \frac{3gf_T}{4\pi r^2}\left[\frac{41g^2 - 2g(9g^2 - 8) + \zeta_7(1 + 2g)}{r^2(2g - 1)(3g - 2)}\right] + \zeta_8,$$

where H_1, ζ_5, ζ_6, ζ_7, ζ_8 has value stated in Appendix I. It is indicated from above equations that e^{ν} exhibit positive nature when $g > \frac{2}{3}$. This result in presence of EMF and $f(T)$ gravity are well matched in [1] and the generating functions for this model are as

$$z = \frac{(g - 1)}{r(2g - 1)} - \frac{r}{4f_T(2g - 1)^2} - \frac{\check{q}^2}{r^3};$$

$$\Pi(r) = -\left\{\frac{3gf_T}{4\pi r^2}\left[\frac{41g^2 - 2g(9g^2 - 8) + \zeta_7(1 + 2g)}{r^2(2g - 1)(3g - 2)}\right] + \zeta_8\right\}.$$

6.2 Model with diminishing complexity factor

In this section, we analyze the model matching with condition $Y_{TF} = 0$, which is a diminishing complexity factor condition. Due to the existence of an infinite possible solution, we need to emphasize certain conditions to attain a specific one. For this, we impose that P_r=0. Satisfying above condition, Eq. (13) turn out as

$$\nu' = -\left\{\frac{2g}{r(2g - 1)} + \frac{(Tf_T - f)r}{2(2g - 1)^2 f_T} + \frac{\check{q}^2}{r^3}\right\}, \quad (45)$$

where again using $2g-1 = e^{-\lambda}$ for g and taking into account of constraint Y_{TF}=0 in Eq. (38), we observe

$$m_T = (m_T)_{\Psi e}(\frac{r}{r_{\Psi}})^3 + \frac{cosh\pi - 1}{2} \times \int_r^{r_{\Psi e}} f_T \times \frac{e^{\nu+\lambda}}{\tilde{r}}\left(\frac{\check{q}^2}{\tilde{r}^4} + \varsigma_8\right)d\tilde{r}. \quad (46)$$

The value of e^{ν} is attainted by taking into account Eqs. (27), (45) and (46) observe as

$$e^{\nu} = \frac{1}{(cosh\pi - 1)^2} \times \frac{1}{(2g - 1) \times 4r^2g^2 + 2\check{q}^2 g + \frac{(2g-1)\check{q}^4}{r^3}} + (m_T)_{\Psi e}\left(\frac{r}{r_{\Psi}}\right)^3 + \frac{cosh\pi - 1}{2}\int_r^{r_{\Psi e}} f_T \times \frac{e^{\nu+\lambda}}{\tilde{r}}\left(\frac{\check{q}^2}{\tilde{r}^4} + \varsigma_8\right)d\tilde{r}.$$

The implication of constraint Y_{TF} also yield

$$r \times g'(1 - g) + g \times (5g - 2) + (2g - 1)r^2\zeta_9 = 0.$$

The value of ζ_9 is stated in Appendix I. The integration of the above equation turn into

$$A_4 r^{10} = \frac{g^5}{(5g - 2)^6} + r^{10}\int \zeta_{10} dr, \quad (47)$$

here, constant of integration is labeled as A_4 and value of ζ_{10} is stated as $\zeta_{10} = \frac{(2g-1)\times g^4}{(5g-2)}\zeta_9$. The physical variables in this case possess the subsequent relations

$$|\mu| = \frac{3 \times f_T}{4\pi r^2}\left(\frac{3g - r^2\zeta_9}{6}\right) - \Theta_{00}^{(T)};$$

$$P_{\perp} = -\frac{3g^2 \times f_T}{8\pi r^2} \times \frac{1}{(1 - g)} + (2g - 1)\zeta_9 - \frac{\Theta_{00}^{(T)}}{r^2}.$$

The results in presence of EMF components and $f(T)$ gravity are well-matched in [1] and the generating functions for this model are as

$$z=-\frac{r}{4f_T\times(2g-1)^2}+\frac{(g-1)}{(2g-1)r}-\frac{\check{q}^2}{2r^3},$$
$$\Pi(r)=\frac{3g^2f_T}{8\pi r^2}\times\frac{1}{(1-g)}-(2g-1)\zeta_9+\frac{\Theta^{(T)}_{00}}{r^2}.$$

6.3 Stiff matter

This part of the article is concerned to evaluate few solutions which fulfilled the stiff matter equation of state (SMEQ's). It chronically relates an increase in pressure with an increase in density. That is the reason the material that possesses this property is hard to compress and furnish more assistance opposite to gravity. This concept was first brought up by [70], where pressure and energy density are retained equal. This argument turn Eq. (15) into

$$P_r^{'}+\frac{h_1}{(8\pi)^2}+\frac{2\Pi}{r}-\frac{\check{q}\check{q}'}{4\pi r^4}=0. \quad (48)$$

To achieve solution, we imposed certain constraints. One as $P_\perp=0$ and afterward $Y_{TF}=0$ so that

- At $P_\perp=0$. Imposing this constraint on Eq. (48) and afterward integrating, we attain

$$P_r=\frac{rh_1}{3(8\pi)^2}+\frac{B_1}{r^2}-\frac{r^2}{(8\pi)^2}\int_0^r\frac{r^3h_1'}{3}dr+\frac{1}{r^2}\int_0^r\frac{\check{q}\check{q}'}{4\pi r^4}dr\Longrightarrow|\mu|=\frac{rh_1}{3(8\pi)^2}+\frac{B_1}{r^2}-\frac{r^2}{(8\pi)^2}\int_0^r\frac{r^3h_1'}{3}dr+\frac{1}{r^2}\int_0^r\frac{\check{q}\check{q}'}{4\pi r^4}dr,$$

where B_1 is constant of integration. The previous equations in association with Eqs. (16), (17) and (19) observed

$$m=\frac{4\pi B_1 r}{f_T}-\int_0^r\frac{\check{q}q'}{r}dr+\chi_1;$$
$$e^{-\lambda}=\frac{8\pi B_1 r}{f_T}-\frac{2}{r}\int_0^r\frac{\check{q}q'}{r}dr-1-\frac{\check{q}^2}{r^2}+\frac{2\chi_1}{r},$$

the value of χ_1 is stated in Appendix I. The generating functions in terms of this model turn out as

$$\Pi=\frac{rh_1}{3(8\pi)^2}+\frac{B_1}{r^2}-\frac{r^2}{(8\pi)^2}\int_0^r\frac{r^3h_1'}{3}dr+\frac{1}{r^2}\int_0^r\frac{\check{q}\check{q}'}{4\pi r^4}dr;\quad z=\frac{1}{r}+\frac{\nu'}{2}.$$

where $\nu'=\frac{2\left\{4\pi f_T(P_r r^4-(2m-r)r^2\Theta^T_{11}-\frac{\check{q}^2}{r})-m\right\}}{r(\frac{8H\pi r}{f_T}+2\chi_1)}$.

- At $Y_{TF}=0$.
Now, the addition of diminishing complexity factor in SMEQ's will be analyzed. For this purpose, we make use of this constraint into Eq. (37) and surrogate into Eq. (48), so that

$$P_r''+\frac{3P_r'}{r}+\frac{5\chi_2'}{r}-\frac{2\chi_2}{r^2}+\frac{h_1'}{(8\pi)^2}+\theta_1=0,$$

here, the value of θ_1 and χ_2 is stated in Appendix I. The solution of above equation becomes

$$P_r=-c+\frac{d}{r^2}+\theta^\star-\chi_3,$$

where, c and d serve as constant of integration while χ_3 is stated in Appendix I. Now, matching Eqs. (16) and (17) which acknowledge the above conditions, and provide a pave to calculate the value of λ as stated

$$m=-\frac{4\pi r}{f_T}\left(\frac{r^2c}{3}-d\right)+4\pi\int_0^r\left(\theta^\star r^2+\frac{\check{q}\check{q}'}{4\pi r}\right)dr+\chi_4;$$
$$\lambda=-ln\left[\frac{8\pi}{f_T}\times\left(-\frac{r^2c}{3}+d\right)-1+\frac{2r}{\chi_4}\right].$$

Taking into account the matter content enclosed from outward surface Ψ^e, we acquire

$$P_r=-d\left(\frac{1}{r^2_{\Psi^e}}-\frac{1}{r^2}\right)+2\chi_3;$$
$$m=-\frac{4\pi rd}{3r^2_{\Psi^e}}\left(r^2-3r^2_{\Psi^e}\right)+\int_0^r\frac{\check{q}\check{q}'}{r}dr+\chi_5.$$

The value of χ_5 is stated in Appendix I. Now, we build appealing relationship among r_{Ψ^e}, $f(T)$, EMF and P_r as follows

$$\frac{4\pi P_r r^3}{f_T}-r^2(2m-r)\Theta^{(T)}_{11}-\frac{\check{q}^2}{r}=\frac{4\pi rd}{f_T}(1-2r^2\Theta^{(T)}_{11})-\frac{4\pi r^3d}{f_Tr^2_{\Psi^e}}\left(1-\frac{2r^2\Theta^{(T)}_{11}}{3}\right)+\frac{8\pi r^3\chi_3}{f_T}-2\chi_5r^2\Theta^{(T)}_{11}+r^3\Theta^{(T)}_{11}-\frac{\check{q}^2}{r}-\int_0^r\frac{\check{q}\check{q}'}{r}dr.$$

While the tangential pressure in this way secure the following form

$$P_\perp=-\frac{d}{r^2_{\Psi^e}}-\frac{\check{q}\check{q}'}{4\pi r^3}+\chi_6,$$

where the value of χ_6 is stated in Appendix I.

7 Closing comments

The analysis to explore the features of static matter content equipped with hyperbolical symmetry in presence of EMF and $f(T)$ gravity, where T acts as torsion scalar, is done in this article. We took motivation from Herrera's strategy [1] and elaborate this work in the context of $f(T)$ gravity and EMF. We accounted for inner geometry as static hyperbolically symmetry and combine it with outward geometry viewed as Reissner–Nordström with aid of Darmois constraints computed in [58]. We deeply analyze their physical attributes. First of all, We distinguished the stress–energy tensor by carefully considering the ingredients of the tetrad field in the Minkowski co-ordinate frame, commencing with Modified field equations. We calculated a viable formulation of mass utilizing Tolman and Misner-Sharp mass from the viewpoint of $f(T)$ gravity and along with EMF, and developed some relationships between them. We build up the relationship between the conformal tensor and the intrinsic curvature which provides a pave to evaluate structure scalars in $f(T)$ gravity. We discussed an accurate finding to analyze the impact of the EMF in the composition of $f(T)$ gravity, and found few effective analytical answers. We concluded the following remarks:

- The first observation about this sort of matter content is that they possess the negative form of energy density. The anisotropic nature of pressure has a significant character in this study and exhibits the property that stresses are not equal in it. In this scenario, the central space is void as it is not covered by the matter.
- The use of two different masses is analyzed in detail. Both of these two definitions of masses match when they are calculated at the boundary of fluid content. They exhibit different values even when a single piece is inside the fluid excluding isotropic pressure and a homogenous form of energy density.
- The total energy budget (or Tolman mass) possess negative value if $(\frac{4\pi}{f_T}(p_r r^3 - r^2(2m - r)\Theta_{11}^{(T)})) - \frac{\check{q}^2}{r} < m)$ as WEC invade in our framework. Also, many astrophysical phenomena possess negative energy density. It is deduced that with the advent of negative density, this sort of fluid is supposed to surpass extreme physical conditions, enabling quantum impacts to be detected.
- The relationship between the conformal tensor and the intrinsic curvature has significance in this analysis. The conformal tensor infers knowledge about the tidal forces. Among 256 components of the conformal tensor, 10 are only independent of each other in 4D. With the aid of a four-velocity vector, these 10 tensors are incorporated into a two-second rank tensor named as electric and magnetic part of the conformal tensor. In our case, the only non-vanishing constituent is electric and it provided help in the establishment of the relationship between the conformal tensor and the intrinsic curvature.
- We used orthogonal division of the intrinsic curvature to construct the structure scalars in $f(T)$ gravity in presence of EMF. First of all, we found the three tensors and then break them into their trace-free and trace components of the form X_{TF}, Y_{TF}, X_T and Y_T, respectively. As structure scalars provide information about the features of fluid contents. So, in our framework, we developed the relationship of the total energy budget in terms of Y_{TF} and Y_T. We inferred that homogeneity and inhomogeneity of energy density, presence of EMF, and dark source components of $f(T)$ have substantial impact on our analysis.

We wrapped this discussion as, a detailed description to analyze the features of static fluid content which are equipped with hyperbolical symmetry in the framework of $f(T)$ gravity and in presence of EMF. For this concern, generating functions are used. We found few effective analytical answers and the relations of physical quantities in the framework of $f(T)$ gravity and in presence of EMF. The substantial impact of EMF and $f(T)$ dark source components are analyzed. All the cases deal with a void central region in this sort of fluid content.

Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors' comment: All data generated or analysed during this study are included in this accepted manuscript.]

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Appendix I

The value of dark source term h_1, $\Theta_{11}^{(T)}$, $\Theta_{00}^{(T)}$, $\Theta_{22}^{(T)}$ and $k_1(r)$ are as below

$$h_1 = \frac{e^{-2\lambda}\nu'}{32\pi}\left(\frac{2}{r} - \frac{\nu'}{4}\right)T' - \frac{e^{-\lambda}\lambda'}{16\pi}$$

$$\times\left\{(Tf_T-f)-\{Tf_T-f\}_{,1}\right\}$$
$$-\frac{e^{-2\lambda}}{r}\left\{\frac{f_{TT}}{16\pi}\left(\nu'-\frac{3}{r}\right)\right\},$$
$$\Theta_{00}^{(T)}=\frac{e^{\nu}}{16\pi}\left\{(Tf_T-f)+e^{-\lambda}f_{TT}\left(\frac{2}{r}-\frac{\nu'}{4}\right)T'\right\};$$
$$\Theta_{11}^{(T)}=-\frac{e^{\lambda}}{8\pi}\left(\frac{Tf_T-f}{2}\right),$$
$$\Theta_{22}^{(T)}=\frac{r^2}{16\pi}\left\{-(Tf_T-f)+\frac{f_{TT}}{2}\left(\nu'e^{\lambda}-\frac{3}{r}e^{-\lambda}\right)T'\right\};$$
$$K_1(r)=e^{\frac{(\nu+\lambda)}{2}}\left(\Theta_{11}^{(T)}+\Theta_{00}^{(T)}+\Theta_{22}^{(T)}\right)d\tilde{r}^3.$$

The value of ς_1, ς_2, ς_3, ς_4, ς_5, ς_6, ς_7 and ς_{7I} is listed below

$$\varsigma_1=\frac{4\pi}{f_T}\int_0^r\left(\Theta_{11}^{(T)}-2\Theta_{00}^{(T)}-\Theta_{22}^{(T)}\right)dr-4\pi\Theta_{22}^{(T)}$$
$$-4\pi\Theta_{00}^{(T)}-\frac{1}{4\pi r^4}\left\{\int_0^r\{k_1(r)d\tilde{r}\}_{,1}+k_1(r)\right\},$$
$$\varsigma_2=\frac{1}{2}\left\{\left(\frac{Tf_T-f}{2}\right)\delta_{\tau\phi}+f_{TT}S_{\tau\phi\lambda}\nabla_\lambda T\right\}$$
$$-\frac{1}{3}\left\{f_{TT}S^{\tau\lambda}\nabla_\lambda T-\left(\frac{Tf_T-f}{2}\right)\right\}h_{\tau\phi},$$
$$\varsigma_3=\frac{1}{2}\left\{\left(\frac{Tf_T-f}{2}\right)g_{\tau\phi}-f_{TT}S^{\lambda}_{\phi\tau}\nabla_\lambda T\right.$$
$$\left.+\frac{f_{TT}}{2}(V_\phi+V_\tau-g_{\tau\phi}V^\sigma)S^{\lambda}_{\tau}\nabla_\lambda T\right\}$$
$$-\frac{h_{\tau\phi}}{2}\left\{\left(\frac{Tf_T-f}{2}\right)-f_{TT}S^{\tau\lambda}\nabla_\lambda T\right\},$$
$$\varsigma_4=-\frac{1}{9}\left\{f_{TT}S^{\tau\lambda}\nabla_\lambda T-\left(\frac{Tf_T-f}{2}\right)\right\};$$
$$\varsigma_5=\frac{1}{2}\left\{f_{TT}S_{\tau\phi\tau}\nabla_\lambda T+\left(\frac{Tf_T-f}{2}\right)\delta_{\tau\phi}\right\},$$
$$\varsigma_6=\frac{1}{6}\left\{f_{TT}S^{\tau\lambda}\nabla_\lambda T-\left(\frac{Tf_T-f}{2}\right)\right\};$$
$$\varsigma_7=\frac{1}{2}\left\{\left(\frac{Tf_T-f}{2}\right)g_{\tau\phi}-f_{TT}S^{\lambda}_{\phi\tau}\nabla_\lambda T\right.$$
$$\left.+\frac{f_{TT}}{2}(V_\phi+V_\tau-g_{\tau\phi}V^\sigma)S^{\lambda}_{\tau}\nabla_\lambda T\right\},$$
$$\varsigma_{7I}=-4\pi\Theta_{22}^{(T)}-4\pi\Theta_{00}^{(T)}$$
$$-\frac{1}{4\pi r^4}\left\{+\int_0^r k_1(r)d\tilde{r}+k_1(r)\right\},$$

where the value of ζ_1, ζ_3, ζ_4, ζ_5, ζ_6, ζ_7, ζ_8, ζ_9 and H_1 is as under

$$\zeta_1=\left\{\left(Tf_T-f\right)+\frac{f_{TT}}{2}\left(3-2(2z-1\right)\right.$$
$$\left.\times\left(\frac{2m}{r}-1\right)^2T'\right\},$$
$$\zeta_3=-r^2\left(r-1+\frac{3r\lambda'}{2}\times\frac{-2g'}{(2g-1)}\right)\frac{Tf_T-f}{f_T}-\frac{r^4}{2f_T^2}$$
$$\times\left(\frac{Tf_T-f}{2g-1}\right)^2-r^3\left(\frac{Tf_T-f}{f_T}\right)'$$
$$+g^2\left(\frac{4\check{q}^4}{r^4}-\frac{8\check{q}^2}{r^2}-\frac{16\check{q}\check{q}'}{r}\right)$$
$$-g'\frac{2\check{q}^2}{r}-\left(\frac{4\check{q}\check{q}'}{r}-\frac{\check{q}^4}{r^4}-\frac{8\check{q}^2}{r^2}\right)$$
$$-g\left(\frac{4\check{q}^4}{r^4}-\frac{16\check{q}\check{q}'}{r}+\frac{12\check{q}^2}{r^2}\right),$$
$$\zeta_4=\frac{24}{r^4(9g-4)^2}\left[\left(\frac{Tf_T-f}{f_T}\right)'+2\left(\frac{Tf_T-f}{f_T}\right)\right]$$
$$+\frac{6}{r^3f_T(9g-4)^2(2g-1)}$$
$$\times\left[\frac{(Tf_T-f)^2}{f_T}-\frac{2(Tf_T-f)gg'}{r}\right]$$
$$+24r^2g^2\left[g^2\left(\frac{4\check{q}^4}{r^4}-\frac{8\check{q}^2}{r^2}-\frac{16\check{q}\check{q}'}{r}\right)\right.$$
$$-g'\frac{2\check{q}^2}{r}-\left(\frac{4\check{q}\check{q}'}{r}-\frac{\check{q}^4}{r^4}-\frac{8\check{q}^2}{r^2}\right)$$
$$\left.-g\left(\frac{4\check{q}^4}{r^4}-\frac{16\check{q}\check{q}'}{r}+\frac{12\check{q}^2}{r^2}\right)\right],$$
$$\zeta_5=\frac{(Tf_T-f)r^2}{2f_T}\left\{3g+\frac{(Tf_T-f)}{8f_T(2g-1)}-1\right\};$$
$$\zeta_6=-\frac{1}{16\pi}\{(Tf_T-f)+(2g-1)f_{TT}$$
$$\times\left(\frac{2}{r}-\frac{\zeta_5}{4(2g-1)\beta_1g^2}\right)T'\},$$
$$\zeta_7=-r^2\left(r-\frac{3r\lambda'}{2}\times\frac{2g'}{(2g-1)}-1\right)\left(\frac{Tf_T-f}{f_T}\right)$$
$$-\left(\frac{Tf_T-f}{f_T}\right)'r^3-\left(\frac{Tf_T-f}{2g-1}\right)^2\times\frac{r^4}{2f_T^2},$$
$$\zeta_8=\left(g(g-1)+\frac{gr}{(2g-1)^2}-\frac{rg'}{4(2g-1)^2}\right)$$
$$-\left(\frac{Tf_T-f}{f_T}\right)'\times\frac{r}{4(2g-1)}+\frac{Tf_T-f}{f_T}$$
$$+\left(\frac{Tf_T-f}{f_T}\right)^2\times\frac{r^2}{8(2g-1)^3}$$
$$+\frac{r^2}{f_T}\left\{(2g-1)\times\frac{f_{TT}}{2}\left(+\frac{2g}{(2g-1)r}+\frac{3}{r}\right.\right.$$
$$\left.\left.+\frac{r(Tf_T-f)}{2f_T(2g-1)^2}\right)T'+(Tf_T-f)\right\}$$
$$+\frac{\check{q}^2(9g^2-4g+H_1)}{(3g-2)r^4}-\frac{(2g-1)}{2}$$
$$\times\left\{\frac{2\check{q}\check{q}'}{r^3}+\frac{\check{q}^2}{r^3}-\frac{3\check{q}^2}{r^4}-\frac{3\check{q}^4}{2r^6}-\frac{2g\check{q}^2}{(2g-1)r^4}\right\},$$

$$
\begin{aligned}
H_1 =& g^2\left(\frac{4\check{q}^4}{r^4}-\frac{8\check{q}^2}{r^2}-\frac{16\check{q}\check{q}'}{r}\right)-g'\frac{2\check{q}^2}{r}\\
&-\left(\frac{4\check{q}\check{q}'}{r}-\frac{\check{q}^4}{r^4}-\frac{8\check{q}^2}{r^2}\right)\\
&-g\left(\frac{4\check{q}^4}{r^4}-\frac{16\check{q}\check{q}'}{r}+\frac{12\check{q}^2}{r^2}\right),\\
\zeta_9 =& \frac{f_T}{8\pi}\left[\left(\frac{Tf_T-f}{f_T}\right)'\times\frac{-r}{2(2g-1)}+\left(\frac{Tf_T-f}{f_T}\right)\right.\\
&\times\left(\frac{1}{2(2g-1)}+\frac{r^2}{8f_T(2g-1)^4}+2rg'(2g-1)\right.\\
&\left.\left.-\frac{1}{(2g-1)^2}\right)\right]+\frac{\check{q}^2}{r^4}\\
&-(2g-1)\left\{\frac{g\check{q}^2}{2r^4}+\frac{3\check{q}^2}{2r^4}-\frac{\check{q}\check{q}'}{r^3}-\frac{\check{q}^4}{4r^6}-\frac{\check{q}^2}{r^3}\right\}-\Theta_{00}^{(T)}.
\end{aligned}
$$

The value of $\chi_1, \chi_2, \chi_3, \chi_4, \chi_5, \chi_6, \theta_1$ and $\theta^\star$ is as given

$$
\begin{aligned}
\chi_1 &= -\frac{4\pi r^2}{f_T}\int_0^r\left(\frac{h_1 r}{3(8\pi)^2}-\frac{r^2}{(8\pi)^2}\int_0^r\frac{h_1' r^3}{3}dr+\frac{\Theta_{00}^{(T)}r^2}{f_T}\right)dr;\\
\chi_2 &= \frac{h_1 r}{3(8\pi)^2}-\frac{r^2}{(8\pi)^2}\int_0^r\frac{h_1' r^3}{3}dr,\\
\chi_3 &= \int\left[\frac{8\chi_2}{r}+6\int\left(\frac{\chi_2}{r^2}+\frac{h_1'}{(8\pi)^2}\right)dr\right];\\
\chi_4 &= 4\pi\int_0^r\left(\frac{r^2\Theta_{00}^{(T)}}{f_T}-\frac{r^2\chi_3}{f_T}+\left(\frac{1}{f_T}\right)'\left(\frac{r^3 c}{3}-rd\right)\right)dr,\\
\chi_5 &= -\frac{4\pi d}{r_{\Psi^e}^2}\int_0^r\left(\frac{1}{f_T}\right)'\left(rr_{\Psi^e}^2-\frac{r^3}{3}\right)dr\\
&\quad+4\pi d\int_0^r\left(\frac{r^2\Theta_{00}^{(T)}}{f_T}+2\chi_3\right)dr;\\
\chi_6 &= r(\chi_3'+\chi_3)-\frac{d}{r_{\Psi^e}^2}+\frac{h_1 r}{2(8\pi)^2},\\
\theta_1 &= \frac{\check{q}\check{q}'}{r^5\pi}-\frac{7\check{q}^2}{8r^6\pi}-\frac{\check{q}\check{q}''}{4r^4\pi}-\frac{\check{q}'^2}{4r^4\pi};\\
\theta^\star &= -c-\int\frac{1}{r^3}\left(\int r^3\theta_1\right)dr
\end{aligned}
$$

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