Informational interpretation on volume operator and physical bound on information

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Abstract

From informational viewpoint on volume operator in loop quantum gravity, following conjecture is suggested: a logic gate must occupy finite volume which has the minimum value, and number of logic gates contained within a region of space are bounded by the volume of the region.

1 Introduction

Since information is registered and processed by physical system, it is necessarily restricted by physical law. Several physical bounds on information are well-known. The most famous example is causality: information cannot travel faster than the speed of light. As for thermodynamics, it must consume free energy $kT \ln 2$ to erase a bit [1, 2]. In addition, entropy (or information) that can be contained within a region of a space are bounded by the area of the region as $S \leq A/4$, so-called holographic bound (see *e.g.* [3]). Margolus-Levitin theorem gives the bound on the maximum speed of dynamical evolution (*i.e.* bound on information processing rates) [4]. This bound depends on the energy of the system. Using this, Lloyd estimated the bound for the total number of elementary operations in the universe [5].

In this article, a new physical bound on information is conjectured: a logic gate must occupy finite volume and this volume has the minimum value. Then, number of logic gates contained within a region of a space are bounded by the volume of the region.

2 Informational viewpoint on loop quantum gravity

At the beginning, we review some facts about loop quantum gravity (for detail, see *e.g.* [6]). Loop quantum gravity is the nonperturbative quantization of the general relativity as a diffeomorphism-invariant gauge theory. Its quantum states are gauge invariant and diffeo-invariant. The spin network states satisfy such invariances, and they span the state space of loop quantum gravity as the ortho-normal bases. A spin network state is labeled by a spin network. A spin network is a graph with edges labeled by half-integer spin and can be interpreted as a linear combination of some loops. As a functional of connection field, a spin network state takes value of the trace of the holonomy along its spin network.

In loop quantum gravity, area and volume are defined as operators. These operators geometrically operate on spin network states, and spin network states are eigen states of area and volume operators. Area operator for a surface is constructed from area integral with regularization. Its discrete eigenvalue is the form of $A = 8\pi\gamma l_{\text{Planck}}^2 \sum_{n=1}^{N} \sqrt{j_n(j_n+1)}$. Where $n = 1 \dots N$ labels the edges (of the spin network) crossing the surface, and each edge has spin j_n. The area of the surface is sum over the area of the edges crossing the surface. Volume operator for a region is also constructed from volume integral with regularization. The volume of the region is sum over the volume of vertices within the region. It has also discrete eigenvalues: 3-valent vertex has 0 volume, and 4(or more)-valent vertex has finite volume.

Now, let us see the loop quantum gravity from informational viewpoint. Consider a spin network state with spin 1/2 edge. How is this state different from the state without such edge as a functional of connection? The difference is (trace of) holonomy along that edge. By existence of the edge, this state can probe the connection there like as spin 1/2 degree of freedom. It seems there is 1 bit information along spin 1/2 edge.

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Then, consider the situation that the spin 1/2 edge across some surface. There is 1 bit information along the spin 1/2 edge and this edge produces area on the surface. It seems that a channel where 1 bit information can pass must have the area $\frac{\sqrt{3}}{4}l_0^2$ where $l_0^2 = 8\pi\gamma l_{\text{Planck}}^2$. This is a kind of holographic bound.

Finally, let us reinterpret the volume operator from informational viewpoint. The spin 1/2 edges can be regarded as 1 bit channels and an invertible logic gate such as controlled-NOT has at least 2 inputs and 2 outputs. Then, can 4-valent vertices be regarded as invertible logic gates? 4-valent vertices produce finite volumes under operation of the volume operator. Thus, the logic gate must occupy finite volume at least $\sqrt{\frac{\sqrt{3}}{8}}l_0^3$.

3 Summary and discussion

A new kind of physical bound on information is conjectured. A logic gate must occupy finite volume and this volume has the minimum value. Then, number of logic gates contained within a region of a space are bounded by the volume of the region.

Roughly estimation shows that computation with only 10^{183} logic gates can be executed in the universe. Of course this bound is not so severe on present CPUs. They have about 10^9 gates in about $10^{-6}m^3$ volume. It takes about 600 years to become effective even if "Moore' s law" is correct.

It should be mentioned that the dynamical feature is not treated here. The spin foam model might show the bound for executable operations in a finite spacetime region.

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