PTEP

Prog. Theor. Exp. Phys. **2022** 091E01(7 pages)
DOI: 10.1093/ptep/ptac114

A possible solution to the helium anomaly of EMPRESS VIII by cuscuton gravity theory

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Received July 15, 2022; Revised August 12, 2022; Accepted August 12, 2022; Published August 27, 2022

We discuss cosmology based on the cuscuton gravity theory to resolve the anomaly of the observational ^{4}He abundance reported by the EMPRESS collaboration. We find that the gravitational constant $G_{\rm cos}$ in the Friedmann equation should be smaller than Newton's constant $G_{\rm N}$ such that $\Delta G_{\rm N}/G_{\rm N} \equiv (G_{\rm cos} - G_{\rm N})/G_{\rm N} = -0.085^{+0.026}_{-0.028}$ (68% C.L.) in terms of big-bang nucleosynthesis, which excludes $\Delta G_{\rm N} = 0$ at more than 95% C.L. To fit the data, we obtain a negative mass squared of a non-dynamical scalar field with the Planck-mass scale as $\sim -\mathcal{O}(1)M_{\rm PL}^2(\mu/0.5M_{\rm PL})^4$ with the cuscuton mass parameter μ. This fact could suggest the need for modified gravity theories such as the cuscuton gravity theory with a quadratic potential, which can be regarded as the low-energy Hořava–Lifshitz gravity, and might give a hint of quantum gravity.

Subject Index E03

1. Introduction

Recently, the EMPRESS collaboration (EMPRESS VIII) newly observed 10 extremely metal-poor galaxies (EMPGs) with metallicity ($< 0.1Z_\odot$) using the Subaru telescope, and obtained data of the ^{4}He to hydrogen ratio (^{4}He/H) by measuring the He i λ10830 near-infrared emission [1]. By analyzing the data of 64 galaxies in total with 13 EMPGs (including the 10 new EMPGs), they estimated the primordial mass fraction of ^{4}He to be $Y_p = 0.2379^{+0.0031}_{-0.0030}$ by extrapolating the data into the value at zero metallicity (the oxygen to hydrogen ratio O/H $\to$ 0).

Comparing the data with theoretical predictions in the standard big-bang nucleosynthesis (BBN), they obtained the effective number of neutrino species to be $N_{\nu,\rm eff} = 2.41^{+0.19}_{-0.21}$ at 68% C.L. It is remarkable that this means that the standard value of $N_{\nu,\rm eff}$ predicted in the big-bang cosmology ($= N_{\nu,\rm eff,std} \simeq$ 3.044–3.046 [2–7]), is observationally excluded at more than 2σ.

Taking this discrepancy of $N_{\nu,\rm eff}$ in the standard big-bang cosmology seriously, we must consider modified theories beyond the standard model. The EMPRESS collaboration extended their framework to a new theory beyond the standard model by adding one more free parameter, the so-called "degeneracy parameter" ξ_{ν_e}, which means a non-zero lepton number in the electron neutrino sector. It has been known for a long time that a positive ξ_{ν_e} can reduce Y_p without changing the number of neutrino species much [8], by which the helium anomaly can be solved this time. The best-fit value is $\xi_{\nu_e} \sim 0.05$, excluding $\xi_{\nu_e} = 0$ at more than $1\,\sigma$ [1] (see also Refs. [9,10] for a similar analysis with detailed discussions about dependencies on nuclear-reaction rates in BBN). Theoretically, such a large lepton number can be produced even after the cosmic temperature is smaller than the weak scale $\mathcal{O}(10^2)$ GeV in models with Q-balls (L-balls) [11,12], late-time resonant leptogenesis [13], oscillating sterile neutrinos [14], etc. In future, we can measure ξ_{ν_e} more precisely by planned observations of 21 cm and the cosmic microwave background (CMB) down to errors of $\Delta\xi_{\nu_e} \sim 5 \times 10^{-3}$ [15].

There is another way to solve this anomaly, which is a modification of the Einstein gravity. In terms of a modified gravity theory, recently an interesting model to realize dark energy has been proposed, called cuscuton gravity [16,17], along with an extended version [18–20]. In the context of beyond-Horndeski theories, the original cuscuton gravity theory was extended to be a generalized one [21], in which the second-order time derivatives of a scalar field in the equation of motion disappear. Thus, the scalar field appearing in the theories is just a non-dynamical shadowy mode. There is a new type of minimally modified gravity theory, which also has only two gravitational degrees of freedom [22]. It is called VCDM, includes a cuscuton gravity theory, and gives the equivalence in cosmological models [23]. As shown in Refs. [23,24], both theories are related to each other.

As an attractive feature in the models of the cuscuton gravity theory, it is notable that the gravitational constant $G_{\rm cos}$ which appears in the Friedmann equations can be different from Newton's constant $G_{\rm N}$.

In this letter we discuss how we can resolve the ^{4}He anomaly in cuscuton gravity theory models. A modification of $N_{\nu,{\rm eff}}$ from its standard value $N_{\nu,{\rm eff,std}} = 3.044$ effectively has an identical effect to a modification of the gravitational constant without changing $N_{\nu,{\rm eff}}$ in the Friedmann equations. Thus, we can look for a solution to the ^{4}He anomaly by modifying the gravitational constant in the cuscuton gravity theory. In particular, we show that we can concretely constrain the parameters in cuscuton gravity theory models from the observations.

2. Bounds from big-bang nucleosynthesis

In Ref. [1], the EMPRESS collaboration reported the primordial mass fraction of ^{4}He,

$$Y_p = 0.2379^{+0.0031}_{-0.0030}, \tag{1}$$

at 68% C.L. According to the theoretical predictions in the BBN computation, this gives the effective number of neutrino species as

$$N_{\nu,{\rm eff}} = 2.41^{+0.19}_{-0.21}, \tag{2}$$

which excludes the standard value, $N_{\nu,{\rm eff,std}} = 3.044$, predicted in the big-bang cosmology by more than $2\,\sigma$.

The Friedmann equation in the ΛCDM model with vacuum energy V_0 is given by

$$H^2 = \frac{8\pi G_{\rm cos}}{3}(\rho + V_0)\,. \tag{3}$$

Since we can ignore the vacuum energy at the BBN stage, the Hubble parameter H is represented by the product of the energy density of the universe ρ and $G_{\rm cos}$, which is the "effective" gravitational constant appearing in the Friedmann equations. It is notable that $G_{\rm cos}$ can be potentially different from Newton's constant, $G_{\rm N}$, and we may write the difference as $\Delta G_{\rm N} \equiv G_{\rm cos} - G_{\rm N}$.

Then, we obtain the approximate relation

$$\frac{\Delta G_{\rm N}}{G_{\rm N}} = \frac{7}{7N_{\nu,\rm eff,std} + \sqrt[3]{2 \cdot 11^4}} \Delta N_{\nu,\rm eff}, \tag{4}$$

with $\Delta N_{\nu,\rm eff} \equiv N_{\nu,\rm eff} - N_{\nu,\rm eff,std}$; its prefactor gives approximately $7/(7N_{\nu,\rm eff,std} + \sqrt[3]{2 \cdot 11^4}) \simeq 0.1343$ for $T \lesssim m_e$, with m_e being the electron mass. On the other hand, we may have another value of the prefactor ($\simeq 0.1628$) in the case of $T \gtrsim m_e$ with another value of $N_{\nu,\rm eff,std}$ ($= 3$ for $T \gtrsim m_e$). The difference between the former and latter values comes from the neutrinos being decoupled from the thermal bath just before $T \sim m_e$, and only the photon was heated by the e^+e^- annihilation at around $T \sim m_e$. In this study, the decoupling temperature of the weak interaction between neutron and proton, $T_{\rm dec}$, which mainly determines Y_p, tends to get delayed compared to the one in the standard big-bang cosmology ($T_{\rm dec} \sim 0.8$ MeV) due to $N_{\nu,\rm eff} < N_{\nu,\rm eff,std}$. Thus, we adopt the former value ($= 0.1343$) in this study, which also gives a more conservative absolute value of the magnitude of $|\Delta G_{\rm N}/G_{\rm N}|$.

From the observational data in Eq. (2), we obtain the bound on $G_{\rm cos}$ to be

$$\left.\frac{G_{\rm cos}}{G_{\rm N}}\right|_{\rm BBN} = 0.915^{+0.026}_{-0.028} \quad (68\% \text{ C.L.}), \tag{5}$$

or equivalently, $\Delta G_{\rm N}/G_{\rm N} = -0.085^{+0.026}_{-0.028}$ (68% C.L.).

3. Models of the cuscuton gravity theory

The action of the cuscuton gravity theory is represented by [16,17]

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}M_{\rm PL}^2 R + \epsilon\mu^2\sqrt{-X} - V(\phi)\right] + S_M(g_{\mu\nu}, \psi_M), \tag{6}$$

with $M_{\rm PL}$ being the reduced Planck mass ($\simeq 2.436 \times 10^{18}$ GeV) and R, $V = V(\phi)$, and $S_M(g_{\mu\nu}, \psi_M)$ being the Ricci scalar, the potential energy of a scalar field ϕ, and the action of the matter field(s) ψ_M, respectively. $\epsilon = \pm 1$ correspond to two branches of cuscuton gravity theory [25]. Here, the kinetic term X is defined by

$$X \equiv g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi. \tag{7}$$

and μ is the mass parameter of the cuscuton gravity theory.

In the cuscuton gravity theory, the potential $V(\phi)$ can be arbitrary, but if we assume the quadratic form of the potential such that

$$V = V_0 + \frac{1}{2}\alpha\phi^2, \tag{8}$$

which can be regarded as the low-energy Hořava–Lifshitz gravity [26], we find the ΛCDM cosmology with a modification of the gravitational constant [17]. Here, α is the mass squared parameter which has either a positive or negative signature. Then, the flat Friedmann equation is written by Eq. (3).

Note that cosmology in the VCDM theory is equivalent to that in the cuscuton gravity [23]. We can also show that the cuscuton gravity with a quadratic potential is equivalent to the Einstein-aether theory [27] with only one coupling constant c_2 [25].[1] In both theories (VCDM

[1]This model is a very special case of the Einstein-aether theory because aether field modes do not appear in the perturbation equations (S. Mukohyama, private communication), which is consistent with the fact that the system has only two degrees of freedom.

and Einstein-aether), we can take the Newtonian limit, which shows that the Newtonian gravitational constant G_N is given by the reduced Planck mass M_{PL} as $G_N = (8\pi M_{PL}^2)^{-1}$.

In those theories, the effective gravitational constant in the Friedmann equation is given by

$$\frac{G_{\cos}}{G_N} = \left(1 - \frac{3}{2}\frac{\mu^4}{\alpha M_{PL}^2}\right)^{-1} = \left(1 + \frac{3}{2}c_2\right)^{-1} = 1 - \frac{3}{2}\beta_2, \tag{9}$$

where the "potential" of the VCDM scalar field φ, which has mass dimension two, is chosen as

$$\mathcal{V}_{\rm VCDM} = \frac{V_0}{M_{PL}^2} + \frac{1}{2}\beta_2\varphi^2, \tag{10}$$

with the dimensionless mass parameter β_2.

In order to find $G_{\cos} < G_N$, each parameter should satisfy $\alpha < 0$,[2] $c_2 > 0$, and $\beta_2 > 0$. Note that we have the relation between these parameters as

$$\frac{\mu^4}{\alpha M_{PL}^2} = -c_2 \ \text{ and } \ \beta_2 = \frac{c_2}{1 + \frac{3}{2}c_2}.$$

In what follows, we first discuss the constraints on c_2 just for simplicity, but we can translate them into the constraints on the other parameters.

From the observational bound from EMPRESS VIII on $G_{\cos}/G_N$ shown in Eq. (5), we obtain the bound on c_2,

$$c_2 = 0.0620^{+0.0232}_{-0.0198} \ (68\% \text{ C.L.}), \tag{11}$$

with the BBN, which gives $0.0235 \leq c_2 \leq 0.1099$ at 95% C.L. It is remarkable that $c_2 = 0$ is excluded at more than 95% C.L. by the BBN. Provided we assume no other change in the standard cosmology, e.g., without assuming any change of $N_{\nu,\rm eff}$ (and/or ξ_{ν_e}), this may imply rejecting general relativity.

We summarize the constraints on the parameters as follows. From the EMPRESS VIII data, we find

$$\begin{aligned} 0.0235 \leq c_2 \leq 0.1099, \\ -42.55 \leq \frac{\alpha M_{PL}^2}{\mu^4} \leq -9.099, \\ 0.0227 \leq \beta_2 \leq 0.0943 \end{aligned}$$

at 95% C.L.

In Fig. 1 we show the white region allowed by EMPRESS VIII with the BBN at 95% C.L. in the α–μ^2 plane for the cuscuton gravity model. In other words, the red shaded regions are excluded by observations. It is remarkable that the line of $\mu = 0$, which corresponds to general relativity with a cosmological constant, is excluded by EMPRESS VIII with the BBN at 95% C.L.

One may wonder about the negative value of α, because the potential is unbounded from below. However, it does not give any instability because the scalar field ϕ is non-dynamical.

4. Conclusion

We have studied a cosmological model in the cuscuton gravity theory with a quadratic potential $V = V_0 + \frac{1}{2}\alpha\phi^2$ to resolve the anomaly of the observational ^{4}He abundance reported by

[2]When $\alpha < 0$, the branch of $\epsilon = -1$ is required to find a consistent ΛCDM expanding universe. We would like to thank Tsutomu Kobayashi, who pointed this out.

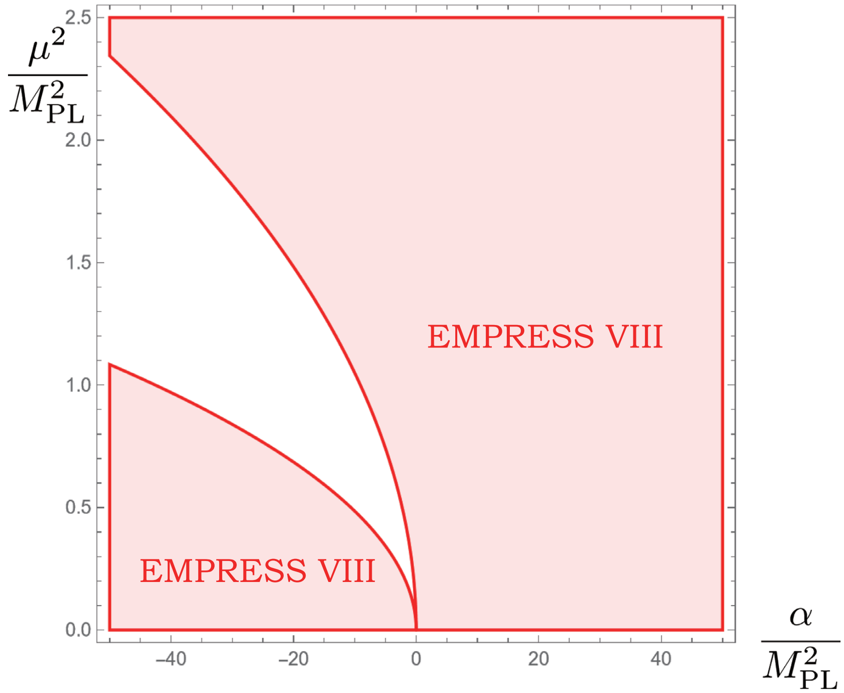

Fig. 1. Regions excluded by EMPRESS VIII with the BBN (red) at 95% C.L. in the α–μ^2 plane. Here, μ^2 and α are plotted in units of the reduced Planck mass M_{PL}. The line of $\mu = 0$, which corresponds to general relativity with a cosmological constant, is excluded at 95% C.L. by EMPRESS VIII with the BBN.

the EMPRESS collaboration. This model is equivalent not only to the VCDM theory with a quadratic potential with the dimensionless coefficient β_2, but also to the Einstein-aether theory with only one coupling constant c_2.

For the mass squared parameter α in the cuscuton gravity theory, we have obtained the allowed region as $-42.55 \leq (\alpha/M_{\mathrm{PL}}^2)(\mu/M_{\mathrm{PL}})^{-4} \leq -9.099$, or equivalently $0.0227 \leq \beta_2 \leq 0.0943$ in the VCDM theory and $0.0235 \leq c_2 \leq 0.1099$ in the Einstein-aether theory. General relativity is excluded in the present approach. Thus, this could suggest the need for modified gravity theories such as the cuscuton gravity theory with a quadratic potential, which can be regarded as the low-energy Hořava–Lifshitz gravity.

In addition to the bound obtained by the BBN, the modification of the gravitational constant can also be constrained by observations of fluctuation and polarization of the CMB. Here we remark on the invalidity of the translation of a bound on $N_{\nu,\mathrm{eff}}$ from the CMB observation to the one on $G_{\mathrm{cos}}/G_{\mathrm{N}}$. Because background neutrinos are thermally produced and have adiabatic fluctuations, a bound on $N_{\nu,\mathrm{eff}}$ from the CMB is obtained by both the total energy density and the evolution of adiabatic curvature perturbation. Therefore, there is no simple one-to-one mapping among the bounds on $N_{\nu,\mathrm{eff}}$ and G_{cos}, although an order-of-magnitude discussion would still be possible.

Using the data released by the Planck collaboration in 2018, Ref. [28] reported an observational bound on $G_{\mathrm{cos}}/G_{\mathrm{N}}$ based on models of scalar-tensor theories obtained by the CMB and the baryon acoustic oscillation (BAO). Although we may need further detailed analysis in the present model, their bound would be approximately applied to the current case in the cuscuton gravity model only by an order-of-magnitude discussion to be $|G_{\mathrm{cos}}/G_{\mathrm{N}}|_{\mathrm{CMB+BAO}} - 1| \lesssim \mathcal{O}(0.1)$ (95% C.L.), which would give $-\mathcal{O}(0.1) \lesssim c_2 \lesssim \mathcal{O}(0.1)$ at 95% C.L. This range of the error covers most of the parameter range in Eq. (11) allowed by EMPRESS VIII with the BBN due to its larger error bars than those of Eq. (11). Thus, contrary to the case of the BBN, general relativity ($c_2 = 0$) is even allowed only by taking the data of CMB+BAO.

In the future, $N_{\nu,\mathrm{eff}}$ can be measured more precisely by planned observations of 21 cm + CMB down to errors of $\Delta N_{\nu,\mathrm{eff}} \sim \mathcal{O}(10^{-2})$ [15]. Then, we will test the gravitational constant to a precision within the order of $\Delta G_\mathrm{N}/G_\mathrm{N} \sim \mathcal{O}(10^{-3})$, which might give a hint of quantum gravity.

We may briefly discuss a possible way to discriminate the effect of the change in $N_{\nu,\mathrm{eff}}$ from the one in G_{cos}. When we change $N_{\nu,\mathrm{eff}}$ as suggested by Refs. [1,12], there are two effects that are measured in the CMB and 21 cm observations. They are changes in energy density and cosmological perturbation. Therefore, precise CMB and 21 cm observations could be able to distinguish the difference between whether the contribution is due to a change in energy density or a change in perturbation.

There could then be a way to use the observational data to clearly show the difference in principle if we devise a way to analyze the data. For example, in data analysis, we propose to compare the two cases where the CMB and 21 cm data are analyzed with the fluctuation effect in the theoretical model and the case with the fluctuation effect cut off in the theoretical model. If there is no sizable change in the allowed region of $N_{\nu,\mathrm{eff}}$ between the two, then we can conclude that we need the contribution of the change mainly in energy density due to the change in $N_{\nu,\mathrm{eff}}$. We might interpret this result as the change dominantly in G_{cos}.

On the other hand, if a sizable change is produced, we understand that the change in energy density alone cannot explain the observational data. Thus, the change in G_{cos} alone does not work, which discriminates the effect of the change in $N_{\nu,\mathrm{eff}}$ from the one in G_{cos}.

However, as far as we know such an analysis is not available yet, nor have any future experiments with sufficient sensitivity been proposed. We hope that with the development of future observations, a suitable experiment will be proposed that will distinguish between the two cases.

Acknowledgments

We thank Niyaesh Afshordi, Tsutomu Kobayashi, and Shinji Mukohyama for useful discussions. K.M. also acknowledges the Yukawa Institute for Theoretical Physics at Kyoto University, where the present work was completed during the Visitors Program of FY2022. This work was supported in part by JSPS KAKENHI Grant Numbers JP17H01131 (K.K.), JP17H06359, and JP19K03857 (K.M.), and by MEXT KAKENHI Grant Numbers JP20H04750, JP22H05270 (K.K.).

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