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Mass of a_1 Meson from Lattice QCD with the Truncated Overlap Fermions

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We report our recent study of the a_1 meson using a quenched lattice quantum chromodynamics simulation with the truncated overlap fermions formalism based on the domain wall fermions. The obtained lightest mass of the a_1 meson, 1272(45) MeV, is consistent with the experimental value for $a_1(1260)$. Thus, $a_1(1260)$ can be identified to have a simple two-body constituent-quark structure. Our quenched simulation result of $a_1(1420)$ cannot explain the experimental mass value, which suggests $a_1(1420)$ is not a simple $q\bar{q}$ two quark state

KEYWORDS: lattice QCD, meson spectroscopy, a_1 meson

1. Introduction

The classification of light axial-vector mesons (a_1) is a long-standing problem of meson spectroscopy. Currently the particle data group lists [1] three a_1 mesons: $a_1(1260)$, $a_1(1420)$, and $a_1(1640)$; however, this is a richer spectrum than that in the prediction of usual $q\bar{q}$ mesons in a constituent quark model. In general, the $J^{PC} = 1^{++}$ and $I = 1$, meson is realized in $^3P_1$ state in the quark model. If $a_1(1260)$ is assumed to be the ground state for a_1 meson, mass of the next radial excitation becomes at least 1.7 GeV [2]. This shows that the radial excitation of $a_1(1260)$ may not be $a_1(1420)$, but $a_1(1640)$. In other words, the structure of $a_1(1420)$ cannot be understood as a simple $q\bar{q}$ state. It is a possible candidate for the dynamical effect due to a singularity in the triangle diagram [3–6] or the exotic multi-quark state [7–10]. The structure of $a_1(1420)$ is still not well understood. Similarly, the structure of the $a_1(1260)$ meson is unclear. Whereas the $a_1(1260)$ meson is the chiral partner of the ρ meson, which is described as $q\bar{q}$ in the Nambu-Jona-Lasinio model [11], the $a_1(1260)$ meson could also be interpreted as the gauge boson of the hidden local symmetry [12–15]. Another interpretation of $a_1(1260)$ is the dynamically generated resonance in $\pi\rho$ scattering in coupled-channel approaches based on chiral effective theory [16]. Nagahiro *et al.* discussed the mixing properties of $a_1(1260)$ of the quark composite state and the hadronic composite state [17].

Here, we investigate the structure of the lightest a_1 meson determined from lattice quantum chromodynamics (QCD). Lattice QCD is a first-principles approach for hadron physics. In particular, we are interested in the possible relation between the nature of the a_1 meson, which is the chiral partner of the ρ meson, and the dynamical breaking of chiral symmetry. It offers the key to an understanding of the relation between the σ meson, which is the chiral partner of the pion, and the dynamical breaking [11]. Therefore, we employ the truncated overlap fermion formalism of lattice chiral fermions [18], which have good chiral symmetry. The truncated overlap fermion formalism is classified into lattice chiral fermions [19–22].

Lattice simulations of the a_1 meson have been previously performed. Wingate *et al.* measured the mass of the a_1 meson using lattice QCD with two flavors of dynamical staggered quarks [23]. Their result was in agreement with the experimental value for $a_1(1260)$. Recently, Prelovsek *et al.* presented the results for the mass of a_1 meson and the coupling constant [24]. These simulations were performed in the full QCD with clover-improved Wilson quarks. This work was continued in Ref. [25], they extracted the resonance mass of the ground state for a_1 meson, $m_{a_1}^{res} = 1.435(53)(^{+0}_{-109})$ GeV and the coupling $g_{a_1\pi\rho} = 1.71(39)$ GeV by simulating the corresponding scattering channel $\pi\rho$. Their obtained value of the a_1 meson mass is higher than the experimental result [1].

Our collaboration performs quenched simulations for the a_1 meson using the truncated overlap fermion formalism with $q\bar{q}$ interpolating operators [26]. This suggest that the lightest a_1 meson is the $q\bar{q}$ state composed of u and d quarks.

2. Lattice calculation

We adopt the following interpolating operator for creating the a_1 meson with $I = 1$ and $J^{PC} = 1^{++}$,

$$O_{a_1} = \bar{q}\gamma_\mu\gamma_5 q \, , \tag{1}$$

where q denotes the u or d quark operator. Using the truncated overlap fermions with the plaquette gauge action, we perform a quenched simulation on an $8^3 \times 24$ lattice, the length of fifth dimension $N_5 = 32$, the five dimensional mass $m_5 = 1.65$, and $\beta = 5.7$. We use point sources and sinks when calculating hadron propagators, which leads to larger masses on a relatively small lattice because of a mixture of higher mass states. The masses obtained in our simulation should thus be considered as the upper limits. The a_1 meson propagator is more noisy than those of π and ρ mesons, and therefore more statistics are required.

We generate gauge configurations based on the plaquette gauge action by using the pseudo heat-bath method. After 20000 thermalization iterations, we start to save gauge configurations every 1000 sweeps. We calculate meson propagators on the stored gauge configurations for each of the quark mass values, $m_f a = 0.08$, 0.06, and 0.04, where a is the lattice spacing. We use 3000 (7964) configurations for the calculation of the meson propagators with $m_f a = 0.08$ and 0.06 ($m_f a = 0.04$).

Table I. Masses of π, ρ, and a_1 mesons, mass ratios and numbers of configurations. Number of configurations are separated by 1000 sweeps.

$m_f a$	$m_\pi a$	$m_\rho a$	m_π/m_ρ	m_{a_1}/m_ρ	N_{config}
0.08	0.667(1)	0.950(2)	0.702(2)	1.480(13)	3000
0.06	0.589(1)	0.904(2)	0.652(3)	1.511(19)	3000
0.04	0.503(1)	0.861(2)	0.584(2)	1.540(19)	7964

The effective masses, $m_{\rm eff} a$, of these mesons are displayed in Fig. 1, which are determined as

$$\frac{G(t)}{G(t+1)} = \frac{e^{-m_{\rm eff}(t)t} + e^{-m_{\rm eff}(t)(T-t)}}{e^{-m_{\rm eff}(t)(t+1)} + e^{-m_{\rm eff}(t)(T-(t+1))}} \, , \tag{2}$$

where $G(t)$ represents the propagators of the mesons. We estimate the statistical errors using the jackknife method. Thanks to the large enough statistics, we obtain very clear propagators and effective masses for the a_1 meson. The masses of the π, ρ, and a_1 mesons for $m_f a = 0.80$, 0.06, and 0.04 are listed in Table I. The π and ρ masses are evaluated from effective masses in the range of $6 \le t/a \le 9$. The a_1 mass, on the other hand, is obtained in the range of $5 \le t/a \le 8$, because the effective masses of a_1 suffer from large errors at large t. The masses of π and ρ mesons obtained in our simulation on a small lattice give good agreement with that Blum *et al.* [27] on a large lattice ($8^3 \times 32$), though our results are less than 2 percent higher than their results.

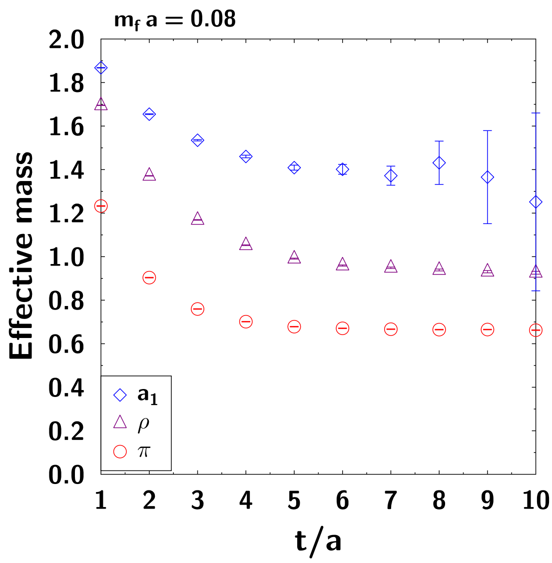

Fig. 1. Time dependence of the effective masses at $m_f a = 0.08$. Open circles, triangles, and diamonds stand for the propagators of π meson, ρ meson, and a_1 meson, respectively.

We linearly extrapolate the lattice results of the meson masses to the chiral limit, $(m_\pi a)^2 = 0$. Using $m_\rho = 775$ MeV as the input, we obtain $a = 0.190(2)$ fm. In this limit, the difference between the chiral extrapolations $m_f a \to 0$ and $m_f a \to -m_{\mathrm{res}} a$ is negligible due to the smallness of $m_{\mathrm{res}} a = 1.27 \times 10^{-2}$, where m_{res} is the residual mass. Therefore, we apply $m_f a \to 0$. We estimate the mass ratio m_{a_1}/m_ρ to be 1.64(6) and the mass of the a_1 meson to be $m_{a_1} = 1272(45)$ MeV. Our result is consistent with the experimental value of $a_1(1260)$, 1230(40) MeV [1].

3. Summary

We studied the lightest a_1 meson based on a quenched approximation using truncated overlap fermions. We estimated the mass of the a_1 meson to be 1272(45) MeV, which is in good agreement with the experimental value of $a_1(1260)$ [1]. The masses obtained in our simulation should be considered as the upper limits. Our results are consistent with those of Wingate *et al.* [23] who employed a full QCD simulation without chiral symmetry. Our simulation used truncated overlap fermions, and thus respects chiral symmetry, but in the quenched approximation.

Our lattice study and quark model analyses support that the simple two-body constituent-quark structure of $a_1(1260)$ is consistent with the experimentally observed $a_1(1260)$. Our a_1 meson does not agree with $a_1(1420)$. A quenched simulation is a clean theoretical experiment in which virtual intermediate states such as $qq\bar{q}\bar{q}$ are highly suppressed.

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References

[1] M. Tanabashi *et al.* [Particle Data Group], "The Review of Particle Physics", Phys. Rev. D **98**, 030001 (2018).
[2] D. Ebert, R. N. Faustov and V. O. Galkin, Phys. Rev. D **79** (2009) 114029 [arXiv:0903.5183 [hep-ph]].
[3] M. Mikhasenko, B. Ketzer and A. Sarantsev, Phys. Rev. D **91**, no. 9, 094015 (2015) [arXiv:1501.07023 [hep-ph]].
[4] F. Aceti, L. R. Dai and E. Oset, Phys. Rev. D **94**, no. 9, 096015 (2016) [arXiv:1606.06893 [hep-ph]].
[5] J. L. Basdevant and E. L. Berger, Phys. Rev. Lett. **114**, no. 19, 192001 (2015) [arXiv:1504.05955 [hep-ph]].
[6] W. Wang and Z. X. Zhao, Eur. Phys. J. C **76**, no. 2, 59 (2016) [arXiv:1511.06998 [hep-ph]].
[7] H. X. Chen, E. L. Cui, W. Chen, T. G. Steele, X. Liu and S. L. Zhu, Phys. Rev. D **91**, 094022 (2015) [arXiv:1503.02597 [hep-ph]].
[8] T. Gutsche, V. E. Lyubovitskij and I. Schmidt, Phys. Rev. D **96**, no. 3, 034030 (2017) [arXiv:1706.07716 [hep-ph]].
[9] T. Gutsche, M. A. Ivanov, J. G. Körner, V. E. Lyubovitskij and K. Xu, Phys. Rev. D **96**, no. 11, 114004 (2017) [arXiv:1710.02357 [hep-ph]].
[10] H. Sundu, S. S. Agaev and K. Azizi, Phys. Rev. D **97**, no. 5, 054001 (2018) [arXiv:1711.05977 [hep-ph]].
[11] Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961); Phys. Rev. **124**, 246 (1961).
[12] M. Bando, T. Kugo and K. Yamawaki, Nucl. Phys. B **259**, 493 (1985).
[13] K. Yamawaki, Phys. Rev. D **35**, 412 (1987).
[14] M. Bando, T. Fujiwara and K. Yamawaki, Prog. Theor. Phys. **79**, 1140 (1988).
[15] N. Kaiser and U. G. Meissner, Nucl. Phys. A **519**, 671 (1990).
[16] L. Roca, E. Oset and J. Singh, Phys. Rev. D **72**, 014002 (2005) [hep-ph/0503273].
[17] H. Nagahiro, K. Nawa, S. Ozaki, D. Jido and A. Hosaka, Phys. Rev. D **83**, 111504(R) (2011) [arXiv:1101.3623 [hep-ph]].
[18] A. Boriçi, Nucl. Phys. Proc. Suppl. **83**, 771 (2000) [hep-lat/9909057].
[19] D. B. Kaplan, Phys. Lett. B **288**, 342 (1992) [hep-lat/9206013].
[20] V. Furman and Y. Shamir, Nucl. Phys. B **439**, 54 (1995) [hep-lat/9405004].
[21] R. Narayanan and H. Neuberger, Phys. Rev. Lett. **71**, no. 20, 3251 (1993) [hep-lat/9308011].
[22] R. C. Brower, H. Neff and K. Orginos, Nucl. Phys. Proc. Suppl. **140**, 686 (2005) [hep-lat/0409118].
[23] M. Wingate, T. A. DeGrand, S. Collins and U. M. Heller, Phys. Rev. Lett. **74**, 4596 (1995) [hep-ph/9502274].
[24] S. Prelovsek, C. B. Lang, D. Mohler and M. Vidmar, PoS LATTICE **2011**, 137 (2011) doi:10.22323/1.139.0137 [arXiv:1111.0409 [hep-lat]].
[25] C. B. Lang, L. Leskovec, D. Mohler and S. Prelovsek, JHEP **1404**, 162 (2014) doi:10.1007/JHEP04(2014)162 [arXiv:1401.2088 [hep-lat]].
[26] Y. Murakami, S. Muroya, A. Nakamura, C. Nonaka, M. Sekiguchi, H. Wada and M. Wakayama, arXiv:1812.07765 [hep-lat].
[27] T. Blum *et al.*, Phys. Rev. D **69**, 074502 (2004) [hep-lat/0007038].