IMPACT OF TEV-SCALE STERILE NEUTRINOS ON PRECISION LOW-ENERGY OBSERVABLES

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We study the impact of TeV-scale sterile neutrinos on electroweak precision observables and lepton number and flavour violating decays in the framework of a type-I see-saw extension of the Standard Model. At tree level sterile neutrinos manifest themselves via non-unitarity of the PMNS matrix and at one-loop level they modify the oblique radiative corrections. We perform a numerical fit to the electroweak observables and find regions of parameter space with a sizable active-sterile mixing which provide a better over-all fit compared to the case where the mixing is negligible. Specifically we find improvements of the invisible Z-decay width, the charged-to-neutral-current ratio for neutrino scattering experiments and of the deviation of the W boson mass from the theoretical expectation.

1 Introduction

The Standard Model (SM) is extremely successful and has passed numerous experimental tests. Moreover the last missing piece, the Higgs particle, has recently likely been seen by the ATLAS and CMS collaborations. On the other hand, the SM is incomplete as it fails to explain the tiny active neutrino masses, the baryon asymmetry of the Universe and the existence of dark matter. A simple yet elegant way to solve the first two or even all of these problems is to supplement the SM by three heavy sterile neutrinos:

$$\mathscr{L} = \mathscr{L}_{SM} + \tfrac{1}{2}\bar{N}_i(i\partial\!\!\!/ - M_i)N_i - h_{\alpha i}\bar{\ell}_\alpha\tilde{\phi}N_i - h^\dagger_{i\alpha}\bar{N}_i\tilde{\phi}^\dagger\ell_\alpha\,. \tag{1}$$

The existence of Majorana neutrinos has well known consequences on the phenomenology below the electroweak scale. In particular, the new states can contribute to the amplitude of the neutrinoless double-beta decay [1,2,3,4,5] and induce rare charged lepton decays [6,7]. Furthermore, they can affect the electroweak precision observables (EWPOs) via tree-level as well as loop contributions and thus provide an explanation for anomalies in the experimental data. In particular, the tree-level effects result in non-unitarity of the active neutrino mixing matrix [6] and lead to a suppression of the invisible Z-decay width. This is in agreement with the long standing fact that the LEP measurement of the invisible Z-decay width is two sigma below the value expected in the SM [8]. Furthermore the neutral-to-charged-current ratio in neutrino scattering experiments can be changed thus providing an explanation for the NuTeV anomaly [9]. Also a slight shift of the W boson mass from the value derived from other SM parameters is induced, reducing the tension between the input parameters of the electroweak fit and the experimentally observed value [10].

[a]Presenting author. Talk based on arXiv:1302.1872.

Encouraged by the fact that sterile neutrinos are very well motivated we study their phenomenological impact. Specifically we consider TeV-scale sterile neutrinos with a sizable active-sterile mixing and determine their over-all contributions to the EWPOs and to indirect detection experiments in the framework of the see-saw type-I extension of the SM.

2 Impact on low-energy observables

After the electroweak symmetry breaking the active and sterile *flavor* eigenstates mix. In other words, the light *mass* eigenstates acquire a small sterile component. Simultaneously, the heavy *mass* eigenstates acquire a small active component and couple to the W- and Z-bosons,

$$\begin{aligned}\mathscr{L}_{\text{int}} = &-\frac{e}{2c_w s_w} Z_\mu \textstyle\sum_{i,j=1}^{3+n} \sum_{\alpha=e,\mu,\tau} \bar{\nu}_i \mathbf{U}^\dagger_{i\alpha} \gamma^\mu P_L \mathbf{U}_{\alpha j} \nu_j \\ &-\frac{e}{\sqrt{2} s_w} W_\mu \textstyle\sum_{i=1}^{3+n} \sum_{\alpha=e,\mu,\tau} \bar{\nu}_i \mathbf{U}^\dagger_{i\alpha} \gamma^\mu P_L e_\alpha + \text{h.c.}\,. \end{aligned} \tag{2}$$

This affects the low-energy observables in two ways. First, this introduces additional processes with the heavy neutrinos in the intermediate state. Second, because the light mass eigenstates acquire a small sterile component their couplings to the W- and Z-bosons are smaller than assumed in the Standard Model.

Lepton universality constraints. The second effect has an immediate impact of the probability of the W-decay into a charged lepton of one of the three generations and a light neutrino. First, one can expect that the decay width is smaller than in the Standard Model, just because the coupling is smaller. Second, the decay probabilities for the electron, muon and tau leptons are now in principle different. This goes under the name lepton universality violation. There are relatively stringent experimental constraints on the violation of lepton universality [11],

$$\epsilon_e - \epsilon_\mu = 0.0022 \pm 0.0025\,, \tag{3a}$$
$$\epsilon_\mu - \epsilon_\tau = 0.0017 \pm 0.0038\,, \tag{3b}$$
$$\epsilon_e - \epsilon_\tau = 0.0039 \pm 0.0040\,, \tag{3c}$$

where $\epsilon_\alpha \equiv \sum_{i\geq 4} |\mathbf{U}_{\alpha i}|^2$. Note that $\epsilon_\alpha \neq 0$ implies non-unitarity of the PMNS matrix, i.e. of the 3×3 mixing matrix of the light eigenstates.

W-boson mass. The modification of the couplings to the W-boson also affects another very important observable. Because of the non-unitarity the Fermi constant measured in the muon decay differs from the Fermi constant measured in experiments with semi-leptonic processes. And because the muon decay width is used as input in the Standard Model fit, this modification influences many observables. In particular it changes the theoretical expectation for the W-boson mass whose experimental value, $M_W = 80.385 \pm 0.015$, is roughly one sigma away from the Standard Model expectation, 80.359 ± 0.011 GeV.

Invisible Z-decay width. The existence of the heavy neutrinos also affects couplings to the Z-boson. Because we now have two neutrino lines, the effect is roughly speaking twice as strong as for the W-boson. Typically, adding new particles to a theory means larger decay widths, simply because there are more states to decay into. Surprisingly, this is not what happens with the invisible Z-decay width once we add heavy neutrinos. It becomes smaller instead. Here is the reason. On the one hand, the non-unitarity of the PMNS matrix makes couplings of the light neutrinos to the Z-boson smaller. This automatically makes the decay width into these states smaller. On the other hand, because the Z-boson is lighter than the heavy neutrinos, it simple cannot decay into the new states for kinematical reasons. As a result, the invisible Z-decay width is smaller than expected in the Standard Model. Put in another way, this means that the effective number of neutrinos measured by LEP should be slightly smaller than three. This is in qualitative agreement with the experimental result $\Gamma_{\text{inv}}/\Gamma_{\text{lept}} = 5.942 \pm 0.016$, which is roughly two sigma away from the Standard Model expectation 5.9721 ± 0.0002.

Charged to neutral current ratio. The existence of the heavy neutrinos makes coupling of the light ones to the W- and Z-bosons smaller. The coupling to the Z-boson is affected roughly twice as strong as the coupling to the W-boson. The immediate implication is that the neutral current is suppressed stronger than the charged current. This conclusion is qualitatively consistent with the results of the NuTeV experiment. After including a recent NNLO analysis[9,12] the experimental values for g_L and g_R are given by $g_{\rm L}^2 = 0.3026 \pm 0.0012$ and $g_{\rm R}^2 = 0.0303 \pm 0.0010$, whereas the Standard Model expectations are 0.3040 ± 0.0002 and 0.0300 ± 0.0002 respectively.

Lepton flavor violating decays. Recall now that the heavy neutrinos affect the low energy observables not only because of non-unitarity, but also because they appear as intermediate states in the Feynman diagrams. The prime example where both effects play a role is a lepton flavor violating decay $\mu \to e\gamma$. The contribution of the light neutrinos is completely negligible. Taking into account the unitarity of the full mixing matrix $\mathbf{U}$ we find for the contribution of the heavy states, $\delta_\nu = 2\sum_{i=4}^{3+n} \mathbf{U}_{ei}^* \mathbf{U}_{\mu i} \left[g\left(m_i^2/M_W^2\right) - 5/3\right]$, where the second term in the square brackets comes from non-unitarity of the PMNS matrix, whereas the first one is induced by the intermediate heavy neutrinos. The recent limit on this branching ratio obtained by the MEG collaboration[13] is $\mathrm{BR}(\mu^+ \to e^+\gamma) \leq 5.7 \cdot 10^{-13}$ at 90% confidence level.

Neutrinoless double beta decay. Another example is neutrinoless double beta decay. The effective electron neutrino mass is given by $|\langle m_{ee}\rangle| \approx \left|\sum_{i=1}^{3} \mathbf{U}_{ei}^2 m_i - \sum_{i=4}^{3+n} F(A, M_i)\mathbf{U}_{ei}^2 m_i\right|$. Typically, one takes into account only the contributions of the light neutrinos, this is the first term, but the heavy neutrinos can also contribute, this is the second term. The experimental bound has also been recently updated[14] by the GERDA collaboration, $|\langle m_{ee}\rangle| < 0.2 - 0.4$ eV.

STU parameters. Last but no definitely not least, the heavy neutrinos can also appear in the self-energy loops of the W- and Z-bosons and affect theoretical predictions for the low-energy observables we have discussed so far. These loop corrections can be taken into account using the STU parameters developed by Peskin and Takeuchi.

Combination of the tree-level and loop corrections. Explicit expressions for the corrections to the electroweak observables read,

$$\frac{\Gamma_{\rm lept}}{[\Gamma_{\rm lept}]_{\rm SM}} = 1 + 0.6\,(\epsilon_e + \epsilon_\mu + 0.0145\,T) - 0.0021\,S\,, \tag{4a}$$

$$\frac{\Gamma_{\rm inv}/\Gamma_{\rm lept}}{[\Gamma_{\rm inv}/\Gamma_{\rm lept}]_{\rm SM}} = 1 - 0.67\,(\epsilon_e + \epsilon_\mu + \epsilon_\tau) + 0.0021\,S - 0.0015\,T\,, \tag{4b}$$

$$\frac{\sin^2\theta_{\rm w}^{\rm lept}}{\left[\sin^2\theta_{\rm w}^{\rm lept}\right]_{\rm SM}} = 1 - 0.72\,(\epsilon_e + \epsilon_\mu + 0.0145\,T) + 0.0016\,S\,, \tag{4c}$$

$$\frac{g_L^2}{\left[g_L^2\right]_{\rm SM}} = 1 + 0.41\,\epsilon_e - 0.59\,\epsilon_\mu - 0.0090\,S + 0.0022\,T\,, \tag{4d}$$

$$\frac{g_R^2}{\left[g_R^2\right]_{\rm SM}} = 1 - 1.4\,\epsilon_e - 2.4\,\epsilon_\mu + 0.031\,S - 0.0067\,T\,, \tag{4e}$$

$$\frac{M_{\rm W}}{[M_{\rm W}]_{\rm SM}} = 1 + 0.11\,\epsilon_e + 0.11\,\epsilon_\mu - 0.0036\,S + 0.0056\,T + 0.0042\,U\,, \tag{4f}$$

where S, T, and U are the STU parameters which encode the loop corrections, and ϵ_e, ϵ_μ and ϵ_τ encode the tree-level non-unitarity effects. Importantly, the loop corrections can be as large as the tree-level ones and therefore we can have partial cancellation of the tree-level and loop corrections. If this cancellation happens or not of course depends on the values of the model parameters and this is where we approach the question of parameter scan.

Parameter scan. The contribution of the heavy neutrinos to most of the observables may be small, but it is decisive as far as masses and mixing angles of the light neutrinos are concerned. Within the past fifteen years there has been an enormous progress in this field. On the one hand most of the neutrino parameters have been measured experimentally. On the other hand, and this is very important for the parameter scan, Casas and Ibarra have developed a very handy parametrization [15] of the full six-by-six neutrino mixing matrix in terms of the experimentally measurable quantities and a few unknown parameters,

$$\mathscr{R} = -i\mathcal{U}\,\hat{m}_{\text{light}}^{\frac{1}{2}}\,O^*\hat{m}_{\text{heavy}}^{-\frac{1}{2}}\,, \tag{5a}$$

$$\mathscr{U} = \left(1 - \mathscr{R}\mathscr{R}^\dagger\right)^{\frac{1}{2}}\mathcal{U}\,, \tag{5b}$$

with O being an arbitrary complex orthogonal matrix, $\mathcal{U}$ the *unitary* matrix diagonalizing $\hat{m}_\nu$, $\hat{m}_{\text{heavy}}$ the diagonal mass matrix of the heavy neutrinos and $\hat{m}_{\text{light}}$ that of the light neutrinos. Now comes the question of the number of degrees of freedom. In principle, we start with nine variables: three masses of the heavy neutrinos plus three complex angles in the matrix O. However, the corrections are expressed in terms of only six quantities: the three parameters of non-unitarity plus the three STU parameters. In other words, the initial nine degrees of freedom map to six. Out of the STU parameters, the S and U parameters are negligibly small. So effectively, the initial degrees of freedom map to four parameters: ϵ_e, ϵ_μ, ϵ_τ and T. I would like to emphasize that our goal was not to perform a full parameter scan, but rather to find examples of regions in the parameter space where the fit is improved with respect to the Standard Model. For every considered point in the parameter space we have checked that it is compatible with the $\mu \to e\gamma$ and $0\nu\beta\beta$ constraints.

Rare processes. If the point passed this test, as is the case for all points in figure 1, then we computed the values of the other low-energy observables and used them to calculate χ^2 using

$$\chi^2_{\text{EWPO}} = \sum_i \frac{(O_i - O_{i,\text{SM}})^2}{(\delta O_i)^2 + (\delta O_{i,\text{SM}})^2}\,, \tag{6}$$

where $O_{i,SM}$ denotes the predictions of the SM, $\delta O_{i,SM}$ are the theoretical errors and δO_i are the experimental errors. If the heavy neutrinos had absolutely no impact on the low-energy

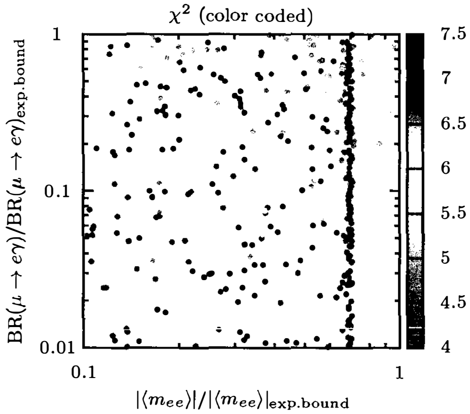

Figure 1: χ^2 for four d.o.f. as a function of the ratios of the $\mu \to e\gamma$ branching ratio and $|\langle m_{ee}\rangle|$ to the corresponding experimental bounds (NH). Here ϵ_μ is suppressed.

observables, then $\chi^2 \approx 7.5$. Such points are color-coded by red on this plot. For the best fit points we get a moderate decrease of χ^2 to 4. These points are color-coded by green. The improvement of χ^2 per new degree of freedom is roughly one. Which of the electroweak observables are responsible for this improvement?

Electroweak fit. The answer can be inferred from figure 2. The main improvement is due to the charged-to-neutral current ratio and due to the W-mass. The improvement of the invisible

Z-decay width is rather minor. Here the tree-level corrections are largely compensated by the loop ones. Note that figure 2 assumes normal hierarchy of the light neutrino masses. For

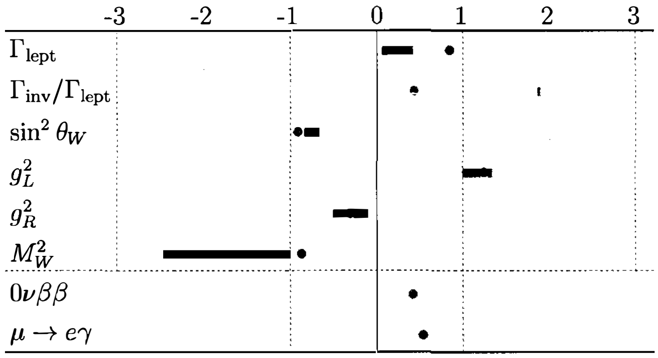

Figure 2: EWPOs calculated at the best-fit point for NH and suppressed ϵ_μ (green dots) compared to the experimentally observed values, denoted by the zero line. The colored lines stand for the respective experimental sigma deviations, thus the displacement of the predicted values form the observations is presented in units of the experimental error. Note that for the $0\nu\beta\beta$ and $\mu \to e\gamma$ constraints we present only the one sigma exclusion limits. The theoretical predictions of the SM with their theoretical uncertainties are displayed as well (blue bars). (The best-fit point is at $M_1 = 20.3$ TeV, $M_2 = 14.1$ TeV, $M_3 = 21.0$ TeV, $\epsilon_e = 2.1 \cdot 10^{-3}$, $\epsilon_\mu = 3.0 \cdot 10^{-6}$ and $\epsilon_\tau = 4.5 \cdot 10^{-3}$.)

the inverted and quasidegenerate mas hierarchies χ^2 at the best-fit points reduces to roughly $\chi^2 \approx 5.5$ and $\chi^2 \approx 5$ respectively [16].

3 Summary

To summarize, sterile neutrinos with masses at the TeV-scale and a sizable active-sterile mixing affect electroweak precision observables as well as $\mu \to e\gamma$ and $0\nu\beta\beta$ processes. The effect is twofold. On the one hand, the coupling of the light mass eigenstates to the W- and Z-bosons is smaller than expected in the Standard Model. On the other hand, the heavy mass eigenstates also couple to the W- and Z-bosons and can contribute as intermediate states. Given that there some discrepancies between the predictions of the Standard Model and the experimental data, corrections induced by the sterile neutrinos are more than welcome. Accepting some fine-tuning we can improve the fit of the neutral-to-charged current ratio, of the W-mass, and to a lesser extent of the invisible decay width.

Acknowledgments

The work of A.K. has been supported by the German Science Foundation (DFG) under Grant KA-3274/1-1 "Systematic analysis of baryogenesis in non-equilibrium quantum field theory". The authors would like to thank M. Blennow, T. Schwetz , J. Heeck, J. Barry and S. Antusch for helpful discussions.

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