MODELING BRANES IN WARPED EXTRA-DIMENSION*

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We have presented a generalization of the second Randall–Sundrum model in which different bulk cosmological constants on each side of the brane are allowed. A smooth version of the model was introduced and discussed. We have also considered a scenario with two thick branes which allows to address the issue of the hierarchy problem.

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1. Introduction

Randall and Sundrum constructed a model (RS1) [1] which offers a possibility of explaining the hierarchy between the Planck scale $M_{\rm Pl} \simeq 10^{18}$ GeV and the electroweak scale $m_W \simeq 10^2$ GeV. In their second seminal paper (RS2), they proposed an attractive alternative to compactification of the extra dimension [2]. Both models suffer from the presence of infinitesimally thin structures, the so-called D3 branes. In addition, the RS1 requires the presence of a brane with negative tension. There were many attempts to regularize thin branes by certain configurations of a scalar field having localized energy density. Unfortunately, it turns out that periodicity constraints the dynamics of those models so strongly that only trivial (constant) configurations of the scalar field are allowed, see [3] and references therein. Therefore, in this paper we are going to limit ourself to the case of the uncompactified

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extra dimension, *à la* RS2. We will consider two scenarios: a generalized version of the RS2 allowing for different cosmological constants on both sides of the brane [4] and a two thick brane scenario which can address the hierarchy problem [3].

2. RS2 generalization

We consider the following action as an extension of the RS2 model [2]

$$\mathcal{S} = \int d^5x\sqrt{-g}\left\{2M^3R - \Lambda_+\Theta(y) - \Lambda_-\Theta(-y) - \lambda\delta(y)\right\}\,, \tag{1}$$

where $\Lambda_\pm$ are 5D cosmological constants for $y \gtrless 0$, the brane is located at $y=0$ and λ is the brane tension. In our convention, the capital Roman indices will refer to 5D objects, *i.e.*, $M, N, \ldots = 0,1,2,3,5$, and the Greek indices label four-dimensional (4D) objects, *i.e.*, $\mu,\nu,\ldots = 0,1,2,3$. In Eq. (1), Θ is the Heaviside theta function and δ is the Dirac delta function.

We are going to look for solutions of the Einstein equations assuming the following form of the 5D metric

$$ds^2 = e^{2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2\,. \tag{2}$$

The Einstein equations following from Eqs. (1) and (2) are

$$6A'^2 = -\frac{1}{4M^3}\Big(\Lambda_+\Theta(y) + \Lambda_-\Theta(-y)\Big)\,, \tag{3}$$

$$3A'' + 6A'^2 = -\frac{1}{4M^3}\Big(\Lambda_+\Theta(y) + \Lambda_-\Theta(-y) + \lambda\delta(y)\Big)\,. \tag{4}$$

The solution of Eq. (3) is given by

$$A(y) = -|y|k_\pm \qquad \text{for} \qquad y \gtrless 0\,, \tag{5}$$

where $k_\pm \equiv \sqrt{-\frac{1}{24M^3}\Lambda_\pm}$. It is important to note that $A'(y)$ is not continuous at $y=0$, so that A'' must be singular there. From the Einstein equation (4), we find that the following relation must hold

$$\lambda = \sqrt{6M^3}\left(\sqrt{-\Lambda_+} + \sqrt{-\Lambda_-}\right)\,. \tag{6}$$

The above equation is an analogue of the RS relation between the bulk cosmological constant and the brane tension [1, 2]. It is important to note that relation (6) is necessary in order to recover the 4D Poincaré invariance on the brane. It can be shown that these background solutions are stable w.r.t. the small perturbations and also the 4D Plank mass is finite, hence the 4D gravity is recovered without large corrections to Newton's law [4]. In the following subsection, we consider the smooth version of generalized RS2 model.

2.1. Thick brane version of the generalized RS2

Here we will extend the solution found above with a singular D3-brane to a thick (smooth) brane scenario in which the thick brane is dynamically generated by a scalar field $\phi = \phi(y)$. The action for a 5D scalar field minimally coupled to the Einstein–Hilbert gravity is

$$\mathcal{S} = \int dx^5 \sqrt{-g} \left\{ 2M^3 R - \tfrac{1}{2} g^{MN} \nabla_M \phi \nabla_N \phi - V(\phi) \right\} . \tag{7}$$

The Einstein equations resulting from the action (7) and the metric ansatz (2) are

$$24M^3 \left(A'\right)^2 = \tfrac{1}{2} \left(\phi'\right)^2 - V(\phi) \,, \tag{8}$$

$$12M^3 A'' + 24M^3 \left(A'\right)^2 = -\tfrac{1}{2} \left(\phi'\right)^2 - V(\phi) \,. \tag{9}$$

We assume that the scalar potential $V(\phi)$ could be expressed in terms of the superpotential $W(\phi)$ [3, 5] as follows

$$V(\phi) = \frac{1}{2} \left(\frac{\partial W(\phi)}{\partial \phi} \right)^2 - \frac{1}{6M^3} W(\phi)^2 \,, \tag{10}$$

where $W(\phi)$ satisfies the following relations:

$$\phi' = \frac{\partial W(\phi)}{\partial \phi} \quad \text{and} \quad A' = -\frac{1}{12M^3} W(\phi) \,. \tag{11}$$

We are interested in a kink-like profile[1] scalar field

$$\phi(y) = \frac{\kappa}{\sqrt{\beta}} \tanh(\beta(y) \,, \tag{12}$$

where β is the thickness regulator and κ parameterizes tension of the brane in the so-called *brane limit*: $\beta \to \infty$. We will find solutions which mimic a positive-tension brane along with two different cosmological constants on either side of the brane. If the scalar field $\phi(y)$ is known, then the superpotential $W(\phi)$ can be obtained from Eq. (11) as

$$W(y) = \int_{y_0}^{y} \left(\phi'(y)\right)^2 dy + W_0 \,, \tag{13}$$

[1] The scalar field $\phi(y)$ profile could be different than the standard kink but in using the superpotential method the essential concept must hold, *i.e.*, the profile should be monotonic (invertible) and $\phi'^2(y)$ should be integrable [4].

where W_0 is a constant of integration. Since $\phi(y)$ is invertible, therefore we can write the superpotential $W(\phi)$ and the potential $V(\phi)$ as (see Fig. 1)

$$W(\phi) = \kappa\sqrt{\beta}\phi\left(1-\frac{\beta}{3\kappa^2}\phi^2\right)+W_0\,, \tag{14}$$

$$V(\phi) = \frac{\beta^3}{2\kappa^2}\left(\phi^2-\frac{\kappa^2}{\beta}\right)^2-\frac{1}{54M^3}\frac{\beta^3}{\kappa^2}\phi^2\left(\phi^2-3\frac{\kappa^2}{\beta}\right)^2 + \frac{1}{9M^3}\frac{\beta^{3/2}}{\kappa}\phi\left(\phi^2-3\frac{\kappa^2}{\beta}\right)W_0-\frac{1}{6M^3}W_0^2\,. \tag{15}$$

The non-zero value of W_0 turns out to be essential to reproduce, in the brane limit, the generalized RS2 model presented in the previous section. Note that for $W_0=0$ the solution for $A(y)$ is symmetric under $y\leftrightarrow -y$ and it corresponds to the standard RS2 in the brane limit [5].

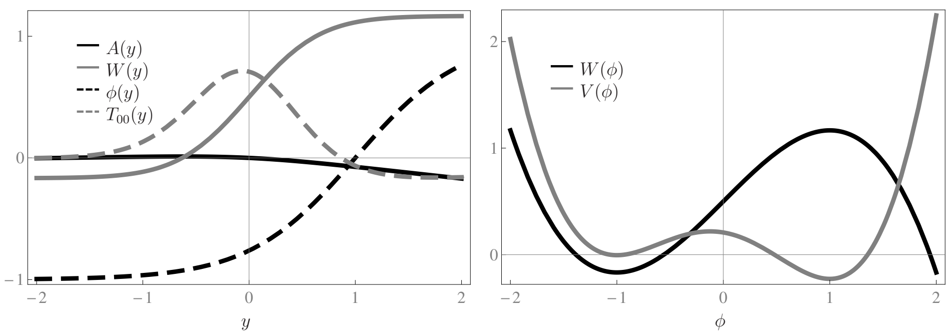

Fig. 1. For thick brane version of generalized RS2 model, the left graph shows the behavior of $A(y)$, $W(y)$, $\phi(y)$ and $T_{00}(y)$ as a function of y, whereas, the right graph presents the superpotential $W(\phi)$ and the potential $V(\phi)$ as a function of the scalar fields ϕ for $W_0=0.5M^4$ and $\kappa=\beta=M=1$.

It is straightforward to calculate the warp function $A(y)$ by integrating the second equation in Eq. (11) w.r.t. y. The result reads

$$A(y) = -\frac{\kappa^2}{72M^3\beta}\left(\tanh^2(\beta y)+\ln\cosh^4(\beta y)\right)-\frac{W_0}{12M^3}y\,. \tag{16}$$

The integration constant above was fixed by the condition $A(0)=0$. Note that far away from the thick brane the warp function approaches the generalized RS2 form (5) with

$$k_\pm = \frac{1}{24M^3}\frac{4}{3}\kappa^2+\frac{W_0}{12M^3}\mathrm{sgn}(y) = \frac{1}{24M^3}\lambda\pm\frac{W_0}{12M^3}$$

for $\lambda\equiv\frac{4}{3}\kappa^2$. It is also important to note that we get the same behavior of $A(y)$ (5), for all values of y in the brane limit *i.e.* when $\beta\to\infty$.

3. Two thick branes and the hierarchy problem

In this section, we consider the following double kink scalar field profile

$$\phi(y) = \frac{\kappa_1}{\sqrt{\beta}} \tanh(\beta(y - y_1)) + \frac{\kappa_2}{\sqrt{\beta}} \tanh(\beta(y - y_2)) \,. \tag{17}$$

We can find the superpotential $W(\phi)$ by a similar method as the one described in Sec. 2.1

$$\begin{aligned} W(y) =& \kappa_1^2 \left\{\tanh[\beta(y - y_1)] - \tfrac{1}{3}\tanh^3[\beta(y - y_1)]\right\} \\ &+ \kappa_2^2 \left\{\tanh[\beta(y - y_2)] - \tfrac{1}{3}\tanh^3[\beta(y - y_2)]\right\} + W_0 \,. \end{aligned} \tag{18}$$

The integration constant W_0 can be fixed by the requirement that $A(y)$ has a maximum at $y = y_1$ such that $A'(y_1) = 0$. After obtaining the superpotential $W(y)$, we find the warped function from Eq. (11)

$$\begin{aligned} A(y) = & \frac{1}{72M^3\beta} \left\{ \kappa_1^2 \left(\frac{1}{\cosh^2(\beta(y - y_1))} - \ln\cosh^4(\beta(y - y_1)) \right) \right. \\ & \left. + \kappa_2^2 \left(\frac{1}{\cosh^2(\beta(y - y_2))} - \ln\cosh^4(\beta(y - y_2)) \right) \right\} + \frac{1}{12M^3} W_0 y + A_0 \,, \end{aligned} \tag{19}$$

where A_0 is a constant of integration. Note that far away from the thick branes or in the brane limit ($\beta \to \infty$) the warp function approaches the RS form [1, 2]

$$A(y \to \infty) \sim -k|y| \,, \qquad |y| \gg |y_1|, |y_2| \,, \tag{20}$$

where $k = \frac{1}{24M^3}(\lambda_1 + \lambda_2 - W_0)$ with $\lambda_{1,2} = \frac{4}{3}\kappa_{1,2}^2$.

As it is seen from Fig. 2 to the left and to the right of the branes, the warp factor is quickly vanishing, which implies the localization of gravity [5]. If the branes are sufficiently thin (or well separated), then in between of them the warping is also nearly exponential so that the hierarchy problem could be addressed. We will call the branes located at y_1 and y_2 as UV and IR branes, respectively. To see the consequences of the warped background geometry lets assume that the Higgs field is bounded at the IR brane and its action can be written as

$$\mathcal{S}_H = -\int d^4x \sqrt{-\hat{g}} \left\{ \hat{g}_{\rm IR}^{\mu\nu} \partial_\mu H^\dagger \partial_\nu H - m^2 |H|^2 + \lambda |H|^4 \right\} , \tag{21}$$

where $\hat{g}_{\rm IR}^{\mu\nu}$ is the 4D metric induced on the IR brane, $\hat{g}_{\rm IR}^{\mu\nu} = e^{-2A(y_2)}\eta^{\mu\nu}$, with $A(y_2)$ being the value of warped factor at the IR brane and m is the 5D Higgs mass parameter (of the order of 5D Planck mass). After rescaling

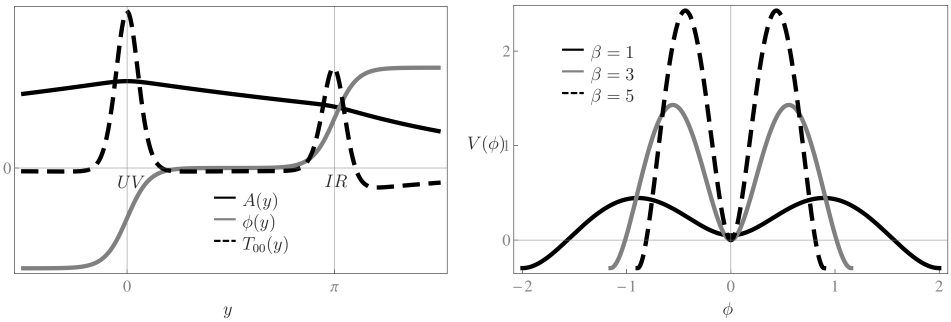

Fig. 2. The left graph shows the behavior of $e^{2A(y)}$, $\phi(y)$ and $T_{00}(y)$ as a function of y, whereas, the right graph presents the potential $V(\phi)$ as a function of the scalar fields ϕ for different values of β.

$H \to e^{-A} H$, we obtain the following action for canonically normalized Higgs field

$$\mathcal{S}_H = - \int d^4x \left\{ \eta^{\mu\nu} \partial_\mu H^\dagger \partial_\nu H - \mu^2 |H|^2 + \lambda |H|^4 \right\}, \tag{22}$$

where $\mu = m e^{A(y_2)}$ is the effective Higgs mass parameter as viewed on the IR brane. If we assume that the fundamental mass scale of the 5D theory is the Planck mass, then we can require the value of warped factor at IR brane such that we get the effective 4D Higgs mass parameter $\mu \sim$ TeV. It is easy to see that if $k \equiv \frac{1}{24M^3} (\lambda_1 + \lambda_2) y_2 \sim 30$, then the hierarchy problem could be solved. Furthermore, it was shown in Ref. [3], in such a scenario there exits a normalizable zero-mode which corresponds to the 4D graviton localized on the UV brane.

4. Summary and conclusion

We have discussed a generalized version of the Randall–Sundrum model 2 in which we allow for different cosmological constants on the two sides of the brane. We have considered a thick version of the model, in which the singular brane has been modeled by a configuration of a scalar field. Properties of the thick brane solution have been discussed. In order to address the hierarchy problem, we have introduced a scalar field profile composed of two kinks. This set-up in the brane limit corresponds to a model with two thin branes both having a positive tension. The presence of two branes allows to address the issue of the hierarchy problem which could be solved by the standard warping of the four dimensional metric provided the Higgs field is properly localized.

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