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THE SYMMETRY GROUP OF VECTOR AND AXIAL VECTOR CURRENTS*

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Abstract

We review, modify slightly, generalize, and attempt to apply a theory proposed earlier of a higher broken symmetry than the eightfold way. The integrals of the time components of the vector and axial vector current octets are assumed to generate, under equal time commutation, the algebra of SU(3) × SU(3). *The energy density of the strong interactions is assumed to consist of a piece invariant under the algebra, a piece that violates conservation of the axial vector currents only and belongs to the representation* (3, 3*) *and* (3*, 3), *and a piece that violates the eightfold way and probably belongs to* (1, 8) *and* (8, 1). *Assuming the algebraic structure is exactly correct, there is still the question of whether one can assign particles approximately to super-supermultiplets. The pseudoscalar meson octet, together with a pseudoscalar singlet, a scalar octet, and a scalar singlet, may belong to* (3, 3*) *and* (3*, 3). *The vector meson octet, together with an axial vector octet, may belong to* (1, 8) *and* (8, 1). *The baryon octet with* $J = 1/2^+$, *together with a singlet with* $J = 1/2^-$, *may belong to* (3, 3*) *and* (3*, 3), *as suggested before. Several crude coupling patterns and mass rules emerge, to zeroth or first order in the symmetry violations. Some are roughly in agreement with experiment, but certain predictions, like that of the existence of a scalar octet, have not been verified. Whether or not they are useful as an approximate symmetry, the equal time commutation rules fix the scale of the weak interaction matrix elements. Further rules of this kind are found to hold in certain Lagrangian field theory models and may be true in reality. In particular, we encounter an algebraic system based on* SU(6) *that relates quantities with different kinds of behavior under Lorentz transformations.*

I. Introduction

THE "eightfold way" theory of a broken higher symmetry for strong interactions was proposed [1, 2] at a time when the value of a badly violated symmetry was unclear for two reasons:

(1) It was not obvious what real significance could be assigned to the algebraic properties of the higher symmetry.

(2) It was not known whether the particle spectrum would show unmistakable evidence of the higher symmetry.

A solution was offered to the first problem when we pointed out [3] that the weak vector currents with $|\Delta \underline{I}| = 1/2$, $\Delta Y/\Delta Q = +1$, and $|\Delta \underline{I}| = 1$, $\Delta Y = 0$ generate an algebraic system through the equal-time commutation relations of their time-components and that this algebra is preserved even though the conservation of the strangeness-changing currents is violated. We assumed that the algebra in question is that of SU(3); no matter how badly the eightfold way is broken, the vector current octet is then the current of the F-spin. (This result was a simple generalization of the conserved vector current hypothesis, that the $\Delta Y = 0$ vector

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current is the current of the I-spin.) We went on to consider the equal-time commutation relations of the vector currents with the energy density, which interacts with gravitation. The assumption of the broken eightfold way is that the energy density is the sum of two pieces, one of which is invariant under the F-spin, and the other of which transforms like one component (by definition, the eighth) of an octet:

$$\mathscr{H} = \mathscr{H}_{\text{inv}} - c\,\phi_8. \tag{1}$$

The second problem arose when we attempted to arrange the isotopic multiplets of strongly interacting particles in supermultiplets that correspond, in order c^0, to irreducible representations of SU(3). We then derived, to zeroth and first order in c, coupling patterns and mass rules that were to be compared with experiment. We had no guarantee, however, even if our algebraic system was the right one, that these rules would be sufficiently well obeyed to show traces of the higher symmetry and the manner in which it is violated. Fortunately, ample evidence is now available to support the eightfold way symmetry, together with the pattern of violation in equation (1). In fact, the mass rules derived to first order in c are surprisingly accurate.

The success of the broken eightfold way, despite the large violations of symmetry involved, suggests that it may be worthwhile to study in detail the still higher symmetry associated with the axial vector currents. We proposed [3] that the axial vector currents with $|\Delta \underline{I}| = 1/2$, $\Delta Y/\Delta O = +1$, and $|\Delta \underline{I}| = 1$, $\Delta Y = 0$ belong to an octet with respect to F-spin and that the time-components of the vector and axial vector octets together generate, under equal time commutation, the algebra SU(3) × SU(3). Moreover, we suggested that the term $\mathscr{H}_{\text{inv}}$ in the energy density in equation (1) consists of two parts

$$\mathscr{H}_{\text{inv}} = \overline{\mathscr{H}} - \lambda\,u_0, \tag{2}$$

where $\overline{\mathscr{H}}$ is invariant under the full algebra and leaves both vector and axial vector currents conserved, while the term $-\lambda u_0$, transforming like a particular pair of representations of SU(3) × SU(3), violates conservation of the axial vector currents, while still commuting with the F-spin. These algebraic statements do not, of course, depend for their proposed validity on the smallness of the parameter λ.

We did not take altogether seriously, in ref. 3, the idea that these algebraic relations might be applied, like those of the eightfold way, by trying to assign particles to irreducible representations and finding rules to zeroth and first order in λ, to be compared with experiment. The success of the broken eightfold way, however, now makes it less ridiculous to see whether we can find traces of this even more badly broken symmetry. We would try to group the strongly interacting particle supermultiplets into super-supermultiplets, usually including particles of both parities, and to check by experiment very crude relations derived to low order in λ. In Section IV we describe the most plausible scheme of this kind, and note that an octet of scalar mesons is required for its success. Since no such octet has been clearly established at this time, we must reserve judgment on whether the approximation of small λ is of any use in describing the strong interactions.

It was mentioned in ref. 3 that an algebra is generated by the time-components of the vector and axial vector currents together with the symmetry-breaking term u_0 in the energy density. In Section VII we follow up this idea and discuss the possibility that the "extended algebra" may be rather small; we show that in a special model it is the algebra of SU(6) and suggest that such may be the case in reality. It is interesting that the extended algebra ties together quantities with different Lorentz transformation properties, such as the scalar u_0 and the four-vector currents.

The algebra [presumably SU(3) × SU(3)] of the vector and axial vector currents and the extended algebra [possibly SU(6)] can be used, in the form of equal time commutation relations, to supplement dispersion relations in the calculation of weak current matrix elements. It may also be true that more and more information about the strongly interacting particles can be expressed in algebraic language by repeated use of the notion of equal-time commutation relations.

II. Review of the Theory

We treat the strong interactions exactly, and the electromagnetic, weak, and gravitational interactions in lowest order. Even though we discuss the scattering amplitudes for strongly interacting particles on the mass shell only (by the method of dispersion relations or "S-matrix theory") we must still acknowledge that in lowest order the matrix elements of the weak and electromagnetic interactions can be determined for arbitrary mementum transfer by measurement and by analytic continuation of the measurable amplitudes. Presumably the same is true in principle of gravitation. Thus we may deal with electromagnetic and weak current operators and a stress-energy-momentum tensor operator $\theta_{\alpha\beta}$, all functions of a space-time variable x, with the matrix elements for any momentum transfer k given by Fourier transform and with time derivatives given by commutation with the space integral of the energy density $\mathscr{H} = -\theta_{44}$.

Incidentally, we notice in this way that the "S-matrix theory" of strong interactions, with electromagnetism, weak interactions, and gravitation treated as small purturbations, is just a branch of abstract field theory, since the current operators and $\theta_{\alpha\beta}$ are all field operators.

The weak current may be broken up, according to quantum numbers conserved by the strong interactions, first into a vector and an axial vector part, and then into pieces characterized by different values of $|\Delta \underline{I}|$ and $\Delta Y/\Delta Q$. We restrict our attention here to the familiar terms with $|\Delta \underline{I}| = 1$, $\Delta Y = 0$, and $|\Delta \underline{I}| = 1/2$, $\Delta Y/\Delta Q = +1$. If there are others, they may lead to bigger algebras than we have here, but need not invalidate our conclusions.

The integrals of the time components of all these currents generate some minimal algebraic system under equal time commutation. For those currents that are conserved, the corresponding integrals (like the electric charge) are constant operators; the others vary with time. But the structure constants of the algebra remain unchanged under all conditions and correspond to a law of nature that specifies the minimal algebra of the vector and axial vector currents that we are studying.

In ref. 3, we made three assumptions that determine the algebra: (a) The vector weak current, like the electromagnetic current, is a component of the F-spin current $\mathscr{F}_{i\alpha}(x)$, where $i = 1\ldots 8$ and α is a Lorentz index. We have, then,

$$F_i(t) = -i\int \mathscr{F}_{i4}\, d^3x, \tag{3}$$

$$[F_i(t), F_j(t)] = i f_{ijk} F_k(t). \tag{4}$$

Of course, F_1, F_2, and F_3 are conserved by the strong interactions and are just the components of the isotopic spin; thus the conserved vector current hypothesis is included here. The components F_4, F_5, F_6, and F_7 actually vary with time. (b) The axial vector weak current is the same component of another current $\mathscr{F}_{i\alpha}^5(x)$ that transforms like an octet with respect to F-spin. We have

$$F_i^5(t) = -i\int \mathscr{F}_{i4}^5\, d^3x, \tag{5}$$

$$[F_i(t), F_j^5(t)] = i f_{ijk} F_k^5(t). \tag{6}$$

(c) The commutation rules of the operators $F_i^5(t)$ close the algebraic system by giving

$$[F_i^5(t), F_j^5(t)] = i f_{ijk} F_k(t). \tag{7}$$

We now define

$$2F_i^{\pm}(t) \equiv F_i(t) \pm F_i^5(t) \tag{8}$$

and notice that F_i^+ and F_i^- are two commuting F-spins, so that we are really dealing with the algebra of SU(3) × SU(3). The two sets of operators, which we may think of as "left-handed" and "right-handed" F-spins respectively, are connected by parity:

$$PF_i^{\pm}P^{-1} = F_i^{\mp}. \tag{9}$$

The total F-spin itself is, according to equation (8), just the sum of the left- and right-handed parts:

$$F_i = F_i^+ + F_i^-. \tag{10}$$

We will be concerned with irreducible representations of the system consisting of F_i^+, F_i^-, and P. We indicate [3] the behavior with respect to (F_i^+, F_j^-) by a pair of representations, such as $(\underline{3}, \underline{3}^*)$, $(\underline{8}, \underline{8})$, etc. Since parity interchanges F_i^+ and F_i^-, an irreducible representation with respect to parity and the two F-spins will have such forms as $(\underline{8}, \underline{8})$ or $(\underline{3}, \underline{3}^*)$ and $(\underline{3}^*, \underline{3})$ or $(\underline{1}, \underline{8})$ and $(\underline{8}, \underline{1})$.

If a representation of F_i^+, F_i^-, and P is to contain a component invariant under F_i, it must have the form $(\underline{1}, \underline{1})$ or $(\underline{3}, \underline{3}^*)$ and $(\underline{3}^*, \underline{3})$ or $(\underline{8}, \underline{8})$, etc., so that the product of the two indicated representations will contain $\underline{1}$.

The simplest choice, then, for the term λu_0 in equation (2), which violates the conservation of F_i^+ and F_i^- separately while conserving F_i, is to have it belong to $(\underline{3}, \underline{3}^*)$ and $(\underline{3}^*, \underline{3})$, as proposed in ref. 3. The operator u_0 thus belongs to a set of nine scalar and nine pseudoscalar quantities, in each case forming an octet and a singlet with respect to total F-spin. The scalar octet is labeled $u_1 \ldots u_8$ and the singlet u_0, while the pseudoscalar octet and singlet are labeled $v_1 \ldots v_8$ and v_0 respectively.

To specify the transformation properties of the u's and v's, we introduce [3] a generalization of the symbols f_{ijk} and d_{ijk} to the case $i = 0, 1, \ldots 8$ instead of $i = 1 \ldots 8$. To the 3×3 matrices λ_i $(i = 1 \ldots 8)$ we adjoin the matrix $\lambda_0 = (2/3)^{1/2}\, 1$ and obtain the rules

$$[\lambda_i, \lambda_j] = 2 i f_{ijk} \lambda_k, \tag{11}$$

$$\{\lambda_i, \lambda_j\} = 2 d_{ijk} \lambda_k, \tag{12}$$

$$\mathrm{Tr}\, \lambda_i \lambda_j = 2 \delta_{ij}, \tag{13}$$

where i, j, and k run from 0 to 8. Here, f_{ijk} vanishes when any index is zero, and d_{ijk} equals $(2/3)^{1/2} \delta_{ij}$ when k is zero, etc.

We then obtain, for the transformation properties of the u_i and v_i, the results [3]

$$\begin{aligned} [F_i, u_j] &= i f_{ijk} u_k, \\ [F_i, v_j] &= i f_{ijk} v_k, \\ [F_i^5, u_j] &= -i d_{ijk} v_k, \\ [F_i^5, v_j] &= i d_{ijk} u_k. \end{aligned} \tag{14}$$

We note that equations (4), (6), and (7) indicate the representation to which the currents $\mathscr{F}_{i\alpha}(x)$ and $\mathscr{F}_{i\alpha}^5(x)$ belong, namely $(\underline{1}, \underline{8})$ and $(\underline{8}, \underline{1})$, with $i = 1 \ldots 8$. At equal times, we have

$$\begin{aligned} [F_i, \mathscr{F}_{j\alpha}] &= i f_{ijk} \mathscr{F}_{k\alpha}, \\ [F_i, \mathscr{F}_{j\alpha}^5] &= i f_{ijk} \mathscr{F}_{k\alpha}^5, \\ [F_i^5, \mathscr{F}_{j\alpha}] &= i f_{ijk} \mathscr{F}_{k\alpha}^5, \\ [F_i^5, \mathscr{F}_{k\alpha}^5] &= i f_{ijk} \mathscr{F}_{k\alpha}. \end{aligned} \tag{15}$$

The essential physics of the theory is contained in the equations written so far and is taken over directly from ref. 3. Two more points need to be added, however, which are modifications of the corresponding points in the earlier article. One of these concerns the component of $\mathscr{F}_{i\alpha}^+$ utilized for the weak current, and is discussed in the next section. The other point is connected with the transformation properties under SU(3) $\times$ SU(3) of the term in the energy density that violates the eightfold way, namely ϕ_8 in equation (1). The simplest possibilities for a unitary octet are, of course, $[(\underline{3}, \underline{3}^*)$ and $(\underline{3}^*, \underline{3})]$ and $[(\underline{1}, \underline{8})$ and $(\underline{8}, \underline{1})]$. It now appears that the latter may be more nearly satisfactory than the former, as we shall see in the next section. We had previously assumed not only that ϕ_8 transformed as $[(\underline{3}, \underline{3}^*)$ and $(\underline{3}^*, \underline{3})]$, i.e., like u_8, but also that ϕ_8 was equal to u_8.

III. Universality of the Weak Interactions

We know that the electromagnetic current of the strongly interacting particles (or "hadrons" to use Okun's expression) is given by the formula

$$j_\alpha = e\left(\mathscr{F}_{3\alpha} + \frac{1}{\sqrt{3}}\mathscr{F}_{8\alpha}\right). \tag{16}$$

(A constant term may have to be added if there exist hadrons corresponding to certain kinds of spinor representations of SU(3), as discussed in Section VI.)

What about the hadron weak current coupled to leptons? It must be a linear combination of $\mathscr{F}_{1\alpha}^{+} + i\mathscr{F}_{2\alpha}^{+}$ with $\Delta Y = 0$ and $\mathscr{F}_{4\alpha}^{+} + i\mathscr{F}_{5\alpha}^{+}$ with $\Delta Y/\Delta Q = +1$, if we stick to the assumptions (a) and (b) of Section II. The choice of the linear combination is motivated in part by the requirement of universality of the weak interactions: the algebraic properties of the total weak current should be the same for leptons and for hadrons [4, 5].

We write the effective weak interaction in the local approximation (or at least the part coming from a product of charged currents) as

$$\frac{G}{\sqrt{2}} J_\alpha^{+} J_\alpha, \tag{17}$$

where $J_\alpha = J_\alpha(\text{leptons}) + J_\alpha(\text{hadrons})$. The situation is then the following, as described in ref. 4.

For the now obsolete case of one neutrino for electron and muon, J_α(leptons) has the form

$$\bar{\nu}\gamma_\alpha(1+\gamma_5)e + \bar{\nu}\gamma_\alpha(1+\gamma_5)\mu = 2\sqrt{2}\,\bar{\nu}\gamma_\alpha\frac{(1+\gamma_5)}{2}\frac{(e+\mu)}{\sqrt{2}}.$$

The "weak charge" $-i\int d^3x\, J_4(\text{leptons})$ evidently may be written in the form $2\sqrt{2}\,(K_1 + iK_2)$, where K_1, K_2, and $-i[K_1, K_2]$ have the commutation rules of an angular momentum or an isotopic spin. The weak charge and its hermitian conjugate, for leptons, generate the algebra of SU(2), with $\frac{1+\gamma_5}{2}\frac{e+\mu}{\sqrt{2}}$ and $\frac{1+\gamma_5}{2}\nu$ appearing as the lower and upper components of a spinor.

With distinct neutrinos for electron and muon, as in the real situation, J_α(leptons) becomes

$$\bar{\nu}_e\gamma_\alpha(1+\gamma_5)e + \bar{\nu}_\mu\gamma_\alpha(1+\gamma_5)\mu = 2\bar{\nu}_e\gamma_\alpha\frac{1+\gamma_5}{2}e + 2\bar{\nu}_\mu\gamma_\alpha\frac{1+\gamma_5}{2}\mu. \tag{18}$$

This time the leptonic weak charge has the form $2(K_1 + iK_2)$, where again K_1 and K_2 are the first two components of an angular momentum and the algebra of SU(2) is generated. Now $\frac{1+\gamma_5}{2}e$ and $\frac{1+\gamma_5}{2}\nu_e$ form a spinor and so do $\frac{1+\gamma_5}{2}\mu$ and $\frac{1+\gamma_5}{2}\nu_\mu$.

Let us now demand universality for the weak interactions. In the one-neutrino case, we would require that the weak charge for hadrons have the form $2\sqrt{2}\,(K_1 + iK_2)$ and in the two-neutrino case that it have the form $2(K_1 + iK_2)$, where K_1 and K_2 are the first two components of an angular momentum. Writing the weak charge for hadrons in the general form

$$A(F_1^{+} + iF_2^{+})\cos\theta + A(F_4^{+} + iF_5^{+})\sin\theta,$$

we may verify that we have $A(K_1 + iK_2)$, where

$$K_1 = F_1^+ \cos\theta + F_4^+ \sin\theta,$$

$$K_2 = F_2^+ \cos\theta + F_5^+ \sin\theta,$$

$$K_3 = F_3^+ \cos^2\theta + \left(\frac{\sqrt{3}}{2}F_8^+ + \frac{1}{2}F_3^+\right)\sin^2\theta - F_6^+ \sin\theta\cos\theta.$$

Now from the approximate equality of vector coupling constants in the decay of the muon and the decay of the nucleus 0^{14}, we know that $A \cos\theta$ is around 2.

Clearly, then, universality in the one-neutrino case gives us $A = 2\sqrt{2}$, $\theta = 45°$ or

$$J_\alpha = 2(\mathscr{F}_{1\alpha}^+ + i\mathscr{F}_{2\alpha}^+ + \mathscr{F}_{4\alpha}^+ + i\mathscr{F}_{5\alpha}^+)$$

as in ref. 3. However, for the actual case of two neutrinos, we must take $A = 2$ with θ small and have

$$J_\alpha = 2\cos\theta(\mathscr{F}_{1\alpha}^+ + i\mathscr{F}_{2\alpha}^+) + 2\sin\theta(\mathscr{F}_{4\alpha}^+ + i\mathscr{F}_{5\alpha}^+). \tag{19}$$

In a recent paper, Cabibbo [6] has combined our assumptions (a) and (b) quoted in Section II with the choice (18) of current components suitable for universality in the two-neutrino case and has shown that such a theory is in reasonable agreement with present information on leptonic decays of hadrons, with $\theta \approx 0.26$. We may therefore adopt equation (18) with some confidence, provided experimental leptonic and hadronic weak interactions exhibit no further complications.

The weak charge in general thus has the form $2(K_1 + iK_2)$. Moreover, the electric charge in units of e has the general form $K_3 + K_0$, where K_0 commutes with K_1, K_2, and K_3. The weak and electric charge operators, for both leptons and hadrons, thus generate the algebra of U(1) × SU(2). It is only when we take these charges for hadrons and break them up according to the quantum numbers conserved by the strong interactions that we get the group SU(3) × SU(3).

IV. Crude Results for Small λ

We now attempt to make use of the broken symmetry model for rough predictions about the strongly interacting particles. For the most part we shall put $c = 0$ and forget about violations of the eightfold way, concentrating on axial vector current conservation and *its* violation. In the limit $\lambda \longrightarrow 0$, where all the axial vector currents are conserved, if the axial vector β-decay coupling constant is not to vanish, we must have either vanishing baryon masses or vanishing pseudoscalar meson masses. We choose vanishing baryon masses, and thus the point of view expressed here differs from that of many authors [7]. Under these conditions, the "renormalization constant" for the axial vector current becomes unity in the limit $\lambda \longrightarrow 0$. The β-decay interaction in the zero-momentum transfer case is thus completely fixed, in the limit $\lambda \longrightarrow 0$, by the symmetry pattern.

Since we are going to try assigning dominant representations to the particles, let us begin with the eight baryons having $J = 1/2^+$. For convenience we describe them, in the limit $\lambda \longrightarrow 0$, by "fields" $\psi_1 \ldots \psi_8$. If they belonged to $(\underline{1}, \underline{8})$ and $(\underline{8}, \underline{1})$, then ψ_j and $\gamma_5\psi_j$ would transform under F_i and F_i^5 in the same way that $\mathscr{F}_{j\alpha}$ and $\mathscr{F}_{j\alpha}^5$ respectively transform in equation (15). In the zero momentum transfer case, then, in the limit $\lambda \longrightarrow 0$, we would have the following pattern for the weak current $\mathscr{F}_{i\alpha}^+$:

$$-if_{ijk}\overline{\psi}_j\gamma_\alpha(1 + \gamma_5)\psi_k.$$

Both the vector and axial vector currents would be coupled through F rather than D. This seems to be far from the truth [6].

Instead, we try the other baryon representation suggested in ref. 3, namely $(\underline{3}, \underline{3}^*)$ and $(\underline{3}^*, \underline{3})$. We then have to add a ninth particle, described by ψ_0. Under F_i and F_i^5, ψ_i and $\gamma_5\psi_i$ transform like u_i and iv_i in equation (14). The coupling pattern as $\lambda \longrightarrow 0$, for any momentum transfer, is then

$$-if_{ijk}\overline{\psi}_j\gamma_\alpha\psi_k + d_{ijk}\overline{\psi}_j\gamma_\alpha\gamma_5\psi_k \tag{20}$$

with $i, j, k = 0, 1, \ldots 8$. For the baryon octet, then, the vector current is coupled through F and the axial vector current through D, with equal coefficients. This situation resembles the experimental one [6], the admixture of F in the axial vector pattern being of the order of 30%. There is a single form factor for the whole expression (20) in the limit $\lambda \longrightarrow 0$, and the anomalous magnetic and induced pseudoscalar form factors vanish. Since the vector coupling is through F, the neutron has no electrical interaction in the limit.

The interpretation of the ninth baryon depends on whether the mass associated with ψ_0 is positive or negative. A negative mass would lead us to treat $\gamma_5\,\psi_0$ as the appropriate operator, so that the new particle with positive mass would have $J = 1/2^-$. Now in first order in λ we can compute the ratio of octet and singlet masses, using the transformation property $(\underline{3}, \underline{3}^*)$ and $(\underline{3}^*, \underline{3})$ of the mass term u_0. The result is that the mass associated with the singlet is minus twice that of the octet. Thus $\gamma_5\,\psi_0$ should describe, to first order in λ, a baryon with $J = 1/2^-$ and twice the average mass of the baryon octet with $J = 1/2^+$. If the extra baryon is identified with $\Lambda(1405)$, the spin and parity assignments may well be right but 1405 MeV is a far cry from twice the average mass of the baryon octet. We should perhaps not expect better agreement, however, with an approximation that treats baryon masses in first order.

The pseudoscalar octet should be assigned to the same representation as the divergence of the axial vector current, so that the Goldberger-Treiman relation can have some validity even to lowest order in λ. As explained in ref. 3, we have

$$\int d^3x\, \partial_\alpha \mathscr{F}_{i\alpha}^{5} = \dot{F}_i^{5} = i\left[\int \mathscr{H} d^3x,\, F_i^{5}\right] = \lambda\, i\left[F_i^{5},\, \int u_0\, d^3x\right]$$
$$= \lambda\, d_{i0k}\int v_k\, d^3x = \sqrt{\frac{2}{3}}\,\lambda \int v_i\, d^3x \tag{21}$$

neglecting c. So the divergence of the current, in lowest order, belongs to $(\underline{3}, \underline{3}^*)$ and $(\underline{3}^*, \underline{3})$ and we make the same assignment for the pseudoscalar octet, along with a pseudoscalar singlet, a scalar octet, and a scalar singlet. For convenience, we describe these by "fields" π_i and σ_i, $i = 0, \ldots 8$, transforming like v_i and u_i respectively.

To order λ^0, these eighteen mesons all have a common mass. In order λ, they split according to the following pattern: the scalar singlet has a squared mass equal to $\overline{\mu^2} - 2\Delta$, the pseudoscalar octet has $\overline{\mu^2} - \Delta$, the scalar octet has $\overline{\mu^2} + \Delta$, and the pseudoscalar singlet has $\overline{\mu^2} + 2\Delta$. It is possible that the scalar singlet may be identified with a low-lying $J = 0^+$, $I = 0$ mesonic state decaying quickly into 2π; such states have been reported at various masses and at least one of them may exist. We then expect a scalar octet lying higher than the pseudoscalar one and a pseudoscalar singlet lying still higher.

The matrix elements of the weak current between one of these mesons and another follow the same pattern, in the limit $\lambda \longrightarrow 0$, as for the baryons in equation (20), since these mesons and the nine baryons belong to the same representation:

$$-i f_{ijk}(\pi_j \partial_\alpha \pi_k + \sigma_j \partial_\alpha \sigma_k) + d_{ijk}(\sigma_j \partial_\alpha \pi_k - \pi_j \partial_\alpha \sigma_k). \tag{22}$$

In the limit, there is a common form factor for the whole of (22).

If the scalar singlet meson represented by σ_0 (let us call it σ) really has a lower mass than K^+, then the K_{e4} decay $K^+ \longrightarrow \pi^+ + \pi^- + e^+ + \nu$ should be dominated by the chain $K^+ \longrightarrow \sigma + e^+ + \nu$, $\sigma \longrightarrow \pi^+ + \pi^-$. Treating σ as stable and using the zeroth order pattern (22), we may compute the ratio of the rates of $K^+ \longrightarrow \sigma + e^+ + \nu$ and $K^+ \longrightarrow \pi^0 + e^+ + \nu$:

$$\frac{K^+ \longrightarrow \sigma + e^+ + \nu}{K^+ \longrightarrow \pi^0 + e^+ + \nu} = \frac{8}{3}\,\frac{f(m_\sigma/m_K)}{f(m_\pi/m_K)}, \tag{23}$$

where

$$f(\beta) = 1 - 8\beta^2 + 24\beta^4 \ln \beta^{-1} + 8\beta^6 - \beta^8. \tag{24}$$

Roughly, then, the relative rate of $K^+ \longrightarrow \pi^+ + \pi^- + e^+ + \nu$ should be given by 2/3 of the expression (23) and the mass of the $\pi^+\pi^-$ system should be clustered around m_σ. The experimental data [8] still do not permit any firm conclusions, except that the ratio calculated in (23) must be a few times 10^{-3} if the decay through σ takes place. By contrast, if the σ mass is as low as 310 MeV, our rough formula gives about 0.21, in complete disagreement with observation. With a mass near 400 MeV, agreement is possible.

The symmetry, in the limit $\lambda \longrightarrow 0$, permits trilinear couplings of the scalar and pseudoscalar mesons to one another and to the nine baryons. In each case the allowed coupling pattern is formed with the symbol d'_{ijk}, which equals d_{ijk} except when one index is zero and the other two equal but not zero; the value of d' is then $-\sqrt{1/6}$. The effective couplings are:

$$2g\, d'_{ijk}\bar{\psi}_i(\sigma_j + i\pi_j\gamma_5)\psi_k, \tag{25}$$

$$(h/6)\, d'_{ijk}(\sigma_i + i\pi_i)(\sigma_j + i\pi_j)(\sigma_k + i\pi_k) + \text{hermitian conjugate}. \tag{26}$$

The coupling of the pseudoscalar octet to the baryon octet through D is, of course, in reasonable agreement with experiment, as we might have expected from the Goldberger-Treiman relation and the axial vector current coupling through D.

Other predictions, however, are not in good agreement with the present experimental situation. No scalar octet has been found. The nuclear forces arising from the scalar particles should be gigantic, according to (25), and evidence for such forces is not convincing at the present time. Also the scalar coupling of the ninth baryon $\gamma_5\psi_0$ to the pseudoscalar octet and the baryon octet should also be very strong, according to (25), and lead to a width of several BeV (!) for the ninth baryon if we take the prediction literally. An estimate in the next section of h in equation (26) indicates large widths also for the scalar mesons if they are significantly above the thresholds for decay into two pseudoscalar mesons.

We may thus adopt several different attitudes:

(1) The SU(3) × SU(3) algebraic system, assuming it is correct, does not provide a useful approximate symmetry.
(2) Higher order effects in λ, for example as indicated by the Goldberger-Treiman relation (see next section), may reduce some of the coupling constants.
(3) The scalar mesons will turn up with large couplings, and the ninth baryon will turn up as a very vague bump with a huge width.
(4) Something is wrong with our choices of representations.

At present, it is not easy to choose among these possibilities. They are discussed further in Section V. Meanwhile, we return to the assignment of representations.

The vector meson situation is complicated by the $\phi-\omega$ mixing and will not be fully treated here, but we should expect a vector meson octet that dominates the vector form factors to transform like the currents, i.e., according to $(\underline{1}, \underline{8})$ and $(\underline{8}, \underline{1})$. Thus there should be an axial vector octet nearby. In fact, the splitting between the two octets is of second order in λ, since we cannot make $(\underline{3}, \underline{3}^*)$ and $(\underline{3}^*, \underline{3})$ out of $[(\underline{1}, \underline{8})$ and $(\underline{8}, \underline{1})]$ times itself.

In conclusion, we mention the violation of the eightfold way by the term in ϕ_8. If ϕ_8 were to belong to $(\underline{3}, \underline{3}^*)$ and $(\underline{3}^*, \underline{3})$, then the splitting of the baryon octet with $J = 1/2^+$ would go mainly with D, since to order $\lambda^0 c^1$ the only allowed coupling is analogous to (25). In fact, the splitting is mostly F. We may therefore consider the possibility that ϕ_8 belongs to $(\underline{1}, \underline{8})$ and $(\underline{8}, \underline{1})$. The baryon octet is then split only in order $\lambda^1 c^1$ and both F and D come in.

The scalar and pseudoscalar octets are split open in order $\lambda^0 c^1$ by a term ϕ_8 transforming like $(\underline{1}, \underline{8})$ and $(\underline{8}, \underline{1})$. Moreover, in that order, the spacing is the same for both octets.

V. The Goldberger-Treiman Relation

The Goldberger-Treiman relation states in essence that certain matrix elements of the divergence $\partial_\alpha \mathscr{F}_{i\alpha}^5$ of the axial vector current components obey unsubtracted dispersion relations in the invariant momentum squared carried by the current and that these dispersion relations are dominated by the intermediate state with one pseudoscalar meson [3, 9].

The pion decay amplitude is described by the quantity f_π, which is defined, as in ref. 3, by the formula

$$<0\,|\partial_\alpha \mathscr{F}_{i\alpha}^{5}\,|\,\pi> = \frac{m_\pi^{\,2}}{2 f_\pi}\,\Phi_i, \qquad (i = 1, 2, 3) \tag{27}$$

where Φ_i is the wave function of the pion. The width for the decay $\pi^+ \longrightarrow \mu^+ + \nu_\mu$ is then

$$\Gamma_\pi = G^2 \cos^2\theta\, m_\pi\, m_\mu^{\,2}(1 - m_\mu^{\,2}/m_\pi^{\,2})^2 \left(\frac{f_\pi^{\,2}}{4\pi}\right)^{-1} (64\pi^2)^{-1}, \tag{28}$$

which differs by the factor $\cos^2\theta$ from the corresponding expression in ref. 3. Likewise, we have

$$<0\,|\partial_\alpha \mathscr{F}_{i\alpha}^{5}\,|\,K> = \frac{m_K^{\,2}}{2 f_K}\,\Psi_i, \qquad (i = 4, 5, 6, 7) \tag{29}$$

where Ψ_i is the wave function of the kaon. The width for the decay $K^+ \longrightarrow \mu^+ + \nu_\mu$ is

$$\Gamma_K = G^2 \sin^2\theta\, m_K\, m_\mu^{\,2}(1 - m_\mu^{\,2}/m_K^{\,2})^2 \left(\frac{f_K^{\,2}}{4\pi}\right)^{-1} (64\pi^2)^{-1}, \tag{30}$$

where in ref. 3 we would not have had the factor $\sin^2\theta$. Cabibbo [6] has pointed out that with $\theta \approx 0.26$ we get $f_\pi \approx f_K$. (The attempt [3] to make $f_\pi/f_K \approx m_\pi/m_K$ is thus unnecessary.)

The Goldberger-Treiman relation for neutron β decay now states that

$$2 m_N \left(\frac{-G_A}{G\cos\theta}\right) \approx g_{NN\pi}\, f_\pi^{-1}, \tag{31}$$

where $(-G_A/G\cos\theta)$ is the axial vector "renormalization factor" for the nucleon. The matrix element between neutron and proton of the divergence of the axial vector current has been approximately expressed as the product of the pion-nucleon coupling constant $g_{NN\pi}$ and the pion decay constant f_π^{-1}. Experimentally, equation (31) is satisfied with an error of around 10%.

If we now generalize to the baryon octet and the pseudoscalar meson octet, in the approximation of the eightfold way (which includes $f_\pi = f_K = f$), we have in general a part of g that goes with the D coupling and a part with the F coupling; the same is true of $(-G_A/G\cos\theta)$. Not only should the relation (31) hold, then, but also the F/D ratios should be the same for the meson-baryon coupling and for the axial vector current. Cabibbo's value of 0.30/0.95 for the F/D ratio for the current agrees well with all estimates of F/D for the meson coupling.

If scalar mesons exist, the Goldberger-Treiman relation should apply to the β-decay matrix elements between scalar and pseudoscalar meson states, in relation to the strong coupling constants for the scalar-pseudoscalar-pseudoscalar $(\sigma\,\pi\,\pi)$ vertices.

Now let us examine what happens to the Goldberger-Treiman relation for the $NN\pi$ and $\sigma\pi\pi$ cases when we have approximate conservation of both vector and axial vector currents, i.e., $c = 0$ and $\lambda \longrightarrow 0$, assuming our assignments of both baryons and mesons to $(\underline{3}, \underline{3}^*)$ and $(\underline{3}^*, \underline{3})$ are correct. In the limit $\lambda \longrightarrow 0$, the matrix elements $<0\,|u_i|\,\sigma_i>$ and $<0\,|v_i|\,\pi_i>$ are all non-zero and equal. The quantity f^{-1}, proportional to $<0\,|\partial_\alpha\, F_{i\alpha}^{\,5}\,|\,\pi_i>$, is evidently of order λ. Likewise, the mass of the nucleon is of order λ. The renormalization $(-G_A/G\cos\theta)$ approaches unity, with the octet pattern becoming pure D. The coupling constant $g_{NN\pi}$ remains finite and the octet coupling pattern here too becomes pure D. Evidently, then, the Goldberger-Treiman relation (27) can hold approximately in the limit $\lambda \longrightarrow 0$ with both sides of order λ. We have, then, in the double approximation of $\lambda \longrightarrow 0$ and exact validity of the Goldberger-Treiman relation,

$$2 m_N \approx g f^{-1}, \tag{32}$$

where g multiplies the whole baryon-baryon-meson coupling pattern, as in (25). The analogous equation for the $\sigma\pi\pi$ vertices is

$$2\Delta \approx h f^{-1}, \tag{33}$$

where 2Δ is the difference in mass squared between the scalar and pseudoscalar octets in order λ and h multiplies the whole trilinear meson coupling pattern, as in (26). This double approximation is equivalent to saying that the violation of SU(3) $\times$ SU(3) mass degeneracy is accomplished formally by the displacement

$$\sigma_0 \longrightarrow \sigma_0 - \sqrt{\frac{3}{8}}\, f^{-1} \tag{34}$$

in effective couplings such as (25) and (26).

Now if we consider large violations of symmetry, so that higher order effects in λ are important (and even effects involving c, which violate the eightfold way), we may suppose that the Goldberger-Treiman approximation is still good. For example, as we mentioned above, the actual value of $(-G_A/G \cos\theta)$ is around 1.25, with about 0.95 going with the D pattern and about 0.30 with the F pattern, in contrast with the value 1 and the pure D coupling that we would have in the limit $\lambda \longrightarrow 0$. Likewise, the baryon-baryon-meson coupling departs from the pure D coupling that is acquired in the limit $\lambda \longrightarrow 0$, but these two departures seem to follow the Goldberger-Treiman relation in that the F/D ratio is similar in the two cases.

We may look, for example, at the decay of the ninth baryon into $\Sigma + \pi$. In order λ, the mass difference between the two baryons is the same as m_N and in order 1 the coupling constant is the same as $g_{NN\pi}$, with the appropriate ratio of d'_{ijk} coefficients. If, now, in higher order in λ and c the mass difference becomes quite different, we might expect the coupling constant to change in proportion, keeping the Goldberger-Treiman equation approximately valid. Instead of having the effective coupling constant g for the decay equal to $2m_N f_\pi$, as in (32), we would have approximately $2(m - m_\Sigma)f_\pi r$, where m is the real mass of the ninth baryon and r is the renormalization factor for the axial vector current matrix element between Σ and the ninth baryon. If m is 1405 MeV, for instance, then the coupling constant g in question is reduced by the factor $r(m - m_\Sigma)m_N^{-1} \approx 0.23r$ and the width of the ninth baryon is then

$$\Gamma \approx 4\frac{[2f_\pi(m - m_\Sigma)r]^2}{4\pi}k\frac{m_\Sigma + E}{2m},$$

where k is the decay momentum. The width comes out about 200 MeV(r^2) instead of about 3 BeV(r^2); the actual width of $\Lambda(1405)$ is around 60 MeV.

Similar corrections should be applied to the various coupling constants h for the various $(\sigma\,\pi\,\pi)$ vertices; instead of the completely symmetrical formula (33) we can use similar expressions in which the actual differences of mass squared are inserted in place of the first order pattern based on the single quantity Δ. For a scalar K particle of mass μ, the decay into $K + \pi$ would be regulated by a value of h approximately equal to $f_\pi(\mu^2 - m_K^2)r$, where r is the renormalization factor for the axial vector current matrix element between the scalar K particle and K itself. The decay width for this case is then

$$\Gamma \approx \frac{3}{2}\frac{[f_\pi(\mu^2 - m_K^2)r]^2}{4\pi}\frac{k}{\mu^2},$$

where k is the decay momentum. For $\mu \approx 725$ MeV, for example, Γ comes out around 80 MeV(r^2).

None of this is of much use, of course, if the scalar octet does not exist. If it is not found, we will have to abandon the idea of using the group SU(3) $\times$ SU(3) of the vector and axial vector currents as an approximate symmetry of the strong interactions.

We should mention one intermediate possibility, which involves a different assignment of representation to the pseudoscalar octet, namely $(\underline{1}, \underline{8})$ and $(\underline{8}, \underline{1})$. No scalar or pseudoscalar unitary singlet would be predicted. The Goldberger-Treiman relation could not in this case be approximately valid for small λ, since the two sides would be of different order in λ, with the pseudoscalar octet not transforming like v_i under SU(3) $\times$ SU(3). Conceivably, however, for a particular value of λ, not particularly small, we could have the relation. The coupling of baryons to the pseudoscalar and scalar mesons would be forbidden as $\lambda \longrightarrow 0$ and the pseudoscalar octet would acquire its coupling through the violation of the symmetry. The scalar octet would have the opposite properties under charge conjugation to the scalar octet hitherto discussed. Thus in the limit $c \longrightarrow 0$ of the eightfold way, this scalar octet could have no Yukawa coupling to the baryon octet and no coupling to two mesons of the pseudoscalar octet. When the violation of the eightfold way is turned on, the non-strange members of the scalar octet would still lack these couplings, but the strange members could have them by violating the eightfold way. Such an "abnormal" scalar octet would have very different experimental properties from a normal one, particularly for the $Y = 0$ members, and would be readily identifiable as such.

VI. Triplets, Real and Mathematical

So far, we have concerned ourselves with the assumption that the group of the vector and axial vector octets of currents is SU(3) × SU(3), with the transformation properties under the group of the terms λu_0 and $c\phi_8$ in the energy density, and with the possibility that in a crude approximation λ as well as c might be treated as small. We may, however, go further and ask whether there are additional algebraic relations among the quantities we have introduced. In order to obtain such relations that we may conjecture to be true, we use the method of *abstraction* from a Lagrangian field theory model. In other words, we construct a mathematical theory of the strongly interacting particles, which may or may not have anything to do with reality, find suitable algebraic relations that hold in the model, postulate their validity, and then throw away the model. We may compare this process to a method sometimes employed in French cuisine: a piece of pheasant meat is cooked between two slices of veal, which are then discarded [10].

In ref. 3, the Sakata model was employed in this way. However, certain adjustments had to be made to get to the eightfold way. Instead, we may employ the quark model [11], which gives the eightfold way directly; there are also other models [11], based on a fundamental triplet and a fundamental singlet, that are perfectly compatible with the eightfold way.

Any such field-theoretic model must contain some basic set of entities, with non-zero baryon number, out of which the strongly interacting particles can be made. If this set is a unitary octet, the theory is very clumsy; it is hard to arrange any coupling that will reduce the symmetry from SU(8) to SU(3) without introducing [1, 2] in addition a Yang-Mills octet of fundamental vector mesons. Thus the only reasonably attractive models are based on unitary triplets and perhaps singlets.

If we adopt such a viewpoint, we should say that the correct dynamical description of the strongly interacting particles requires either the bootstrap theory or else a theory based on a fundamental triplet. In neither case do the familiar neutron and proton play any basic role.

It is, of course, a striking fact that no unitary triplets have so far been identified among the strongly interacting particles; however, they may turn up. Their appearance may, of course, be consistent with either the bootstrap theory or a theory with a fundamental triplet. Their non-appearance could certainly be consistent with the bootstrap idea, and also possibly with a theory containing a fundamental triplet which is hidden, i.e., has effectively infinite mass.

Thus, without prejudice to the independent questions of whether the bootstrap idea is right and whether real triplets will be discovered, we may use a mathematical field theory model containing a triplet in order to abstract algebraic relations.

If we want to use just a triplet and no singlet, we must have quarks, with baryon number 1/3 and electric charges $-1/3$, $-1/3$, and $+2/3$. Such particles presumably are not real but we may use then in our field theory model anyway. Since the quark model is mathematically the simplest, we shall in fact employ it in the next section, as in ref. 11, for our process of abstraction.

If we consider a model with a basic triplet t and a singlet b, then we are free to take for these particles integral electric charges and baryon number equal to one; say we do so. The singlet must be neutral, and we can then form the known supermultiplets from (b), $(b\ t\ \bar{t})$, $(b\ \bar{b}\ b\ \bar{t}\ t\ \bar{t}\ t)$, etc. The triplet can have electric charges q, q, $q+1$ or $-q$, $-q$, $-q-1$, where q is any integer. In the former case, the electric charge Q in units of e is given by the relation

$$Q = (q + 1/3)(n_t - n_{\bar{t}}) + F_3 + \frac{F_8}{\sqrt{3}},$$

where $n_t - n_{\bar{t}}$ is the number of triplets minus the number of antitriplets. In the latter case we have the relation

$$Q = -(q + 1/3)(n_t - n_{\bar{t}}) + F_3 + \frac{F_8}{\sqrt{3}}.$$

With integral charges and baryon number one, there is no reason to require any member of a real triplet to be absolutely stable; we may permit them all to decay into ordinary baryons. However, the question arise whether such decays take place by weak interactions or by moderately strong interactions that violate SU(3) but not isotopic spin conservation. If the latter, we want the violation of SU(3) to transform like $\underline{3}$ and $\underline{3}^*$, so that in second order it gives the familiar octet behavior of the violation. But $I = 0$, $Q = 0$ occurs in $\underline{3}$ and $\underline{3}^*$ only when $q = 0$. Thus if the decay of triplets into baryons is to be attributed to moderately strong interactions that give rise, in second order, to the octet violation of SU(3), we must have $q = 0$. This is the model used by Maki [12] and by Tarjanne and Teplitz [13].

With other values of q, we presumably have the triplet decaying into ordinary baryons by the weak interaction. As pointed out previously [11, 14], the most interesting case of this kind is the one with $q = -1$, where the four members of the basic triplet and singlet present a perfect analogy with the known leptons.

Absolute stability of one member of a triplet is of course, a possibility for any value of q.

These kinds of triplets with integral charges are, of course, more likely to correspond to real particles than the quarks, and we may also use them in field theory models to abstract algebraic relations, obtaining essentially the same ones as for the quarks in the next section.

VII. Further Algebraic Relations

We start with the simple Lagrangian model of quarks discussed in ref. 11. There is a triplet t of fermio fields corresponding to three spin 1/2 quarks: the isotopic doublet u and d, with charges 2/3 and $-1/3$ respectively, and the isotopic singlet s, with charge $-1/3$. A neutral vector meson field B_α is introduced, too. The Lagrangian is simply

$$-\bar{t}\gamma_\alpha \partial_\alpha t - \mathscr{L}_B - iF B_\alpha \bar{t}\gamma_\alpha t$$

as $\lambda \longrightarrow 0$ and $c \longrightarrow 0$, where $\mathscr{L}_B$ is the free Lagrangian for the field B and

$$\bar{t}\gamma_\alpha t = \bar{u}\gamma_\alpha u + \bar{d}\gamma_\alpha d + \bar{s}\gamma_\alpha s.$$

Now we may add to the Lagrangian a quark mass term

$$\lambda u_0 = m_0(\bar{u}u + \bar{d}d + \bar{s}s) = m_0 \bar{t}t.$$

The energy density acquires a term that is just the negative of this. In the model we may put

$$u_i = \bar{t}\frac{\lambda_i}{2}t, \qquad v_i = -i\bar{t}\gamma_5\frac{\lambda_i}{2}t, \qquad i = 0, 1, \ldots 8,$$

and $\lambda = \sqrt{6}\, m_0$. Likewise, we have, in the model,

$$\mathscr{F}_{i\alpha} = i\bar{t}\frac{\lambda_i}{2}\gamma_\alpha t, \qquad \mathscr{F}_{i\alpha}^5 = i\bar{t}\frac{\lambda_i}{2}\gamma_\alpha\gamma_5 t.$$

For the moment, we forget the term $c\phi_8$ that breaks the eightfold way.

The non-singularity of the model enables us to generalize [3, 11] the commutation relations (14) and (15) and equation (20) to the local relations

$$[\mathscr{F}_{i4}(\underline{x}, t), u_j(\underline{x}', t)] = -f_{ijk} u_k(\underline{x}, t)\delta(\underline{x} - \underline{x}'), \text{ etc.} \tag{35}$$

$$[\mathscr{F}_{i4}(\underline{x}, t), \mathscr{F}_{j4}(\underline{x}', t)] = -f_{ijk} \mathscr{F}_{k4}(\underline{x}, t)\delta(\underline{x} - \underline{x}'), \text{ etc.,} \tag{36}$$

$$\partial_\alpha \mathscr{F}_{i\alpha}^5 = \sqrt{\frac{2}{3}}\lambda v_i + \mathscr{O}(c). \tag{37}$$

We have proposed that these relations be abstracted from the model and postulated as true. In an exact calculation of the matrix elements of the $\mathscr{F}_{i\alpha}$, u_i, and v_i by means of linear homogeneous dispersion relations without subtractions, the nonlinear relations (35) and (36) supply the scale factors that determine such things as the axial vector current renormalization.

In the quark model, the term $c\,\phi_8$ in the Lagrangian could be put in as a mass difference between singlet and doublet quarks, but ϕ_8 would then be the same as u_8 and would transform like $(\underline{3}, \underline{3}^*)$ and $(\underline{3}^*, \underline{3})$. If we want ϕ_8 to belong to $(\underline{1}, \underline{8})$ and $(\underline{8}, \underline{1})$, we could put it into the model as a coupling of the meson B_α to the current $\mathscr{F}_{8\alpha}$. Such a term is reminiscent of Ne'eman's "Fifth Interaction" [15] or of Sakurai's use [16] of $\phi-\omega$ mixing as a dynamical mechanism for violating the eightfold way.

We mentioned in ref. 3 that in the model there are further commutation relations, besides (35) and (36), which we might or might not take seriously, namely the commutation relations of the u's and v's. It is interesting that when these operators are commuted, in the model, they bring back the operators $\mathscr{F}_{i4}$ and $\mathscr{F}_{i4}^{\,5}$, along with a new operator, the helicity charge density, which we may call $\mathscr{F}_{04}^{\,5}$. The algebra of the u_i, v_i, $\mathscr{F}_{i4}$, and $\mathscr{F}_{i4}^{\,5}$ then closes; we have 18 u's and v's, 8 $\mathscr{F}_{i4}$'s, and 9 $\mathscr{F}_{i4}^{\,5}$'s, corresponding to the 35 generators of the algebra SU(6). In the model, the new current $\mathscr{F}_{0\alpha}^{\,5}$ is just $(i/2)\,\bar{t}\,\lambda_0\,\gamma_\alpha\,t$.

Evidently, we can look upon all 35 operators as generating infinitesimal unitary transformations among the three left-handed quarks and the three right-handed quarks. This algebraic system connects scalars and pseudoscalars with four-vectors and four-pseudovectors and thus represents a new stage in the generalization of symmetry. Whereas F-spin connects only systems of the same parity and behavior under proper Lorentz transformations, the group SU(3) $\times$ SU(3) of the vector and axial vector currents connects systems which may have different parity but must still have the same behavior under the proper Lorentz group, and SU(6) now connects systems with different parity and/or different space-time behavior.

Of course, it is not clear, even in the model, that SU(6) is of any use as an approximate symmetry. If it were, it would arrange particles of various spins and parities in super-super-supermultiplets. However, it does appear to be true that a huge number of special algebraic properties can be abstracted from a field theory model. The situation is reminiscent of the growth of dispersion relations from an obscure equation for forward scattering of light to a huge set of relations among all scattering amplitudes, nearly sufficient to determine the whole S-matrix. Conceivably, the study of algebraic relations will undergo a comparable transformation.

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