

## Progress on NNLO+PS predictions for top-quark pair production and decay

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In this proceeding we report on recent progress toward achieving NNLO+PS predictions for off-shell top-quark production and decay in the fully-leptonic channel within the MiNNLO<sub>PS</sub> framework. We discuss both the computational challenges involved in parton-shower-matched simulations of  $e^+v_e b\mu^- \bar{\nu}_\mu \bar{b}$  production and the conceptual obstacles posed by the lack of the two-loop off-shell amplitude. We present one-loop studies aimed at assessing the validity of possible approximations for the virtual contributions of the dominant double-resonant contribution.

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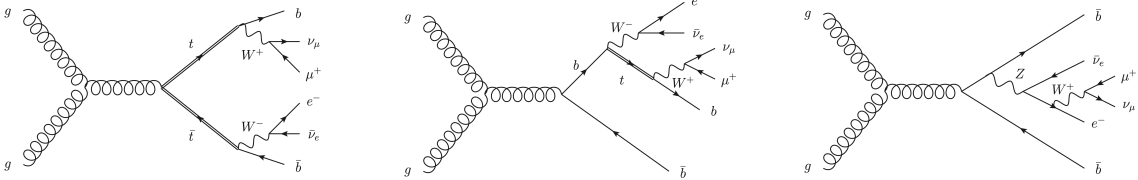
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## 1. Physics motivation

The study of top-quark pair production at modern high-energy colliders provides a fundamental test of the Standard Model (SM) and serves as a window to potential new phenomena. Top-quark production and decay studies are central to the LHC physics programme, enabling stringent tests of the SM and offering sensitivity to possible extensions beyond it. Thanks to the large production rate, detailed measurements of  $t\bar{t}$  kinematics and decay observables can be performed with high statistical precision, allowing for comprehensive comparisons with theoretical predictions. The  $t\bar{t}$  production rate at LHC is known to high accuracy and shows excellent agreement between data and theoretical predictions. Moreover, since the top quark decays before hadronisation, its spin and kinematic properties can be directly reconstructed from its decay products. Top quarks decay almost exclusively into a  $W$  boson and a bottom quark through the electroweak interaction [1]. The subsequent  $W$ -boson decay proceeds either hadronically or leptonically, with the fully leptonic mode offering the cleanest experimental signature. Theoretical predictions for top-pair production have achieved unprecedented precision. NNLO QCD calculations at fixed order (e.g. [2–6]) have been supplemented by logarithmic enhancements from soft radiation, that are resummed to NNLL accuracy (e.g. [7, 8]). The combination of these results with parton-shower algorithms, through approaches like MiNNLO<sub>PS</sub> [9], yields a consistent NNLO+PS description of on-shell top-pair production, which matches experimental data remarkably well across a wide range of observables [10, 11]. Accurate modelling of the decay process is equally essential [12, 13]. Higher-order corrections to the decay  $t \rightarrow bW$  have been computed up to NNLO in QCD, including finite bottom-mass and electroweak effects [14]. To combine the production and decay descriptions, a factorisable treatment, the so-called narrow-width approximation (NWA), is often employed. In this approach, the top quark is treated as an on-shell intermediate state, allowing production and decay corrections to be computed independently and combined multiplicatively. Phenomenological studies have confirmed that for inclusive quantities and observables insensitive to off-shell effects, this approach is highly reliable. Nonetheless, exclusive observables can receive non-negligible finite-width corrections, making full off-shell calculations essential [15].

## 2. Technical challenges



**Figure 1:** Examples of double-resonant (left), single-resonant (center) and non-resonant (right) diagrams.

For a complete off-shell description of the process  $pp \rightarrow e^+ \nu_e b \mu^- \bar{\nu}_\mu \bar{b} + X$ , all resonant, single-resonant, and non-resonant topologies (see Figure 1) must be consistently included, together with their quantum interferences. Theoretical progress over the past decade has made it possible to incorporate these effects in NLO QCD calculations, including the parton-shower matching

through resonance-aware algorithms, such as POWHEG-BOX-RES [19, 20], providing a realistic and unified description of experimental signatures. We will refer to such implementation as `bb4l`. Targeting NNLO precision introduces severe computational challenges related, among the rest, to the high multiplicity of the final state (up to 8 final-state particles) and the modelling of unstable particles that leads to an intricate sub-lists of resonant configurations. Nevertheless, the main limitation remains the lack of full two-loop amplitudes for such processes. At present, calculating the double-virtual contribution involving eight external legs remains beyond reach due to many issues, ranging from the large set of independent invariants, which are required to describe the multi-dimensional phase space, to the algebraic complexity of loop integrals involving numerous massive resonances. In particular, corrections that involved the cross-talk of initial- and final-state coloured partons, usually referred as *non-factorisable* corrections, are the most impractical to tackle. Issues related to the finite bottom-quark mass are not expected to be a major obstacle once the complete massless-bottom amplitude is available, thanks to the massification procedure (e.g. [21, 22, 24]). In this preliminary study, we explore possible approximate representations of the two-loop amplitude by first benchmarking different one-loop approximations against exact results. In particular, we employ the double-pole approximation (DPA), which isolates the dominant double-resonant contributions associated with top-pair production and decay. The validity of the DPA is tested at NLO by comparing approximate and exact results (obtained within the `bb4l` implementation [12]) at inclusive and differential level.

### 3. NLO results for validating approximation techniques

The set up of our results is the following: 13 TeV  $pp$  collisions, NNPDF31 PDF set,  $m_b = 4.92$  GeV,  $m_t = 173.2$  GeV,  $\Gamma_t = 1.4426$  GeV,  $m_Z = 91.19$  GeV,  $\Gamma_Z = 2.4952$  GeV,  $m_W = 80.38$  GeV,  $\Gamma_W = 2.0054$  GeV. We use the  $G_\mu$  input scheme for the EW parameters and complex-mass scheme. We refer to fixed scale when setting  $\mu_F = \mu_R = m_t$ , and to dynamical scale if  $\mu_F = \mu_R = m_{QQF} = m(b\bar{b}e^- \nu_e \mu^+ \nu_\mu)$ . In the following we will further inspect the impact of different scales in various approximations, and assume fixed scale unless differently specified. In the analysis,  $b$ -jets are reconstructed requiring that  $p_{\perp, b\text{-jet}} > 30$  GeV and  $|\eta_{b\text{-jet}}| < 2.4$ .

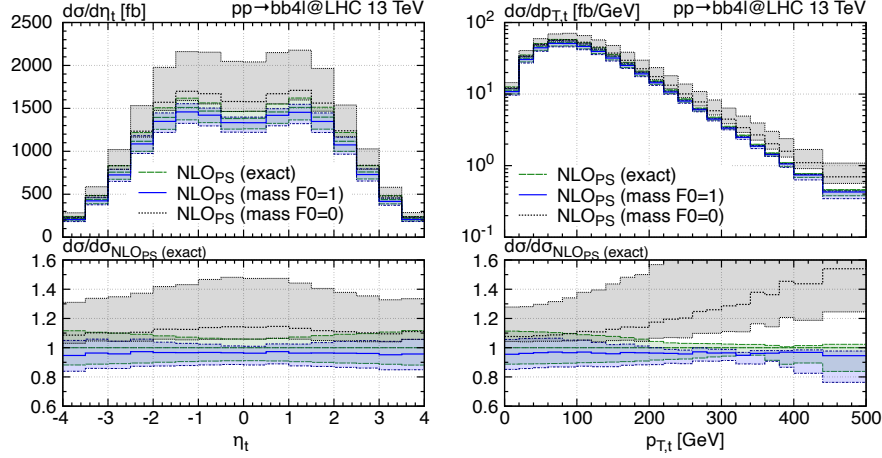
#### 3.1 Massive and massless amplitudes

To obtain a robust approximation of virtual corrections, we first note that the two-loop logarithmic contributions in the bottom-quark mass are known in terms of Born tree-level and one-loop amplitudes [21–24], while the constant piece in  $m_b$  requires the two-loop massless amplitudes. While the power-corrections that are not captured by the *massification formula* are expected to be small, the size of the  $\mathcal{O}(m_b^0)$  term is not predicted. We test the impact of such constant terms by inspecting the corresponding contributions at one-loop. At this perturbative order, the massification formula reads

$$|\mathcal{A}_{m_b \neq 0}^{(1)}(\{p\})\rangle = \mathcal{F}^{(1)} |\mathcal{A}_{m_b=0}^{(0)}(\{p\})\rangle + \mathcal{F}^{(0)} |\mathcal{A}_{m_b=0}^{(1)}(\{p\})\rangle + \mathcal{O}(m_b/\mu_Q), \quad (1)$$

where  $\mathcal{F}$  is a universal operator in colour space with  $\mathcal{F}^{(0)} = 1$ . As a first step, we can investigate the impact of switching off the one-loop massless contribution, i.e.  $\mathcal{F}^{(0)} = 0$ . The comparison

between the exact calculation and the approximated results with the two choices of the  $\mathcal{F}^{(0)}$  factor is presented for some representative observables in Figure 2. The complete massification reproduces



**Figure 2:** Comparison of the exact NLO+PS prediction (green) against the massification approximation with  $\mathcal{F}^{(0)} = 0$  (grey) and  $\mathcal{F}^{(0)} = 1$  (blue). See the text and Eq. (1) for further details.

the dominant physical contribution, with residual differences remaining within the envelope of scale variations. In contrast, neglecting the finite massless component results in an integrated prediction that exceeds the exact total cross section by roughly 16% and exhibits an enlarged scale dependence. Moreover, setting  $\mathcal{F}^{(0)} = 0$  yields moderately flat deviations from the exact result for certain observables (see the left panel of Figure 2), whereas it induces more pronounced distortions in others, for example in the transverse-momentum distribution of the top quark (right plot of Figure 2). The observation above enforces the need for including two-loop effects, at least in some approximated form, and motivates the investigation of different approaches discussed below.

### 3.2 Double pole approximation

A viable method for estimating the virtual contribution is provided by the double-pole approximation (DPA), which offers a practical way to simplify computations involving unstable particles that act as intermediate states. This method relies on expanding the scattering amplitude around the resonance pole of the propagator corresponding to the unstable particle, thereby isolating the leading behaviour close to its physical mass. Within this framework, the resonant component of the amplitude is extracted in a gauge-consistent manner through an on-shell projection. The procedure retains only contributions that exhibit double-resonant configurations while neglecting the others. Since this subset of corrections is not independently gauge invariant, a mapping onto the on-shell phase space is applied to restore consistency. Moreover, the denominators of the top-quark propagators are computed with off-shell kinematics, while numerators are obtained using on-shell momenta. The DPA of the amplitude for the partonic process  $i(x_1 P)\bar{i}(-x_2 P) \rightarrow b(k_3)\bar{b}(k_4)e^-(k_5)\bar{\nu}_e(k_6)\mu^+(k_7)\nu_\mu(k_8)$ , with  $P$  the proton momentum, can be then written schematically as

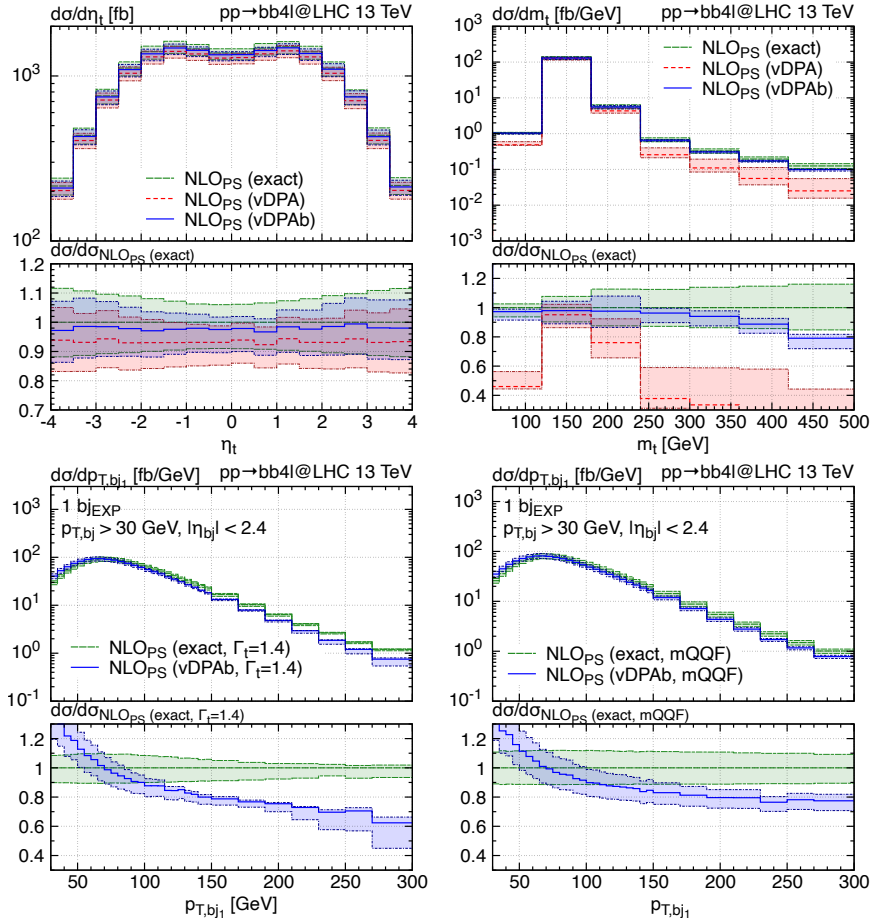
$$\begin{aligned} \mathcal{A}_{\text{res}}(x_1, x_2; \tilde{k}_{3\dots 8}) &= \mathcal{P}(x_1, x_2; \tilde{k}_{\text{fin}}) \mathcal{D}_{W^-}(\tilde{k}_4, \tilde{k}_5, \tilde{k}_6) \\ &\times \frac{-i(\not{k}_{456} + m_t)}{k_{456}^2 - m_t^2 + i\Gamma_t m_t} \frac{i(\not{k}_{378} + m_t)}{k_{378}^2 - m_t^2 + i\Gamma_t m_t} \mathcal{D}_{W^+}(\tilde{k}_3, \tilde{k}_7, \tilde{k}_8), \end{aligned} \quad (2)$$

where  $\mathcal{P}$  represents the sub-amplitude for the  $t\bar{t}$  production, and  $\mathcal{D}_{W^{+(-)}}$  the decay sub-amplitude for the  $W^{+(-)}$  boson. The two remaining factors correspond to the two top-quark propagators, where  $k_{ijk} = k_i + k_j + k_k$ . Finally, the on-shell remapped momenta have to fulfil the condition  $\tilde{k}_{378}^2 = \tilde{k}_{456}^2 = m_t^2$ , but are otherwise arbitrary. We choose the same mapping as the one adopted for diboson studies in POWHEG [25], which does not modify the initial-state energy fractions, and preserves the decay angles. At next-to-leading order and beyond, the DPA incorporates both factorizable and non-factorizable effects. Nonetheless, since two-loop non-factorizable contributions are not expected to become available in the near future, we limit the implementation of the DPA to the factorizable terms at the one-loop level. We then investigate two alternative approaches: *i*) vDPA: the exact real-emission contributions are fully retained, while the virtual correction is computed using RecoLa2 [26, 27] in the DPA and neglecting non-factorisable contributions; *ii*) vDPAb: same as vDPA but reweighting the approximated one-loop amplitude with the exact Born matrix elements. At inclusive level, vDPAb supersedes vDPA, and shows a remarkable agreement with the exact results, with discrepancies remaining below 2.5% of the total cross section. At differential level, vDPAb and vDPA show similar behaviour for observables that are inclusive over the top-quark decay products (top left panel of Fig. 3), while vDPA substantially deviates from the exact result in more exclusive observables (top right panel of Fig. 3). We find that the level of disagreement between the exact and DPA predictions for b-jet observables is highly sensitive to the chosen renormalisation and factorisation scales. In the bottom left panel of Fig. 3 the scale is set to be  $\mu = m_t$ , while in the bottom right panel of Fig. 3, the  $\mu$  is set to the invariant mass of the  $e^+ \nu_e b \mu^- \bar{\nu}_\mu \bar{b}$  system. In the latter set up, the discrepancy between the exact and DPA predictions decreases, remaining below 20%. Nonetheless, the distortion in the shape persists.

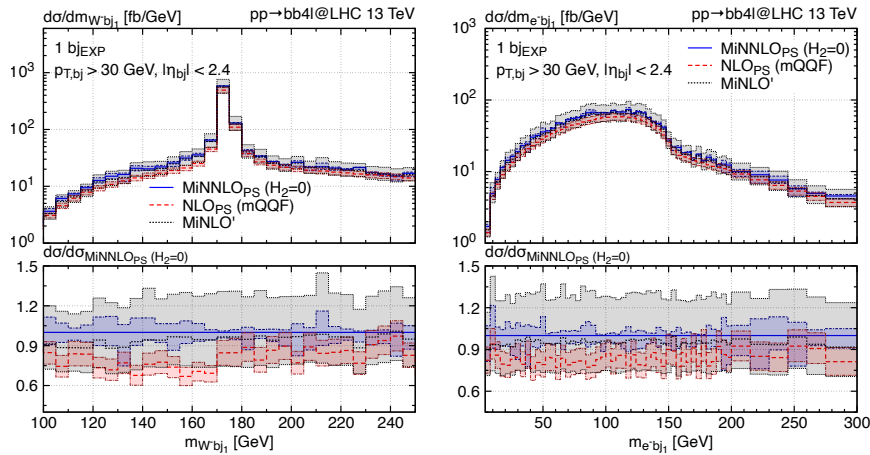
#### 4. First steps towards NNLO+PS corrections

Beside the issue of including the two-loop component, we start to investigate the computational possibilities of performing  $e^+ \nu_e b \mu^- \bar{\nu}_\mu \bar{b}$  simulations at hadron colliders with parton-shower matching within the MiNNLO<sub>PS</sub> framework by providing MiNLO' predictions<sup>1</sup> and preliminary MiNNLO<sub>PS</sub> simulations with the two-loop finite remainder set to zero in the  $\overline{\text{MS}}$  scheme. All relevant NNLO contributions are incorporated in the MiNNLO<sub>PS</sub> simulations, excluding only the finite remainder of potentially approximated two-loop terms while including exact one-loop squared contributions. Using  $m_{Q\bar{Q}F}$  as the scale for the Born-level couplings, we present the first differential predictions, using the exact NLO+PS and MiNLO' results as references. From Fig. 4 we observe that MiNNLO<sub>PS</sub> results exhibit reduced scale uncertainties compared to MiNLO', particularly at low transverse momentum. Observables based on the invariant mass of the charged lepton and the hardest jet retain their shape at higher orders, with MiNNLO<sub>PS</sub> providing positive corrections and smaller scale variation relative to NLO+PS and MiNLO'. While these results are encouraging, further improvements are needed, such as establishing a reliable approximation for the two-loop contributions and improving the computational efficiency of the current implementation.

<sup>1</sup>For computational reasons, the Les Houches events (LHEs) are generated without the finite remainder of IR-subtracted real-virtual contribution, and the correct MiNLO' predictions are obtained via a reweighting procedure.



**Figure 3:** Comparison of NLO+PS exact predictions (green) against NLO+PS results exploiting factorised DPA for the virtual correction (red) and further reweighting with the exact Born matrix element (blue). The second row shows a comparison for an exclusive observable with different scale choices.



**Figure 4:** Preliminary MiNNLO<sub>PS</sub> predictions (blue) are shown for  $e^+\nu_e b\mu^-\bar{\nu}_\mu\bar{b}$  production without the two-loop contribution, alongside the MiNLO' (grey) and NLO+PS (red) predictions.

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