Solution for Little Hierarchy Problem and $b \to s\gamma$ *

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We show that all the parameters which destabilize the weak scale can be taken around the weak scale in the MSSM without conflicting with the SM Higgs boson mass bound set by LEP experiment. The point is if the lightest CP-even Higgs boson h has only a small coupling to Z boson, LEP cannot generate h sufficiently. This time, same bound constrains mass of the heaviest CP-even Higgs boson H. However, it is easy to make H heavy using off-diagonal elements of Higgs mass matrix and consequently smaller stop masses are allowed. This scenario explains two excesses observed at LEP Higgs search. Though all the MSSM Higgs bosons should have the weak scale masses in this scenario, amplitude of $b \to s\gamma$ induced by charged Higgs can naturally be compensated by chargino if we take natural mass parameters by which the little hierarchy problem can be solved.

I. INTRODUCTION

Supersymmetry (SUSY), especially minimal extensions of the standard model (SM) relevant to this symmetry, called as minimal supersymmetric standard model (MSSM), is one of the most promising candidates for new physics. However, this simple model seems to be unsatisfactory at firs glance. The problem is related one of the characteristic features of the MSSM; m_h, mass of the lightest CP-even Higgs boson (h), is always smaller than Z boson mass at tree level [17]. Of course loop corrections to the Higgs potential modify this relation [1] and the largest contributions come from top/stop as logarithmic functions of its masses as,

$$m_h^2 \le m_Z^2 + \Delta_{22}, \qquad (I.1)$$

$$\Delta_{22} \sim \frac{3Y_t^4 \langle H_u \rangle^2}{4\pi^2} \log \frac{m_{\tilde{t}}^2}{m_t^2} \qquad (I.2)$$

(here, Y_t, $m_{\tilde{t}}$ and m_t are top Yukawa coupling, stop mass and top mass, respectively). From this expression, if the lower mass bound of the SM Higgs boson $m_{\varphi_{\rm SM}} > 114.4$GeV (95 % C.L.) set by LEP [2] is naively applied to m_h, $m_{\tilde{t}}$ has to be larger than 500GeV. However, at the same time, stop also contributes to the mass parameter of up-type Higgs field H_u as quadratic form of its mass,

$$m_{H_u}^2 = m_{H_u0}^2 + \Delta m_{H_u}^2, \qquad (I.3)$$

$$\Delta m_{H_u}^2 \sim -\frac{3Y_t^2}{4\pi^2} m_{\tilde{t}}^2 \log \frac{\Lambda}{m_{\tilde{t}}}. \qquad (I.4)$$

Therefore, such a large $m_{\tilde{t}}$ leads to a tuning between tree Higgs mass parameter $m_{H_u0}^2$ and correction $\Delta m_{H_u}^2$, because in order to obtain correct weak boson masses, m_{H_u} must be around the weak scale, $O(m_Z)$. For example, if cutoff scale Λ is taken as the Planck scale, less than a percent tuning is required in this naive analysis. This difficulty is called as "little hierarchy problem" and various solutions have been examined [3].

II. SMALL $Z-Z-h$ COUPLING

As we have seen in the previous section, naive application of the LEP bound to m_h leads to the tuning problem. However, it is not mandatory to apply this mass bound to the lightest Higgs boson of extended Higgs sector, since coupling strength between the SM Higgs boson and Z bosons ($g_{ZZ\varphi_{\rm SM}}$), which was exploited to set $m_{\varphi_{\rm SM}} > 114.4$GeV, does not always equal to that of h and Z bosons (g_{ZZh}). In other words, if g_{ZZh} is sufficiently smaller

*Talk presented by S.-G. Kim at SI2006, based on our recent work, arXiv:hep-ph/0609076.

than $g_{ZZ\varphi_{\rm SM}}$, $m_h < 114.4$GeV can be allowed. Moreover, it was reported that there were 2.3 and 1.7 σ excesses from back ground estimations of Higgs search experiment where the corresponding Higgs boson masses are around 98 and 115GeV, respectively [2]. Notably, the former excess is too small to identify it as productions of the SM Higgs boson, but it can be explained by h with small g_{ZZh} [4, 5]. In this article, we regard this small g_{ZZh} scenario as a way to bypass the little hierarchy problem.

III. REALIZATION OF SMALL g_{ZZh}

First of all, $g_{ZZHiggs}$ coupling originates from $ZZH^\dagger H$ interaction. Therefore, in general, $g_{ZZHiggs}$ is in proportional to the VEV of the corresponding Higgs field. In two Higgs doublets model as in the case of the MSSM, one can take generally linear combinations of two Higgs fields as, h_{VV}, which has a vanishing VEV, and the other combination, h_{SM}, which has a VEV whose value equals to that of the SM Higgs field. They are written as

$$\begin{pmatrix} h_{VV} \\ h_{SM} \end{pmatrix} = \begin{pmatrix} \sin\beta & -\cos\beta \\ \cos\beta & \sin\beta \end{pmatrix} \begin{pmatrix} H_d \\ H_u \end{pmatrix}, \quad \text{(III.1)}$$

where H_d is down-type Higgs field and $\tan\beta \equiv \langle H_u \rangle / \langle H_d \rangle$ (We take $\cos\beta$ and $\sin\beta$ as positive value). It is obvious that h_{VV} has vanishing $g_{ZZh_{VV}}$. Therefore, if main mode of h is h_{VV}, it is hard to find h using $e^+e^- \to Z^* \to Zh$.

Let's see approximate CP-even Higgs mass matrix in terms of CP-odd Higgs boson mass (m_A), $\tan\beta$ and the largest quantum correction to it, Δ_{22}, for the second step.

$$\begin{array}{c} \\ H_d \\ H_u \end{array} \begin{array}{c} \begin{array}{cc} H_d & H_u \end{array} \\ \begin{pmatrix} m_A^2 & -(m_A^2+m_Z^2)\frac{\sin 2\beta}{2} \\ -(m_A^2+m_Z^2)\frac{\sin 2\beta}{2} & m_Z^2+\Delta_{22} \end{pmatrix} \end{array} \quad \text{(III.2)}$$

When $\tan\beta \gg 1$, this matrix becomes diagonal form, and from the previous discussion, these entries correspond to the mass of $H_d \sim h_{VV}$ and $H_u \sim h_{\rm SM}$, since $\tan\beta \gg 1$ means H_u gets almost the same VEV as the SM Higgs field and $\langle H_d \rangle \approx 0$. Therefore if $m_A^2 < m_Z^2 + \Delta_{22}$, we obtain h with small g_{ZZh} coupling. We call this situation as "Inverse case" for the later convenience and also name "Normal case" for the situation where $m_A^2 > m_Z^2 + \Delta_{22}$. Note that in the Inverse case, typical mass scales of the MSSM Higgs bosons are around the weak scale, since m_A is. Moreover, once off-diagonal entries are take into account, it is obvious that comparing to the Normal case, the Inverse case allows smaller stop mass since larger (smaller) eigenvalue becomes always to be larger (smaller) than original larger (smaller) diagonal element. Therefore, in the Inverse case, off-diagonal entries lift the larger eigenvalue (m_H^2), which has to satisfy the LEP constraint, and there is no need for heavy stop. This is the essence of the scenario which can open the way to ease the tension of the MSSM parameters. Since larger $\tan\beta$ leads to smaller off-diagonal component, smaller $\tan\beta$ is preferable for the Inverse case.

IV. NUMERICAL ANALYSIS

We explore the scenario which realizes the Inverse case numerically. Since large SUSY and SUSY breaking parameters entail tuning problem, we take each parameters as low as they don't conflict its own experimental constraint [6]. In the analysis, we assume universal soft masses, m_0(=100GeV) and M(=145GeV), for squarks, sleptons and gauginos at the GUT scale. Resulting masses relevant to the following are $m_{\tilde{t}_L} = 350$GeV, $m_{\tilde{t}_R} = 300$GeV, $M_1 = 60$GeV, $M_2 = 120$GeV, $M_3 = 400$GeV and $\mu = 250$GeV at the weak scale, respectively. We also assume each A_X parameters are proportional to the corresponding Yukawa couplings with uniform factor (A) at the GUT scale and set $A_t = 300$GeV and 325GeV at the weak scale as typical values. These values correspond to $A \sim 0$ and $A \sim 125$GeV at the GUT scale, respectively. Under this assumption, A larger than 250GeV induces charge breaking at the weak scale. Here, we vary not $m_{\tilde{t}}$ but A_t (A) since results are more sensitive to it when naturalness is taken into account. Finally, we assume no constraints for three Higgs mass parameters at GUT scale and treat two of them (m_A and $\tan\beta$) as free at the weak scale as shown in the FIG.1-2.

In the figures, doted-dashed, dashed, thick solid and thin solid lines represent contour lines

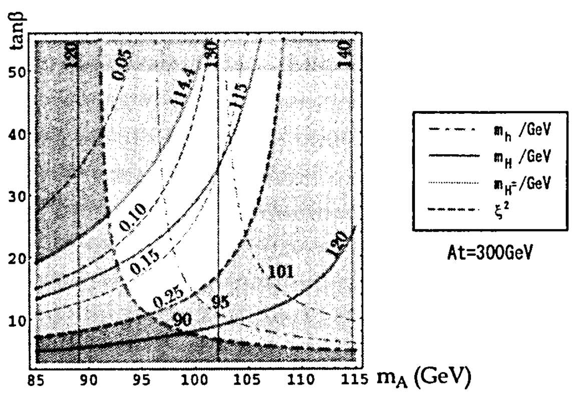

FIG. 1:

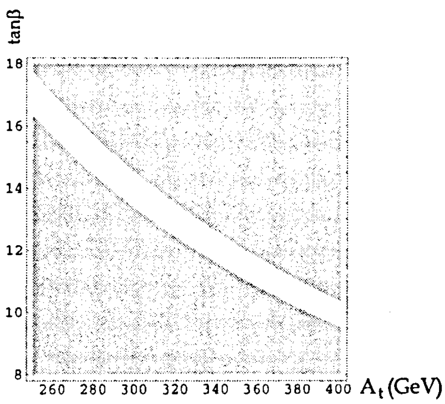

FIG. 3:

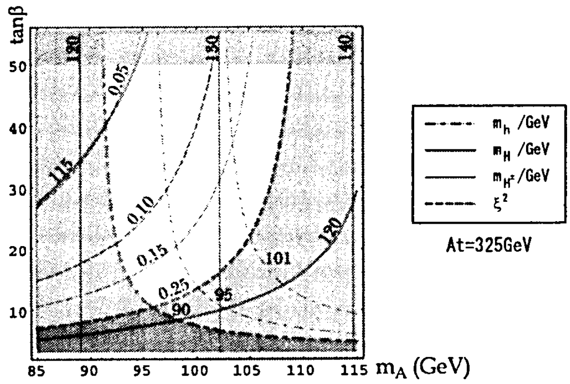

FIG. 2:

of m_h/GeV, $\xi^2(\equiv g^2_{ZZh}/g^2_{ZZ\varphi_{SM}})$, m_H/GeV and $m_{H^\pm}$/GeV, respectively. Each white area is allowed region satisfying conditions of $m_h >$ 90GeV, $\xi^2 < 0.25$, $m_H >$ 114.4GeV and $\tan\beta < 50$. The meanings of these criteria are as follows. First of all, since small g_{ZZh} means large g_{ZhA}, hA production can be enhanced, i.e., arbitrary small m_h is not allowed. However, if m_h is larger than 90GeV, there is almost no constraint because of P-wave suppression [6]. Second, according to the Fig. 10 of [2], upper bound of g_{ZZh} , normalized by $g_{ZZ\varphi_{SM}}$, should be about less than 0.5 ($\xi^2 \equiv g^2_{ZZh}/g^2_{ZZ\varphi_{SM}} \lesssim 0.25$) for $m_h >$ 90GeV at 95% C.L.. Third, since small g_{ZZh} means large g_{ZZH}, m_H has to be larger than 114.4GeV in this case. Finally, since top quark decays into charged Higgs boson and bottom quark in this scenario, there are experimental upper bound for large $\tan\beta$ set by [7]. In addition, when we identify $2.3\,\sigma$ excess at corresponding mass near 98GeV as signals of h, we take narrower region 95GeV< m_h <101GeV. Moreover, since number of the event observed is a tenth of estimated number of corresponding SM Higgs boson, it naively corresponds to $\xi^2 = 0.1$. Here we have used the on-shell top mass m_t = 175GeV and one-loop potential of [8], in which D-term contribution to sfermion masses is neglected. As a whole, we observe that regions which are consistent with LEP experiments are realized with the natural parameter set. Note that we can see the several characteristics of the Inverse case in these figures as discussed in the previous section qualitatively. Especially this scenario predicts a light charged Higgs boson ($m_{H^\pm} \sim$ 130GeV) and this issue is a subject of final section.

V. $b \to s\gamma$ CONSTRAINT

Since charged Higgs boson induced amplitude of $b \to s\gamma$ transition always makes constructive contribution to the SM amplitude [9], current experimental values of this process [10], now in good agreement with the SM predictions [11, 12] gives severe constraint for $m_{H^\pm}$ (e.g., $m_{H^\pm} \geq$ 350GeV for type II 2HDM [11]). However in supersymmetric models, especially in our scenario, another particle, chargino, plays also an important role [9]. The point is, in our scenario, magnitudes of these amplitudes are naturally same order because all particles which contribute this process have the weak scale masses; $m_{H^\pm}$ must be around the weak scale to realize small g_{ZZh}, and chargino and stop masses also must be around the weak scale from naturalness requirement. Moreover, chargino can induce negative amplitude. Fig.3 shows relations

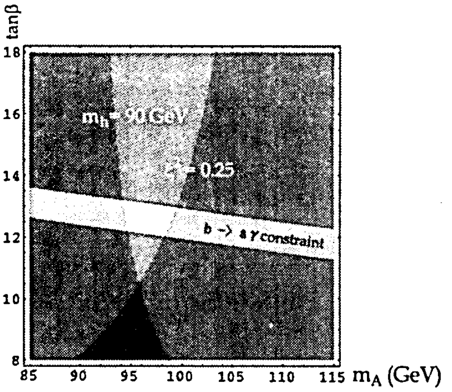

FIG. 4:

between A_t and $\tan\beta$ required by the cancellations. In this analysis we fixed $m_{H^\pm} =$ 125GeV and required sum of charged Higgs and chargino induced amplitudes must be less than 5% of the SM amplitude (This requirement very roughly corresponds to one sigma deviation). A white strip shows parameter regions which realize the suitable cancellation at weak scale. We see that 10% tuning of A_t is sufficient for the cancellation. Fig.4 shows relations between m_A and $\tan\beta$ required by the cancellations at $A_t = 325$GeV. This figure is superposed on the corresponding regions of Fig.2 and show that $\mathcal{B}r(b \to s\gamma)$ severely constrains allowed region of previous section.

VI. DISCUSSION AND SUMMARY

Processes to which light charged Higgs boson contributes at tree level may show one of the signals of this scenario. Especially, $B \to \tau\nu_\tau$ process is interesting because the SM contribution is suppressed by chirality [13] and recently the first evidence for the process was reported by Belle [14]. (Combined) branching ratio already restrict $\tan\beta$ as $\tan\beta < 20$ and $29 < \tan\beta < 37$, when $m_{H^\pm} = 130$GeV [15].

We would like to emphasize that our arguments are quite general one even more rigorous discussions are employed and this is an interesting possibility to solve the little hierarchy problem. We hope this scenario to be tested in future experiments, LHC, ILC or (super-) B factory, etc. [16]. Analysis of dark matter abundance is beyond the scope of this work.

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[1] Y. Okada, M. Yamaguchi and M. Yanagida, Prog. Theor. Phys. Lett. **85**, 1(1990); J. Ellis, G. Ridolfi and F. Zwirner, Phys. Lett. B **257** 83, (1991); **262** 477 (1991).

[2] R. Barate *et al.* [LEP W. G. for Higgs boson searches], Phys. Lett. B **565**, 61 (2003)

[3] E.g., [9] and related proceedings of this SI.

[4] G. L. Kane *et al.*, Phys. Rev. D **71**, 035006 (2005)

[5] M. Drees, Phys. Rev. D **71**, 115006 (2005)

[6] S. Eidelman *et al.*, [Particle Data Group (PDG)], Phys. Lett. B 592, 1 (2004) and W.-M. Yao *et al.*, [PDG], J. Phys. G**33**, 1(2006).

[7] V. M. Abazov *et al.* [D0 Collaboration], Phys. Rev. Lett. **88**, 151803 (2002) and A. Abulencia *et al.* [CDF Collaboration], Phys. Rev. Lett. **96**, 042003 (2006)

[8] M. Drees and M. M. Nojiri, Phys. Rev. D **45**, 2482 (1992).

[9] See the references of arXiv:hep-ph/0609076.

[10] [Heavy Flavor Averaging Group (HFAG)], arXiv:hep-ex/0603003.

[11] P. Gambino and M. Misiak, Nucl. Phys. B **611**, 338 (2001)

[12] A. J. Buras *et al.*, Nucl. Phys. B **631**, 219 (2002), A. J. Buras and M. Misiak, Acta Phys. Polon. B **33**, 2597 (2002) and T. Hurth, Rev. Mod. Phys. **75**, 1159 (2003)

[13] W. S. Hou, Phys. Rev. D **48**, 2342 (1993).

[14] T.Browder, talk presented at ICHEP06, Moscow. *Babar has also reported their result. B. Aubert *et al.*, [BABAR Collab.], arXiv:hep-ex/0608019. See also talk presented by S.J. Sekula at ICHEP06, Moscow.

[15] We use same input parameters quoted in K. Ikado *et al.*, arXiv:hep-ex/0604018.

[16] See, e.g., Q. H. Cao, S. Kanemura and C. P. Yuan, Phys. Rev. D **69**, 075008 (2004) and A. Belyaev *et al.*, arXiv:hep-ph/0609079.

[17] Here, we take usual notations, i.e., the mass of h is always smaller than that of H. CP violation in the Higgs sector which causes the mixing of h, H, and CP-odd Higgs boson A is ignored in the following discussions.