CP-symmetry of order 4: model-building and phenomenology

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We present a multi-Higgs model dubbed CP4 3HDM which, remarkably, combines the minimality in its assumptions with phenomenological richness and predictivity. It is based on a single assumption: the minimal multi-Higgs model to incorporate CP-symmetry of order 4 (CP4) without producing accidental symmetries. It leads to a unique three-doublet model with a constrained scalar potential which can be worked out analytically. We describe two versions of this model: (i) when two extra doublets are inert, CP4 is conserved and leads to a pair of scalar DM candidates with peculiar properties, and (ii) when CP4 is extended to the Yukawa sector, leading to a few very restricted cases, which can, nevertheless, accommodate all fermion masses, mixing, and CP-violation.

1 Building bSM models: balancing between the two extremes

Many aspects of the Standard Model (SM) leave theorists unsatisfied, including absence of dark matter (DM) candidates, its ignorance of the origin of neutrino masses [1] and of CP-violation (CPV) [2], as well as quark and lepton mass and mixing hierarchies. These difficulties arise partly due to the very minimalistic Higgs sector used in the SM, and this is why many models beyond the SM (bSM) are based on extended Higgs sectors [3,4]. When building such models, one often tries to balance two requirements: keeping as few extra assumptions as possible and producing a model well compatible with experiment and sufficiently predictive to be tested in near future. One wants to avoid two extreme cases: when one manages to describe all data at the expense of excessively many new fields and assumptions, and the case when one produces a neat compact bSM model with very few assumptions, which fails when compared to the real world.

A popular way to try to keep this balance is to constrain interactions with extra global discrete symmetries [5,1]. For example, a typical N-Higgs-doublet model (NHDM) has hundreds of free parameters in the scalar and Yukawa sectors. Imposing large non-abelian discrete symmetry groups reduces this number to about a dozen, making the model highly predictive. It turns out, however, that such models almost unavoidably lead to non-physical fermion sectors [6]: for sufficiently large groups, there always remains some flavor symmetry in the vacuum, which either leads to massless or mass-degenerate fermions, or produces insufficient mixing or CPV. On the other hand, imposing smaller symmetry groups such as Z_2 can lead to a good experimental fit but it still leaves very many free parameters, which makes the analysis cumbersome and the whole setting less attractive, see e.g. [7].

Here, we report on a model developed in [8,9] which, remarkably, keeps extra assumptions extremely minimal and, at the same time, produces a perfect fit to fermion masses, mixing, and CPV, and has enough predictivity to be checked in the nearest future. As a bonus, it bears a certain theoretical flair. It incorporates a feature never seen before in bSM models: a complex scalar field which, although being CP-eigenstate, is not CP-even nor CP-odd but is, in a certain sense, CP-half-odd.

2 CP4 3HDM

2.1 The model

The model assumes very little indeed. We just ask for the minimal NHDM implementing a CP-symmetry of order 4 (CP4) without any accidental symmetries—and nothing else. This single requirement leads to the unique 3HDM [10] based on a rather restricted potential $V = V_0 + V_1$, with the phase-insensitive part

$$\begin{aligned} V_0 &= -m_{11}^2(\phi_1^\dagger\phi_1) - m_{22}^2(\phi_2^\dagger\phi_2 + \phi_3^\dagger\phi_3) + \lambda_1(\phi_1^\dagger\phi_1)^2 + \lambda_2\left[(\phi_2^\dagger\phi_2)^2 + (\phi_3^\dagger\phi_3)^2\right] \\ &+ \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2 + \phi_3^\dagger\phi_3) + \lambda_3'(\phi_2^\dagger\phi_2)(\phi_3^\dagger\phi_3) + \lambda_4(|\phi_1^\dagger\phi_2|^2 + |\phi_1^\dagger\phi_3|^2) + \lambda_4'|\phi_2^\dagger\phi_3|^2\,, \quad (1)\end{aligned}$$

with all parameters being real, and the phase-sensitive part

$$V_1 = \lambda_5(\phi_3^\dagger\phi_1)(\phi_2^\dagger\phi_1) + \frac{\lambda_6}{2}\left[(\phi_2^\dagger\phi_1)^2 - (\phi_1^\dagger\phi_3)^2\right] + \lambda_8(\phi_2^\dagger\phi_3)^2 + \lambda_9(\phi_2^\dagger\phi_3)(\phi_2^\dagger\phi_2 - \phi_3^\dagger\phi_3) + h.c. \quad (2)$$

with real λ_5, λ_6, and complex λ_8, λ_9. This potential is invariant under the generalized CP transformation

$$\phi_i \mapsto X_{ij}\phi_j^*\,, \quad X = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & i \\ 0 & -i & 0 \end{pmatrix}. \quad (3)$$

Since $XX^* = \mathrm{diag}(1, -1, -1)$, one needs to apply this CP transformation four times to get the identity transformation, hence the label CP4. Higher-order CP transformations were well known before [11,12,13]. In particular, several versions of the 2HDM incorporate CP4 in the scalar and fermion sector [14,15,16]. However in all such models, the imposition of CP4 automatically led to extra accidental symmetries including the usual CP. Our model is the first example which avoids that, and it reveals new structural features arising exclusively from CP4.

Below, we will outline two variants of this model. One, dubbed DM CP4 3HDM, contains two dark matter candidates coming from two additional inert doublets which are protected against decay by the conserved CP4. The other is flavored CP4 3HDM, in which CP4 is extended to the Yukawa sector but then get spontaneously broken to reproduce the physical fermion sector.

2.2 DM CP4 3HDM

Suppose $\phi_{2,3}$ are inert doublets: they do not contribute to the fermion and W/Z mass generation. Then, their vacuum expectation values (vevs) must be zero: $\langle\phi_1^0\rangle = v/\sqrt{2}$, $\langle\phi_2\rangle = \langle\phi_3\rangle = 0$. The CP4 symmetry remains intact and it protects the lightest inert scalars from decay, making them the DM candidates. By expanding the potential around the vacuum, we get the SM-like Higgs with mass $m_{h_{SM}}^2 = 2\lambda_1 v^2 = 2m_{11}^2$, and a pair of degenerate charged Higgses with $m_{H^\pm}^2 = \lambda_3 v^2/2 - m_{22}^2$. The neutral inert scalar mass matrix splits into two 2×2 blocks with the same eigenvalues $M^2, m^2 = m_{H^\pm}^2 + (\lambda_4 \pm \sqrt{\lambda_5^2+\lambda_6^2})v^2/2$, which are diagonalized by the same rotation angle α defined as $\tan 2\alpha = \lambda_5/\lambda_6$ but in the opposite directions. Denoting the two heavier scalars as H, A and the two lighter scalars as h, a, we find that they are not CP-eigenstates:

$$CP: \quad H \to A\,, \quad A \to -H\,, \quad h \to -a\,, \quad a \to h\,. \tag{4}$$

One can combine them into neutral complex fields, $\Phi = (H - iA)/\sqrt{2}$, $\varphi = (h + ia)/\sqrt{2}$, which *are* CP and mass eigenstates:

$$CP: \quad \Phi(\vec{x}, t) \to i\Phi(-\vec{x}, t)\,, \quad \varphi(\vec{x}, t) \to i\varphi(-\vec{x}, t)\,. \tag{5}$$

One can now quantify the CP properties with the global quantum number q defined modulo 4, and assign $q = +1$ to Φ, φ, and $q = -1$ to their conjugate fields. All other neutral fields are either CP-odd, $q = 2$, or CP-even, $q = 0$. Since CP4 remains intact, any transition between initial and final states conserves total q. In particular, h and a (or φ and φ^*) are stable.

One may wonder if it is legitimate to call the very peculiar transformation law (5) the CP rather than P transformation. The answer is that, while manipulating with the fields, we never change the definition of the symmetry itself. The CP-transformation (5) is the same as in (3) seen in different basis for neutral scalars. The presence or absence of complex conjugation in the definition of how CP acts on complex scalar fields is, in fact, *a basis-dependent feature* in the case of multiple mass-degenerate zero-charge fields, see details in [9].

Concerning the DM evolution in early Universe, this model contains two mass-degenerate DM candidates h and a which cannot coannihilate via Z boson. This is because the Z-inert-inert interaction vertex always picks up the heavy sector scalar: ZHa or ZhA. The two DM candidates can, of course, annihilate into SM particles. In addition, they can rescatter not only via $ha \to ha$ or $hh \leftrightarrow aa$ but also via $aa \leftrightarrow ha \leftrightarrow hh$, as there is no conserved quantum number which counts h's and a's separately. However, the DM scattering off normal matter does not lead to such transitions: h+SM $\not\to$ a+SM.

2.3 Flavored CP4 3HDM

In the second version of the model we extend CP4 to the Yukawa sector. This is only possible if CP4 involves family mixing in the fermion sector as well: $\psi_i \to Y_{ij}\psi_j^{CP}$, where $\psi^{CP} = \gamma^0 C \bar{\psi}^T$. If we want the quark Yukawa lagrangian $\bar{q}_L \Gamma_a d_R \phi_a + \bar{q}_L \Delta_a u_R \phi_a^* + h.c.$ to be invariant under CP4, we need to find such Yukawa matrices Γ_a and Δ_a and such rotation matrices Y^L, Y^d, Y^u in the left and right quark sectors that the following conditions are fulfilled:

$$(Y^L)^\dagger \Gamma_a Y^d X_{ab} = \Gamma_b^*\,, \quad (Y^L)^\dagger \Delta_a Y^u X_{ab}^* = \Delta_b^*\,. \tag{6}$$

We solved these equations [9] under the simplifying assumption $Y^L = Y^d = Y^u$ and later without it [17]. Let us highlight one particular case, in which Y has the same structure as X and

$$\Gamma_1 = \begin{pmatrix} g_{11} & g_{12} & 0 \\ -g_{12}^* & g_{11}^* & 0 \\ 0 & 0 & g_{33} \end{pmatrix}, \quad \Gamma_2 = \begin{pmatrix} 0 & 0 & g_{13} \\ 0 & 0 & g_{23} \\ g_{31} & g_{32} & 0 \end{pmatrix}, \quad \Gamma_3 = \begin{pmatrix} 0 & 0 & -g_{23}^* \\ 0 & 0 & g_{13}^* \\ g_{32}^* & -g_{31}^* & 0 \end{pmatrix}. \tag{7}$$

Here, all parameters apart from g_{33} are complex. When multiplied by vevs (v_1, v_2, v_3), they produce the down-quark mass matrix $M_d = \sum \Gamma_a v_a/\sqrt{2}$. Accompanied with the similar matrix M_u for up-quarks, it will give the quark masses, mixing, and CP-violating phase. Since Γ_1 has degenerate eigenvalues, the unbroken CP4 with vev alignment $(v, 0, 0)$ would lead to degenerate quark masses. Thus, CP4 must be broken spontaneously, and the magnitude of v_2, v_3 cannot be too small as it governs the mass splitting. Γ_2 and Γ_3, being very distinct from Γ_1, generate large FCNC effects. To avoid immediate conflict with experiment, the SM-like Higgs boson must receive only a small contribution from the second and third doublet, so a strong alignment in the Higgs sector seems to be unavoidable. Thus, satisfying all Higgs and flavor physics constraints becomes a non-trivial task within the particular case (7).

We have developed an efficient numerical scan of the scalar and Yukawa parameter space of this model [17]. In the Yukawa sector, about half of our trial points produce models which are able to fit all quark and lepton masses, mixing angles, and CP-violation. We are now implementing it in computer packages for testing the flavor physics observables as well as the Higgs data and the LHC searches constraints.

In summary, we proposed a multi-Higgs model which starts with a single input assumption and leads to a well defined, analytically tractable model with very characteristic DM or flavor sector features. The flavored version of this model is capable of describing all fermion masses, mixing, and CP-violation, and seems to point to sizable FCNC effects, which we are now investigating [17].

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