ANC method: experimental approach and recent results

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Abstract. The Asymptotic Normalization Coefficient (ANC) method, has proven to be useful in retrieving the direct part of the radiative capture cross section for a number of reactions of astrophysical interest. In this work, some of the latest results obtained by the AsFin2 group of the LSN-INFN (Catania) and the OJR of the NPI (Řež) will be briefly discussed.

1 Introduction

One of the biggest problems for nuclear astrophysics has always been to experimentally retrieve the cross section at astrophysical energies: the presence of the Coulomb barrier strongly decreases the overall cross section in the range of interest (usually between some keV and some MeV). Also, many reactions involve radioactive nuclei with a small half-life in the entrance channel, making a direct measurement sometimes almost unfeasible. For this reason, many indirect methods have been developed in the last decades, and among these the Asymptotic Normalization Coefficient (ANC)
textslciteMuc1,Xu one is useful to extract the direct capture cross section for (p, γ) [3], (n,γ) [4] and (α,γ) [5] reactions of astrophysical interest from a suitable one-particle transfer. In the case of highly unstable nuclei with small half-lifes, an extention to the method has been developed involving mirror reactions [6, 7]. In the following work the ANC method will be presented, and recent results regarding the study of the ^{26}Si$(p, \gamma)^{27}$P (performed using the mirror nuclei procedure) [8], the ^{6}Li$(p, \gamma)^{7}$Be [9] and ^{3}He$(\alpha,\gamma)^{7}$Be [10] reactions will be discussed.

2 The ANC method

As stated in the introduction, the ANC method allows to indirectly study direct capture reactions of astrophysical interest from one-particle transfer reactions. In the general case, a direct transfer reaction can be visualized as a $X + A \rightarrow Y + B$ process, where both the X and B nuclei can be considered as $X = Y + a$ and $B = A + a$, a being the transferred particle.
Using the Distorted Wave Born Approximation (DWBA), the matrix element for the transition between the initial and final state can be written as

$$M = \left\langle \chi_f^{(-)} I_{Aa}^{B} \left|\Delta V\right| I_{Ya}^{X} \chi_i^{(+)} \right\rangle \tag{1}$$

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where I^B_{Aa} and I^X_{Ya} are the radial overlap integrals of the two systems $Y + a$ and $A + a$ with the nuclei X and B respectively. The quantities $\chi_i^{(+)}$ and $\chi_f^{(-)}$ are the distorted wave-functions for the entrance and exit channels, while ΔV is the perturbative part of the total potential, composed by a real (V) and an optical (U) part ($\Delta V = V - U$). For a general nuclear process $a + b \rightarrow c$, with $c = a + b$, both the overlap functions in equation 1 can be written using the so-called spectroscopic factor (SF) – in general S$_{ab}$ – that has been widely used in the last fifty years to estimate the occupation probability of the orbitals:

$$I^c_{ab,l_c,j_c}(r_{ab}) = S^{1/2}_{ab,l_c,j_c}\phi_{n_c,l_c,j_c}(r_{ab}) \tag{2}$$

$\phi_{n_c l_c j_c}(r_{ab})$ being the bound-state wave function [12].
The description of the cross section using the SF depends on the binding portential and optical model parameters (OMP) [12] used to describe the reaction: the first one is in fact adjusted according to the binding energy to reproduce the bound state, while the OMP are usually extracted from elastic scattering. The SF is sensitive to the selection of the optical model adopted, and different families of potentials can reproduce the angular distributions retrieved from experimental data [14]. Using the DWBA it is possible to reproduce the experimental angular distribution in the following way:

$$\frac{d\sigma^{exp}}{d\Omega} = \sum_{j_B,j_X} S_{Aa,l_B,j_B} S_{Ya,l_X j_X} \sigma^{DW}_{l_B,j_b,l_X,j_X} \tag{3}$$

Here $\frac{d\sigma^{exp}}{d\Omega}$ and $\sigma^{DW}_{l_B j_b l_X j_X}$ are related by the two SF for the entrance and exit channel: while one of those is usually known, the other is left to be determined.
Mukhamedzhanov and Timofeyuk [11] pointed out that the direct part of the radiative cross section at low energies contains the same radial overlap integral (I^c_{ab,l_c,l_j}) of a direct transfer reaction: this happens because such capture occurs at large distances from the nucleus, where the behaviour can be called asymptotic. The asymptotic behaviour can be therefore used to extract the cross section of reactions of astrophysical interest: in this case is possible to write both the radial overlap functions and bound-state wave functions as

$$I^B_{Aa,l_c,j_c}(r_{ab}) \xrightarrow{r_{ab}>R_n} C^c_{ab,l_b,j_b} \frac{W_{-\eta,l_c+\frac{1}{2}}(2k_{ab}r_{ab})}{r_{ab}} \tag{4}$$

and

$$\phi_{ab,l_c,j_c}(r_{ab}) \xrightarrow{r_{ab}>R_n} b^c_{ab,l_c,j_c} \frac{W_{-\eta,l_c+\frac{1}{2}}(2k_{ab}r_{ab})}{r_{ab}} \tag{5}$$

In both equations (4) and (5) $W_{-\eta,l_c+\frac{1}{2}}$ is the Whittaker function, which asymptotic behaviour can be written as $W_{-\eta,l_c+\frac{1}{2}}(-2k_I r) \rightarrow e^{-k_I r+\eta_I ln(2k_I)}$, while the coefficients b and C are the so-called single-particle ANC (SPANC) and ANC, respectively. By substitution of equations (4) and (5) in (3), we obtain:

$$\frac{d\sigma}{d\Omega} = \sum_{j_B,j_X} \left(C^B_{Aa,l_B,j_B}\right)^2 \left(C^X_{Ya,l_X,j_X}\right)^2 \frac{\sigma^{DWBA}_{l_B,j_B,l_X,j_X}}{b^2_{Aa,l_B,j_B} b^2_{Ya,l_X,j_X}} \tag{6}$$

Using DWBA, we are finally able to extract the ANC's for the direct part of the cross section for reactions of astrophysical interest. This approach is useful because the ANC's have a small dependence on the chosen nucleon binding potential [12], making the cross section nearly independent from b^2. This last statement is not true in the case of tightly bound

core+particle systems: the sensitivity to the potential in the continuum state in fact increases with the separation energies, and the zero-energy S-factor is not uniquely determined by the experimentally determined ground state ANC [13].
More recently [6], an extension to the ANC method has been proposed involving the so-called mirror nuclei. Such a tool has proven to be useful in the case of proton (p) or neutron (n) transfer involving strongly unstable nuclei [6–8]: being $A + p \rightarrow B$ the proton transfer, the proton ANC (C_p^{A+p}) can in fact be extracted from the neutron ANC (C_n^{D+n}) of a suitable mirror partner $D + n \rightarrow E$, where D and E have inverted number of protons and neutrons with respect to A and B, respectively. The two ANC share the relation

$$\left(C_p^{A+p}\right)^2 = R_{mirr}\left(C^{C+n}\right)^2 \tag{7}$$

where R_{mirr} can be calculated as

$$R_{mirr} = \left|\frac{F_l\left(ik_pR_N\right)}{k_pR_nj_l\left(ik_nR_N\right)}\right|^2 \tag{8}$$

F_l being the regular Coulomb wave function, j_l the Bessel function of l–th order, and R_N the radius of the nuclear interior. The quantities k_p and k_n are related to the neutron and proton separation energy via the relation $k = \sqrt{\frac{2\mu\epsilon}{\hbar^2}}$ [6, 7].
In the case of the population of an excited state near the treshold, the mirror state will exist as a resonance. Thus a relation exists that ties the C^2 of the direct capture in the excited state and the Γ of the unbound state in the mirror counterpart [6]:

$$\frac{\Gamma_{p(n)}}{\left|C_{n(p)}\right|^2} = R_\Gamma \approx R_0^{res} = \frac{\hbar^2 k_{p(n)}}{\mu}\left|\frac{F_l\left(ik_{p(n)}R_N\right)}{k_{p(n)}R_N j_l\left(ik_{n(p)}R_N\right)}\right|^2 \tag{9}$$

3 ANC application

3.1 The ^{26}Si(p, γ)^{27}P reaction

^{26}Si production and destruction channels are important in the context of the long-standing problem of ^{26}Al ($T_{1/2}$ = 0.72 Myrs). Regarding this isotope, the 1.809 MeV γ-ray line emitted from the first excited state of ^{26}Mg, to which ^{26}Al decays, has been appointed as a tracer of the recent nucleosynthesis in our Galaxy and its presence has been found spread along Galactic plane. Observations support the idea that this isotope can been formed through nucleosynthesis in massive stars and core-collapse Supernovae [see 15, and references therein], but also Wolf-Rajet objects and AGB-stars [16], Novae [17] and X-ray burst [18] have been proposed as possible nucleosynthesis site. In any of those cases proton capture is allowed, and the ^{26}Al isotope can likely be produced via the reaction chain ^{24}Mg(p,γ)^{25}Al(β^+)^{25}Mg(p,γ)^{26}Al. A clear comprehension of the process is hampered by the presence of an isomeric state ^{26}Al^m($T_{1/2}$ = 6.34 sec), which can be directly fed by the ^{25}Al(p,γ)^{26}Si(β^+)^{26}Al^m chain, reducing the quantity of ^{26}Al in the interstellar medium. The ^{26}Si produced in this way can also be depleted by the ^{26}Si(p,γ)^{27}P reaction, that will therefore interfere with the production of ^{26}Al^m. The study of this reaction can than be useful to understand the ratio between ^{26}Al and ^{26}Al^m. Nonetheless, experiments involving ^{26}Si are challenging due to its short half-life

($T_{1/2}$ = 2.24 sec), and for this reason an indirect measurement of the ^{26}Si$(p,\gamma)^{27}$P has been performed using the ^{26}Mg$(d,p)^{27}$Mg one, applying the method described in Section 2 using mirror nuclei.
The experiment has been performed using the deuteron beam available at the U-120M isochronus cyclotron operated by the CANAM infrastructure of the Czech Academy of Sciences (Řež, Czech Republic) (E_{beam} = 19.2 MeV, I≈ 14 enA). Such a beam has been delivered on a ^{26}MgO (with ^{12}C as backing) target produced by the target laboratory of the Laboratori Nazionali del Sud - Istituto Nazionale di Fisica Nucleare (LNS–INFN).
The experimental apparatus was composed by five ΔE–E telescopes made using thin (250 μm) and thick (5000 μm) anular silicon detectors, covering the 7° – 60° range thanks to a rotating plate.
Once the Optical Model Parameters for the ^{26}Mg$(d,p)^{27}$Mg are extracted by means of the analysis of the angular distribution of the elastic scattering for the entrance channel and of the ^{26}Mg$(d,p)^{27}$Mg for the exit one, the ANC values for the ^{26}Mg + n → ^{27}Mg capture in the ground and first excited states have been found using Eq.6, in which the value of C^2_{pn} = 0.77 fm^{-1} [19] has been used. Once those quantities are known, the procedure for mirror nuclei has been applyied in order to extract the ANC and Γ_p for the ^{26}Si$(p,\gamma)^{27}$P

Table 1. Values found in [8] and compared with [20] and [21]

ref.	$(C^{Mg}_{g.s.})^2$ [fm^{-1}]	$(C^{Mg}_{1^{st}})^2$ [fm^{-1}]	$(C^{p}_{g.s.})^2$ [fm^{-1}]	$\Gamma^p_{1^{st}}$ [MeV]
[8]	28.26 ± 5.30	3.40 ± 0.32	1420 ± 255	$(5.23 \pm 1.05) \times 10^{-9}$
[20]	24.50 ± 4.90	1.1 ± 0.15	1058 ± 273	$(4.04 \pm 0.77) \times 10^{-9}$
[21]	44.00 ± 5.30	3.40 ± 0.32	1840 ± 240	$(5.4 \pm 0.1) \times 10^{-9}$

The results, shown in Table.1 show a substantial agreement between [8] and [20], if the different binding energies (0.807 MeV taken from [22] in [8] and 0.859 MeV in [20], respectively) are considered. Finally, once all the necessary quantities are extracted, the reaction rate for the direct and resonant capture in the ground and first excited state have been calculated using the formulas from [23], finding an enhancement by a factor 1.4 for the ground state and 2.2 for the first excited state contribution. To calculate the latter, the value of Γ_γ has been taken from [24].

3.2 The ^{6}Li(p,γ)^{7}Be and ^{3}He(α,γ)^{7}Be reactions

The ^{3}He(^{6}Li,d)^{7}Be reaction has been used to study both ^{3}He$(\alpha,\gamma)^7$Be [9] and ^{6}Li$(p,\gamma)^7$Be [10] radiative capture reactions. This is possible because ^{6}Li and ^{3}He have a high probability to be found in the cluster configuration $\alpha \oplus d$ and $p \oplus d$, respectively.
The ^{3}He$(\alpha,\gamma)^7$Be reaction is one of the most elusive ones in nuclear astrophysics. Despite the huge number of experimental and theoretical works performed to get information on the cross section during the last decades, its behaviour in the energy region of interest in the core of the Sun (between 10 keV and 30 keV at T = 15 MK) is still largely known only via theoretical extrapolations [25]. This reaction is important in H-burning in the Sun, given that it represents the first reaction of the so called 2nd and 3rd p-p chain branches. Also, a precise knowledge of the ^{3}He$(\alpha,\gamma)^7$Be reaction would be useful in the context the long-standing lithium problem of Big Bang Nucleosynthesis (at energies around 100 keV) [26].
The ^{6}Li$(p,\gamma)^7$Be, on the other hand, represents one of the two main destruction channel for ^{6}Li along with ^{6}Li$(p,\alpha)^3$He [27]. Given that the ^{6}Li is mainly produced via cosmic rays [28],

while ^{7}Li is formed in BBN and and in stellar scenarios [29], the isotopic ratio ^{6}Li/^{7}Li can be used to constraint the galactic enrichment process and lithium poroduction. Unfortunately, available experimental data are extracted at higher energies, leaving the behaviour of the cross section and reaction rate just to extrapolations in the region of astrophysical interest.
For the reason expressed above, both reactions have been studied in the same experimental campaign using the ^{3}He beam available at the 3.5 MV singletron accelerator of the Department of Physics and Astronomy (DFA) of the University of Catania (IT) an at the FN tandem accelerator at the John D. Fox Superconducting Accelerator Laboratory at Florida State University (Tallahassee, Florida - USA). Angular distributions have been studied at two beam energies (E_{beam} = 3 MeV and E_{beam} = 5 MeV), by means of movable silicon ΔE–E telescopes. The beam impinged on ^{6}LiF or on a pure ^{6}Li target evaporated on Formvar backing. At the backward angles in the center-of-mass reference frame, the differential cross section increases with the angle, hinting to the presence of a dominant one-step α-particle exchange mechanism – in this case the $^3\mathrm{He}(\alpha,\gamma)^7\mathrm{Be}$ reaction. In the forward hemisphere, on the other hand, this does not happen, and the one-step proton exchange process appears to be dominant. The ANC's for the $^3\mathrm{He}+\alpha\ \rightarrow^7\mathrm{Be}$ and $^3\mathrm{He}+p\ \rightarrow^7\mathrm{Be}$ have therefore been extracted separetely within the post form of the modified DWBA [12]. Channels coupling effects (CCE) have been taken into account considering the subsystem ^{3}He+^{6}Li, and coupling it with d+^{7}Be to study the proton transfer, and ^{7}Be+d to study the α-transfer. For the $^3\mathrm{He}+\alpha\ \rightarrow^7\mathrm{Be}$, the calculation led to the extraction of $C^2 = 20.84 \pm 1.12$ fm^{-1} for the capture in the ground state and of $C^2 = 12.86 \pm 0.50$ fm^{-1} for the one in the 1st excited state (E_{ex} = 0.429 MeV), in agreement with [30]. The astrophysical S-factor at zero energy has been calculated as $S_{34}(0) = 0.534 \pm 0.025$ keV × b.
The ANC for the $^6\mathrm{Li}+p\ \rightarrow^7\mathrm{Be}$ for the radiative proton capture in the ground and 1st excited state have been also extracted as $C^2 = 4.81 \pm 0.38$ fm^{-1} and $C^2 = 4.29 \pm 0.27$ fm^{-1}. The astrophysical S-factor at zero energy extracted using this values of the ANC is equal to $S(0) = 92 \pm 12$ eV × b, in agreement with [31], but with an uncertainty that is 1.6 times lower.

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