

# Lecture Notes on Gravitational Waves

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**Abstract.** These lectures notes give a overview of gravitational wave astrophysics and the role they play in particle astrophysics and multi-messenger astronomy. The lecture notes are organised into three main topics: the theoretical background of gravitational waves in general relativity, how gravitational waves are detected and identified in the data and what we have learned about both astrophysics and fundamental physics from gravitational waves. The lecture notes are at an introductory level with references to further details and are intended for graduate students in particle astrophysics. The notes are based on lectures presented at the XVIII ISAPP-Baikal Summer School on Particle Physics and Astrophysics, held at Bolshie Koty, Lake Baikal, Russia from 12th until 21st July 2018.

## 1. Introduction

The detection of gravitational waves in 2015 made news headlines around the world. It led to the award of a Special Breakthrough Prize in Fundamental Physics in 2016 and the Nobel Prize in Physics in 2017. But it is important to realise that the detection of gravitational waves by LIGO does not mark the end of its work, but only the beginning. The observatories can now be used to study the astrophysical objects causing the signals. Going forward, different types of signals are expected and it is still possible that something completely unexpected will be seen.

In the field of multi-messenger astronomy, gravitational waves are very much the newcomer. Optical observations of the sky have been performed by humans since prehistoric times. (Perhaps the first recorded optical observations were recordings of the Moon's phases on the Aurignacian bones, from approximately 34,000 years ago, but like a lot of things in prehistoric archeology these interpretations are controversial [1].) The interpretation of meteorites as rocks from outer space was made by Jean-Baptiste Biot at the beginning of the 19th Century. Cosmic rays were first detected in 1912 on high-altitude balloon flights and twenty years later Karl Jansky began detecting astrophysical radio waves. It was not until the era of space flight that X-rays and gamma rays were found to be coming from astrophysical sources and not until the advent of sensitive particle detectors that neutrinos were observed coming from the Sun. Thus we stand at the beginning of a new dawn in gravitational astronomy and these lectures are intended as an introduction to this new field.

The lectures were given to a group of mainly astronomy graduate students in the summer of 2018 as part of a wider school on particle astrophysics. The lectures were split into three topics, reproduced here as three sections. The first section deals with the theoretical background of gravitational waves, emphasising the many theoretical questions that needed to be solved



before the theoretical community accepted that gravitational waves were indeed a prediction of Einstein's theory of general relativity. The second section deals with the question of how gravitational waves are detected. There have been a number of claims of detections in the literature over the years so any claim is worthy of being scrutinised in detail. The third section deals with what can be learnt about astrophysics and fundamental physics from gravitational waves.

The lecture notes are largely self-contained, although some familiarity with relativity and astrophysics is assumed. No attempt is made to cover all topics related to gravitational waves and many further details are available in the references. Textbooks covering the theoretical basis of gravitational waves and astrophysical sources include the volumes by Maggiore [2, 3] and by Creighton and Anderson [4]. A nice guide to the principles of gravitational wave detection is given in Saulson's book [5] and an historical account about the theoretical debate over gravitational waves in [6]. Review articles about many of the topics related to gravitational waves can be found at the Living Reviews in Relativity [relativity.livingreviews.org](http://relativity.livingreviews.org). A discussion of the basic physics of the GW150914 binary black hole event can be found in [7] based on the original work analysing gravitational radiation from point-like Keplerian orbits in [8].

## 2. Theoretical background: What are gravitational waves?

It is a quirk of history that gravitational waves were detected almost exactly 100 years after Einstein first published his theory of gravity (known as general relativity or Einstein gravity) in 1915 [9]. In Newton's theory of gravity, perturbations of the gravitational field propagate at infinite speed. Attempts to modify Newton's gravity for finite propagation speed were attempted, for example by Oliver Heaviside in 1893 [10, 11]. These modifications typically led to instabilities in the Solar System and so astronomers lived, if somewhat uncomfortably, with Newton's infinite propagation. But Einstein's theory of relativity required a relativistic theory of gravitation, which Einstein produced in 1915. It is a curious fact that although Einstein's theory has a finite propagation speed, it does not lead to (major) instabilities in the Solar System due certain cancellations [12].

In a similar fashion to how Maxwell's relativistic electromagnetic field predicts electromagnetic waves, Einstein's relativistic theory of gravity predicts gravitational waves. But putting these ideas of a firm theoretical and mathematical footing took over half a century. This goes to show that general relativity is a complicated theory, with many subtle features that even confused Einstein himself.

### 2.1. Einstein's changing position

Einstein went through many apparent changes with respect to gravitational waves. In a February 1916 letter to Karl Schwarzschild he noted that "there are no gravitational waves, that are analogous to electromagnetic waves" (in electromagnetic theory, a changing dipole is enough to generate electromagnetic waves, in a gravitational theory the equivalent dipole is constant because of conservation of momentum.) By June 1916 he had found that a changing quadrupole moment can produce gravitational waves and attempted to calculate how much energy would be produced by them, concluding that the energy emitted would be negligible in all conceivable cases [13]. While he was correct that the energy emitted by ordinary objects (like the Earth) is negligible, Einstein did not know about the existence of neutron stars and black holes that can pack large amounts of mass in very small objects. It turns out that for the black holes observed by LIGO, the energy emitted in gravitational waves can be very large indeed, sometimes more than the equivalent mass energy of the entire Sun in less than a second.

Einstein's 1916 paper marked the first prediction by Einstein of gravitational waves and made

use of the linearisation of his field equations, which in modern notation we write as

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}. \quad (1)$$

In this formula the full gravitational field,  $g_{\mu\nu}$  is written as a sum of the flat Minkowski metric,  $\eta_{\mu\nu}$ , plus a perturbation,  $h_{\mu\nu}$ . This form of the metric is used in the Einstein field equations and if the perturbation is chosen sufficiently small, then terms that are quadratic in  $h_{\mu\nu}$  and its derivatives can be neglected. The result is a wave equation for the perturbation,  $h_{\mu\nu}$ , which thus describe gravitational waves, or, more colloquially, ripples in spacetime.

But Einstein wasn't fully satisfied with this answer. He knew that his theory of general relativity was non-linear and so non-linear effects could potentially modify the linearised approximation. With a collaborator, Nathan Rosen, he set out to find a fully non-linear solution of his field equations that described gravitational waves. But he found a problem, that he was unable to find a coordinate chart that was not singular somewhere when describing these solutions. Initially, Einstein concluded that such solutions were unphysical. He sent a manuscript to the Physical Review stating that gravitational waves did indeed not exist. The referee of Einstein's paper however found a mistake in Einstein's logic. Einstein was very angry that Physical Review had sent his work to be refereed and vowed never to submit a paper there again. But after talking with colleagues who explained his mistake, Einstein eventually submitted a different paper to a different journal describing gravitational waves as a prediction of his theory [14].

## 2.2. Progress after Einstein

The debate about gravitational waves continued after Einstein's death. At the 1957 Chapel Hill conference, Felix Pirani presented a mathematical formalism for interpreting metric coefficients as having real physical effects. Pirani's idea was to use freely falling masses and measure the distances between them - the same principle that is used in the LIGO interferometers. Richard Feynman also gave his sticky bead argument that gravitational waves carry energy. While Feynman's argument is very straightforward, it neatly sidesteps issues in identifying the energy density of a time-varying vacuum gravitational field. But debate continued on whether masses falling freely under gravity along geodesics emit gravitational waves and how to understand emission in strong fields where the linearised approximation doesn't hold. Work continued on whether the approximation of point masses was reliable, or the matching of strong field and weak field limits was robust [15, 16].

The detection of the binary-pulsar system PSR B1913+16 by Hulse and Taylor in 1974 (which won Hulse and Taylor the Nobel Prize in 1993 and has become known as the Hulse-Taylor binary) did not settle matters entirely. Pulsars had been discovered by Jocelyn Bell and Tony Hewish, and Hulse and Taylor used computer algorithms to search for similar pulsing signals in data of the large (305 metre aperture) radio telescope at Arecibo in Puerto Rico, United States. Nathan Rosen for example, remained skeptical of gravitational waves for many decades afterwards, even after the discovery of the Hulse-Taylor binary [17]. Also, other authors such as Cooperstock were skeptical that gravitational waves carry energy in vacuum [18]. The possibility was also raised that gravitational waves were produced by binaries, but would not propagate sufficiently to be detected on Earth.

## 3. Detecting new signals: How are gravitational waves detected?

As we saw in the first section, it took almost half a century from the publication of Einstein's theory of general relativity until there was broad consensus that gravitational waves were a physical prediction of the theory. This required many subtle questions to be addressed that nowadays we tend to take for granted. The detection in 1974 of the Hulse-Taylor binary showed

that gravitational waves did have real measurable effects on astrophysical systems and the race was on to detect these signals on Earth. As we will see in this section a large number of different technologies needed to be developed before this programme came to fruition. A number of false starts and claims of detections that didn't pan out, underline the importance of checking what is seen in the data is truly coming from gravitational waves.

### 3.1. Weber bars

Attempts to detect gravitational waves on Earth have been ongoing since the mid 1960's and the pioneering work of Joseph Weber. Weber's gravitational wave detectors were based on the principle of resonant bar detectors, large cylinders of metal that would vibrate when a passing gravitational wave excited them. Despite a number of published claims [19] [20, 21, 22, 23, 24] the consensus opinion in the community was that these resonant bar detectors failed to detect gravitational waves [25, 26, 27]. Despite Weber's failure to convince others he had detected gravitational waves, many credit him with starting the experimental search for such signals and one of his bars is on display at the LIGO Hanford Observatory and others are on display in the Weber Memorial Garden outside the Physics Department of the University of Maryland. Weber also developed some of the data analysis techniques that are used in gravitational wave detectors today, such as placing two detectors far apart to rule out local disturbances (Weber placed his detectors in Lemont, Illinois and College Park, Maryland, a distance of almost 1000 kilometers, LIGO has detectors in Hanford, Washington and Livingston, Louisiana, a distance of almost 3000 kilometers), using seismometers, electromagnetic sensors and cosmic ray detectors to rule out other interferences, introducing a time lag in one of the detectors to estimate the rate of spurious coincidences and searching for alternative polarizations of gravitational waves.

### 3.2. Interferometers

A key design improvement over resonant bar detectors was the use of interferometers (Rai Weiss, who won the Nobel Prize in 2017, was one of the early advocates of using interferometers.). This allowed both the sensitivity and the frequency range over which signals could be detected to be greatly improved. The problem was these interferometers needed to be very large to have a chance of detecting gravitational waves. This meant that large numbers of people and resources needed to come together to build and operate them and the LIGO project was started [28].

The LIGO interferometers work on similar principles to those that were used by Michelson and Morley to test the invariance of the speed of light. However, a number of design improvements are needed over a basic Michelson interferometer to make them sensitive enough to detect gravitational waves. The use of Fabry-Perot cavities stores the laser photons in the arms for longer without spatially separating their beams and increases the effective arm length of the detectors, making it more sensitive at the desired frequencies. Mode cleaners are used to spatially filter the initial laser beam and stabilise its frequency. Power recycling allows light that is emitted from the interferometer through the bright port to be returned back into the interferometer. This increases the power in the beams, reducing the shot noise and also passively filtering the input-laser frequency and intensity noises about 1Hz. Active seismic isolation allows known seismic disturbances to be suppressed in the motion of the test masses, which can be further suppressed using passive seismic isolation and suspending the test masses by a chain of very thin pendula fibres [29].

Although a passing gravitational wave will stretch the wavelength of the laser light in the interferometer, just as it stretches the space between the waves, this does not make the interferometers unable to detect gravitational waves [30]. The reason is that what is measured is a phase difference at the beam splitter and not the number of wavelengths of light that fit in a single arm and thus it is sufficient to consider the proper time elapsed by the light in each arm, as measured by an observer at the beam splitter [31].

### 3.3. Future detectors

Future detectors will operate with additional technologies, such as squeezed light [32] to reduce quantum noise, cryogenically cooled test masses to reduce thermal noise and building underground to reduce seismic noise, such as the KAGRA detector in Japan [33]. Proposals exist for triangular detectors with multiple interlocking interferometers and longer arms such as the Einstein Telescope [34] or building interferometers in space such as the ESA-led LISA mission for planned launch in 2034 <https://www.lisamission.org>.

Just like electromagnetic waves, gravitational waves have a full spectrum of different wavelengths, with different classes of sources emitting in different regions of the spectrum. The existing ground-based detectors of LIGO and Virgo are mainly sensitive in the range of 10 Hz to 1000 Hz. Space-based interferometer detectors such as LISA will have much longer arm lengths than are possible on Earth and will be sensitive to much longer wavelengths. Observations of pulsars in the galactic neighbourhood provide even longer effective arm lengths, using the distance between the pulsar and the Earth. Currently a European project EPTA, an American project NanoGrav and an Australian project PPTA are pursuing this method and as of February 2019 they are yet to detect a signal. They occasionally publish combined upper limits [35]. It may even be possible to detect the imprint of gravitational waves on the polarization of the cosmic microwave background and efforts are underway to do this including the BICEP and Keck Array projects at the South Pole [36], <http://bicepkeck.org>.

### 3.4. Processing the data

Once the raw data is produced, we need to know whether it contains any signals. Different types of searches are possible, broadly falling into modelled and unmodeled searches. Modelled searches aim to use expected properties of the signal to improve their efficiency. Unmodelled searches try to be more agnostic, allowing the detection of unexpected signals. The first gravitational wave signal seen at LIGO, GW150914, was first seen by an unmodelled search.

The detectors are designed to suppress certain types of known sources. Other types of noise sources, such as seismic activity, sound waves, temperature fluctuations, magnetic fields and cosmic rays can be monitored using different sensors that are more sensitive to these terrestrial disturbances than the gravitational wave detectors. Further information about them can be found at <http://pem.ligo.org> (LIGO Physical Environment Monitors). A possible environmental explanation for the signal GW150914 was searched for and not found [37].

Gravitational wave detectors are intrinsically challenging to operate, because they cannot be shielded from the signals you are attempting to measure (so you cannot measure a pure background) and it is impossible to construct strong enough gravitational wave sources here on Earth to act as calibration signals for the detectors. The detectors and the associated optics and electronics are very complex and there are still unknown sources of noise in the detectors. These are known to be noise and not gravitational wave signals because they affect the two LIGO detectors separately at different times. The cause of some of the unknown noise sources is eventually discovered and they can be removed or suppressed. Classifying and better understanding these noise sources is one of the aims of the Gravity Spy project, a citizen science project that can be found at <https://www.zooniverse.org/projects/zooniverse/gravity-spy> (GravitySpy).

The most sensitive searches for gravitational wave signals are performed by so-called matched-filter searches [38]. Matched-filtering involves correlating the detector data with a known model waveform of the signal one is searching for. These type of searches work best when the employed models closely resemble the signal being looked for. Other types of searches are also performed by the LIGO scientific and Virgo Collaborations, including unmodeled searches that search for coherent power between different detectors without using detailed models of the assumed signal [39]. The first gravitational wave event, GW150914, was first found by such an unmodeled search

[39] and later confirmed by the matched-filter searches. For well-modeled signals, the unmodeled searches are less sensitive, typically because they are less strict at excluding background events (data features that do not look like the signals being searched for). However, they offer the possibility of detecting unknown, unexpected signals and thus are an important part of analysing new gravitational wave data.

The detection of a gravitational wave signal requires a consistent signal in both detectors. The probability of two unrelated noise artifacts occurring at the same time in the two detectors can be estimated. This is what is calculated in the LIGO analyses that provides the p-values or false-alarm-rates (FARs) or  $p_{\text{astro}}$  for the various events [40]. These results explicitly ignore the possibility of correlated noise sources in the two detectors. To mitigate the possibility of correlated noise sources requires looking into possible sources of such correlations to exclude them. However, note that these analyses do not assume that the noise is Gaussian.

In addition to these background tests, various consistency checks can be performed, such as removing the presumed signal to analyse if the residuals are consistent with noise, or attempting to modify the presumed signal by allowing certain features to vary from the general-relativity predictions to see if this provides a better fit to the data. Results from various versions of these tests can be found in [41, 42, 43]. These types of tests give additional confidence that the signals do indeed come from the gravitational waves of coalescing binaries.

#### 4. Using the waves: What can we learn from gravitational waves?

The first two sections dealt with theoretical and experimental issues related to gravitational waves. Now we turn to the question of what gravitational wave observations can tell us about the universe both in terms of astrophysics and fundamental physics. A catalogue of the compact binary coalescences observed by the LIGO and Virgo teams in their first and second observing runs was recently released [40]. This lists ten binary black hole mergers and one binary neutron star merger. A catalogue constructed using the open data from the first observing run was released in [44]. Details of the three binary black hole events in the first observing run were presented in [45].

##### 4.1. Astrophysics

The 2017 observation of a binary neutron star merger was observed by a wide array of different electromagnetic telescopes from gamma rays to radio waves [46]. The coincident observation of gravitational waves confirmed that this short gamma ray burst was caused by the merger of two neutron stars [47]. With a single event, measuring the luminosity distance from the gravitational wave amplitude and the redshift of the host galaxy NGC 4993 enabled a measurement of the Hubble constant to lie between 58 and 78 kilometers per second per Megaparsec at 68% credible interval [48]. The follow-up of this event also indicated the presence of a kilonova due to the decay of heavy nuclei in the ejecta [46] of between a thousandth and a hundredth of a solar mass [49] - leading to the possibility that the majority of heavy elements, including gold and platinum, are formed from mergers of neutron stars. Measurements of the tidal interactions of the two neutron stars were also able to put strong constraints on their radii and the nuclear equation of state [50].

Predictions for how many signals would be seen in gravitational waves were made before the first detection. These were based both on observations and modelling. In 2010 the LIGO Scientific Collaboration collected some of these predictions together [51]. Now that detections have been made, we can compare the observed rates with the predicted rates. As the population of detections increases, these can be used to further constrain models of how these events are formed. The rate of binary neutron star mergers was predicted to lie between a pessimistic value of 10 per  $\text{Gpc}^3$  per year and 10,000 per  $\text{Gpc}^3$  per year. This has now been measured (with one event detected in approximately 150 days of data in O1 and O2 combined) to be 110 to

3840 per  $\text{Gpc}^3$  per year at 90% confidence [40]. The rate of binary black holes (specifically for 10 solar masses) was predicted to be between a pessimistic value of 0.1 per  $\text{Gpc}^3$  per year and an optimistic rate of 300 per  $\text{Gpc}^3$  per year. This has now been measured (with 10 events in approximately 150 days of data) to be 9.7 and 101 per  $\text{Gpc}^3$  per year at 90% confidence [40]. In the BNS case the pessimistic rate was an order of magnitude lower than what has now been ruled out at 90% confidence and for the BBH case, the pessimistic rate was lower by almost two orders of magnitude. Astronomers can sometimes be too pessimistic!

The observations by LIGO confirm that binary black hole systems do form and they do merge, but currently very little is known about how these systems are formed. Gravitational waves are very inefficient at bringing together stars at typical stellar binary radii and some other process other than gravitational waves is needed to tighten these binaries sufficiently so that they come close enough together for gravitational waves to be efficient at merging them. It is expected that some sort of common envelope phase is required [52]. It is not known whether the binaries formed together as stars or were brought together by some capture event. It is also unknown whether the systems formed in the field of large galaxies where they can remain unperturbed for large periods of time, or in dense globular clusters where stellar interactions are common.

One thing that is known about the heavy black holes observed by LIGO is that they should have been formed in low metallicity environments [53] with metallicities no greater than around half the solar value. This is because stars with higher metallicities have higher solar winds and radiate too much mass during their short lifetimes to produce heavy black holes.

#### *4.2. Fundamental physics*

With the BNS event GW170817, from the arrival time difference of the gravitational waves and the gamma rays and the known distance to the host galaxy NGC 4993, it was possible to put a simple constraint on the speed of gravitational waves relative to the speed of light. This constraint was much stronger than had previously been possible (by 14 orders of magnitude [54]). This result alone ruled out a large class of modified gravity models and put very strong constraints on modified gravity explanations of the accelerated expansion of the universe [55, 56, 57, 58] and dark matter [59].

The LIGO and Virgo collaborations have placed a number of bounds on deviations from general relativity, for the BBH GW150914 [41] and the BNS GW170817 [60]. This includes a bound on the mass of the graviton,  $< 1.2 \times 10^{-22} \text{eV}$ , that is stronger than precision bounds on the mass of the photon [61]. Many of these tests are based on existing methods to interpret signals assuming general relativity. Typically the tests relax one or more of the aspects predicted by general relativity and hence act as consistency tests of the assumptions that go into deriving the standard GR results used for astrophysics. The tests all show that the observations are compatible with general relativity to the accuracy obtained and uncertainties produced by the detector noise. These results provide strong constraints on alternative explanations for the signals observed and some of these arguments can be seen at a very basic level [7].

Deviations from general relativity are expected on curvature scales comparable to the Planck scale. Astrophysical black holes do not attain such strong curvature scales near or at their horizons and thus many people expect all tests with gravitational waves to be consistent with general relativity. However, black holes by definition do attain horizon-forming scales and it is possible (some argue that it is necessary [62]) that this may lead to modified effects. These modified effects are motivated by quantum gravity [63], or the black hole information paradox [64] or quantum field theory on curved spacetimes [65].

If physics is different from the predictions of vacuum general relativity at near-horizon scales, then it is possible that incoming waves will reflect off of some of the structure instead of being completely absorbed as in the case for standard black holes in vacuum general relativity. This reflection of incoming waves is potentially observable with gravitational wave detectors, as it

will lead to a series of reflecting echo signals after the main signal of a compact merger [66].

A number of searches for such echo signals have been performed with different results [67, 68, 69, 70]. I have a vested interest in this debate as it is one of the topics I work on and my position is that the data where echoes have been claimed is consistent with being just noise.

One interesting prediction of general relativity that has yet to be observed with gravitational waves is the non-linear memory effect [71] [72]. This effect is a permanent displacement of spacetime and hence the interferometer arms, after the passing of a gravitational wave. There is an intimate relationship between the memory effect and the vacuum structure of general relativity [73]. This vacuum structure is encoded in the infinite dimensional BMS group and whose discovery goes back to the theoretical debates about whether gravitational waves were real [74]. This highly non-trivial vacuum structure may even be related to a resolution of the black hole information paradox [75]. A number of techniques are being pursued to observe this memory effect [76, 77, 78] but it is likely that success is still some years away [79].

## 5. Summary

The future is bright in gravitational wave science. Detections of several merging black hole have been made and a spectacular multi-messenger event was seen with the merging of two neutron stars in GW170817. The LIGO and Virgo detectors in the USA and Italy have scheduled sensitivity improvements that are currently (February 2019) being implemented and the Japanese detector KAGRA is expected to join in observational runs in the next year or so. From around 2024 another observatory in India is expected to join, completing a five detector world-wide network. From 2034 ESA expects to deploy the space-based LISA detector and other space-based detectors are being planned in Japan and China. A factor of two improvement of detector strain sensitivity will lead to almost an eight-fold increase in the rate of detections and so the near future is likely to see thousands of individual detections. We have still not observed a binary of a black hole and a neutron star or the continuous gravitational pulsations of a single neutron star. With luck we may even catch a nearby supernova or even a previously unexpected signal from a new type of astrophysical object.

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