NLO and NNLO Calculations

V. Del Duca and G. Heinrich

1 The NLO and NNLO program

QCD calculations of multijet rates beyond the leading order (LO) in the strong coupling constant α_s are usually quite involved. Nowadays we know (see Section 1.2) how to perform in general calculations of the next-to-leading order (NLO) corrections to multijet rates, and almost every process of interest has been computed to that accuracy. Instead, the calculation of the next-to-next-to-leading order (NNLO) corrections is still at an organizational stage and represents a main challenge. Why should we perform calculations which are technically so complicated ?

The general motivation is that the calculation of the NLO corrections allows us to estimate reliably a given production rate, while the NNLO corrections allow us to estimate the theoretical uncertainty on the production rate. This is achieved by reducing the dependence of the cross section on the renormalization scale, μ_R, and for processes with strongly-interacting incoming particles the dependence on the factorization scale, μ_F, as well.

An example is the determination of α_s from event shape variables in $e^+e^- \to 3$ jets [1]. Although the NLO contributions to $e^+e^- \to 3\,jets$ have been computed for some time now [2, 3], the NNLO contributions have yet to be obtained. A calculation of these NNLO contributions would be needed to further reduce the theoretical uncertainty in the determination of α_s.

We present in Section 1.1 an additional motivation for performing QCD calculations at NNLO, which is specific to the LHC program, and we outline in Section 1.2 how QCD calculations at NLO are implemented and in Section 1.3 how QCD calculations at NNLO could be performed.

1.1 Higgs production

The main goal of the LHC physics program is the investigation of the mechanism of the electroweak symmetry breaking, and namely the search and detection of the Higgs boson. If the Higgs boson is light (100 GeV $\leq m_H \leq$ 140 GeV), the rare decay channel in two photons, H$\to \gamma\gamma$, provides the best signature [4, 5, 6, 7]. Since the signal-to-background ratio is quite low ($\sim 7\%$), the analysis of this channel promises to be demanding. Our theoretical understanding of signal and background is still preliminary: the NLO QCD corrections to the signal are known to be quite large ($\mathcal{O}(100\%)$) [8]. Also the QCD background $pp \to \gamma\gamma$, given at LO by the parton subprocess $q\bar{q} \to \gamma\gamma$, is known to NLO [9], with the full NLO fragmentation contributions having just been evaluated [10]. However, $pp \to \gamma\gamma$ receives a sizeble contribution from NNLO corrections because of the large gluon luminosity of the subprocess $gg \to \gamma\gamma$ appearing first at NNLO [11]. Thus in order to have a reliable theoretical estimate both the signal and the background need to be determined at least to NNLO accuracy.

In order to improve the signal-to-background ratio, Higgs production in association with a high transverse energy (E_T) jet, $p\,p \to H\,jet \to \gamma\gamma\,jet$, has been considered [12]. This production rate offers the advantage of being more flexible in choosing suitable acceptance cuts to curb the background. $p\,p \to H\,jet$ is known to LO exactly [13], while the NLO corrections [14] have been computed in the infinite top-mass limit. The NLO corrections to the signal are large. However, it is believed that the background, $p\,p \to \gamma\gamma\,jet$, can be more reliably calculated because LO production is dominated by the parton subprocess $q\,g \to q\,\gamma\gamma$, which benefits from the large gluon luminosity, while the subprocess $gg \to g\gamma\gamma$, which is believed to dominate the NNLO contribution, yields a comparatively smaller contribution [11, 15]. Thus, even though the signal, $p\,p \to H\,jet$, likely needs be computed at NNLO accuracy, it should suffice to evaluate the background, $p\,p \to \gamma\gamma\,jet$, at NLO. The NLO corrections to the background, though, have yet to be computed, with the appropriate QCD amplitudes having just been evaluated [16].

1.2 NLO algorithms and one-loop amplitudes

In recent years it has become clear how to construct general-purpose algorithms for the calculation of multijet rates at NLO accuracy. The crucial point is to organise the cancellation of the infrared (i.e. collinear and soft) singularities of the QCD amplitudes in a universal, i.e. process-independent, way. The universal terms in a NLO calculation are given by the tree-level splitting [17] and eikonal [18, 19] functions, and by the universal structure of the poles of the one-loop amplitudes [20, 21, 22]. The universal NLO terms and the process-dependent amplitudes are combined into effective matrix elements, which are devoid of singularities. The various NLO algorithms (phase-space slicing [20, 23], subtraction method [21, 24], dipole formalism [25] and subtraction-improved slicing [26]) provide different methods to construct the effective matrix elements. These can be integrated, analytically or otherwise numerically, in four dimensions. The integration can be performed with arbitrary experimental acceptance cuts.

Then the remaining work to be performed to calculate a production rate at NLO is to compute the appropriate tree and one-loop amplitudes. To compute n-jet production at NLO, two sets of amplitudes are required: *a)* n-particle production amplitudes at tree level and one loop; *b)* $(n+1)$-particle production amplitudes at tree level. If the one-loop amplitudes are regularised through dimensional regularization (DR) by evaluating them in $d = 4-2\epsilon$ dimensions, it suffices at NLO to compute them to $\mathcal{O}(\epsilon^0)$. As an example, in Fig. 1 we show the squared matrix elements which are required to calculate the NLO corrections to $e^+e^- \to 3\,jets$.

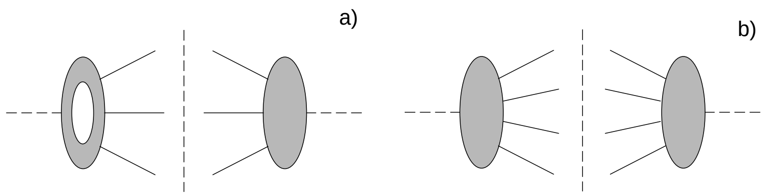

Figure 1: Squared matrix elements which contribute the NLO corrections to $e^+e^- \to 3\,jets$. The dashed line represents a massive vector boson, γ^*, W, Z. $a)$ interference term between one-loop and tree amplitudes. The final-state partons are a $q\bar{q}$ pair and a gluon. $b)$ square of a tree amplitude. The final-state partons are a $q\bar{q}$ pair and two gluons, or two $q\bar{q}$ pairs. In figure $b)$ one of the partons is unresolved.

Efficient methods based on the color decomposition [27, 28, 29, 30] of an amplitude in color-ordered subamplitudes, which are then projected onto the helicity states of the external partons, have largely enhanced the ability of computing tree [31] and one-loop [32] amplitudes. Accordingly, tree amplitudes with up to seven massless partons [31, 33] and with a vector boson and up to five massless partons [34] have been computed analytically. In addition, efficient techniques to evaluate numerically tree multi-parton amplitudes have been introduced [35, 36], and have been used to compute tree amplitudes with up to eleven massless partons [36]. The calculation of one-loop amplitudes can be reduced to the calculation of one-loop n-point scalar integrals [37, 38]. The reduction method [37] allowed the computation of one-loop amplitudes with four massless partons [39] and with a vector boson and three massless partons [40]. However, one-loop scalar integrals present infrared divergences, induced by the massless external legs. For one-loop multi-parton amplitudes, the infrared divergences hinder the reduction methods of ref. [37, 38]. This problem has been overcome in ref. [41]. Accordingly, one-loop amplitudes with five massless partons [42, 43] and with a vector boson and four massless partons [44] have been computed analytically. The reduction procedure of ref. [41] has been generalised in ref. [45], where it has been shown that any one-loop n-point scalar integral, with $n > 4$, can be reduced to box scalar integrals, and that in the reduction of n-point tensor integrals, all higher dimensional ($d > 4 - 2\epsilon$) n-point integrals with $n > 4$ drop out. The calculation of one-loop multi-parton amplitudes thus can be pushed a step further in the near future.

1.3 NNLO calculations

Eventually, a procedure similar to the one followed at NLO will permit the construction of general-purpose algorithms at NNLO accuracy. It is mandatory then to fully investigate the infrared structure of the phase space at NNLO. The universal pieces needed to organise the cancellation of the infrared singularities are given by the tree-level double-splitting [46, 47, 29], double-eikonal [19, 48] and splitting-eikonal [46, 48] functions, by the one-loop splitting [49, 50] and eikonal [49] functions, and by the universal structure of the poles of the two-loop amplitudes [51]. These universal pieces have yet to be assembled together, to show the cancellation of the infrared divergences at NNLO.

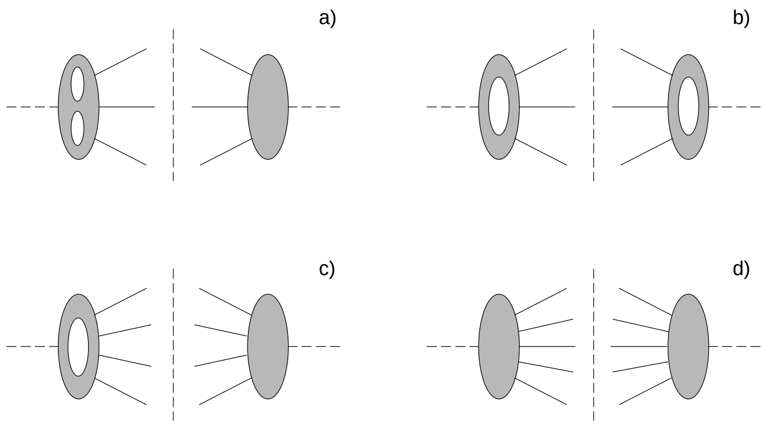

Figure 2: Squared matrix elements which contribute the NNLO corrections to $e^+e^- \to 3\, jets$. The dashed line represents a massive vector boson, γ^*, W, Z. $a)$ interference term between two-loop and tree amplitudes, and $b)$ square of a one-loop amplitude. In figures $a)$ and $b)$ the final-state partons are a $q\bar{q}$ pair and a gluon. $c)$ interference term between one-loop and tree amplitudes. The final-state partons are a $q\bar{q}$ pair and two gluons or two $q\bar{q}$ pairs. One of the partons is unresolved. $d)$ square of a tree amplitude. The final-state partons are a $q\bar{q}$ pair and three gluons, or two $q\bar{q}$ pairs and a gluon. Two of the partons are unresolved.

Then to compute n-jet production at NNLO, three sets of amplitudes are required: $a)$ n-particle production amplitudes at tree level, one loop and two loops; $b)$ $(n+1)$-particle production amplitudes at tree level and one loop; $c)$ $(n+2)$-particle production amplitudes at tree level. In Fig. 2 we show the squared matrix elements which are required to calculate the NNLO corrections to $e^+e^- \to 3\, jets$. In DR at NNLO, the two-loop amplitudes need be computed to $\mathcal{O}(\epsilon^0)$, while the one-loop amplitudes must be evaluated to $\mathcal{O}(\epsilon^2)$ [49, 52]. The main challenge is the calculation of the two-loop amplitudes. At present, the only amplitude known at two loops is the one for $V \leftrightarrow q\bar{q}$ [53], with V a massive vector boson, which depends only on one kinematic variable. It has been used to evaluate the NNLO corrections to Drell-Yan production [54] and to deeply inelastic scattering (DIS) [55]. No two-loop computations exist for configurations involving more than one kinematic variable, except in the case of maximal supersymmetry [56]. One of the main obstacles for configurations involving two kinematic variables is the analytic computation of the two-loop four-point functions with massless external legs, where significant progress has just been achieved. These consist of planar double-box integrals [57], non-planar double-box integrals [58], single-box integrals with a bubble insertion on one of the propagators [59] and single-box integrals with a vertex correction [60]. The two-loop four-point functions with massless external legs are needed for the computation of two-loop amplitudes in parton-parton scattering. Finally, the topical processes considered above, i.e. $e^+e^- \to 3\, jets$ and $p\, p \to H\, jet$ sport configurations involving three kinematic variables and require the analytic computation of two-loop four-point functions with a massive external leg. The two-loop four-point functions of this kind with up to five different denominators have been derived recently [61], while those with six and seven different propagators are still missing. Another obstacle is the color decomposition of two-loop amplitudes, which is not known yet. Substantial progress is expected in the next future on all the issues outlined above, which should make the present note soon outdated.

Finally, we mention that in the factorization of collinear singularities [62] for strongly-interacting incoming particles, the evolution of the parton distribution functions (*pdf*'s) in the jet cross section should be determined to an accuracy matching the one of the parton cross section. For hadroproduction of jets computed at NLO, one needs the NLO, or two-loop, evolution of the *pdf*'s [63, 64, 65]. Accordingly for hadroproduction at NNLO the evolution of the *pdf*'s should be computed to NNLO, or three-loop, accuracy. Except for the lowest five (four) even-integer moments of the three-loop non-singlet (singlet) splitting functions [66], no calculation of the NNLO evolution of the *pdf*'s exists yet. However, NNLO analysis based on the finite set of known moments have been performed for xF_3 [67, 68] and F_2 (non-singlet [69] and singlet [70]). Furthermore, in ref. [71] a quantitative assessment of the importance of the yet unknown higher-order terms has been performed, with the conclusion that they should be numerically significant only for Bjorken-scaling $x < 10^{-2}$.

The computation of the evolution of the *pdf*'s at NNLO accuracy is a main challenge in QCD. The NLO computation was performed with two different methods, one using the operator product expansion (OPE) in a covariant gauge [63], the other using the light-cone axial (LCA) gauge with principal value prescription [64]. However, the prescription used in ref. [64] has certain shortcomings. Accordingly, the calculation has been repeated in the LCA gauge using a generally correct prescription [72], which makes it amenable to extensions beyond NLO. On the other hand, using the OPE method, there had been a problem with operator mixing in the singlet sector, which has been fixed[65] only recently, and the result finally coincides with the one obtained in the LCA gauge in ref. [64]. Thus the calculation of the *pdf* evolution at NLO accuracy is fully under control. Recent proposals for a calculation beyond NLO include extensions of the OPE technique, which have been used to recompute the NNLO corrections to DIS [73], and a computation of the *pdf* evolution by combining the universal gauge-invariant collinear pieces [74]. For the two-loop *pdf* evolution, e.g., they are the collinear pieces mentioned at the beginning of this section.

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