

# The study of the propagation of nonlinear linear wave in QGP: a possible tool to distinguish the extensive and non-extensive thermodynamic background

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## Introduction

Study of perturbation is a useful tool to reveal the properties of a medium. Depending on the magnitude of the perturbations, a perturbation may be called linear(magnitude is very smaller compared to the average value) and nonlinear(magnitude of the order of the average or unperturbed value). In the context of quark gluon plasma(QGP), the response of the medium on propagation of a perturbation may help in characterizing the QGP. The study of linear perturbations are well documented [1], whereas a very few studies can be found for nonlinear perturbations [2, 3]. When a quarks and gluons produced in the early stage of Relativistic Heavy Ion Collision with high momenta, do not equilibrate but propagate as jets through the thermal medium formed due the re-scattering of the low momentum quarks and gluons. These jets while propagating deposit energy in the medium. If the magnitude of the energy density resulting from the jet-medium interaction is comparable to the energy density of the medium then the effects of jet propagation will be treated as nonlinear. In the Refs. [2], the propagation of perturbations are treated as nonlinear waves, maintaining solitonic behaviour, and the background are considered to be of

extensive (Boltzman-Gibbs distribution) type. But for a small system like  $p - p$ , the distribution fits to be of non-extensive(Tsallis distribution) type, suggested by Tsallis [4]. Tsallis distribution contains an extra index, called the  $q$ -parameter, and putting  $q = 1$  gives rise to Boltzman-Gibbs distribution.

In Ref. [3], the nonlinear wave was studied within the purview of an Equation of State(EoS) from Tsallis distribution, but the fluid was treated to follow the extensive ideal hydrodynamics. To get the effect of  $q$ , one should consider the EoS containing  $q$ , as well as the background of  $q$ . Now in Ref. It is shown that  $q$ -ideal hydrodynamics( $q$ -hydro) is equivalent to extensive dissipative hydrodynamics( $d$ -hydrodynamics) [5]. In our study, we considered the  $q$ -EoS and the background we consider follows the extensive dissipative hydrodynamics to study the propagation of nonlinear waves.

## Results and discussion

We considered the order of perturbations up to second order and the equations of propagation of nonlinear waves are found to be Kdv-like(Korteweg-De Vries). Despite of the presence of the relevant transport coefficients, like shear and bulk viscosities( $\eta$  and  $\zeta$ ), the nonlinear waves are found to be solitonic and does not dissipate much within the lifetime of the QGP(see Fig.1). It is also found that if the magnitude and the width of the perturbations

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are larger, the effects of dissipation is very less and the speed of propagation is larger, which is a well known feature of the nonlinear wave, says that nonlinear waves with larger magnitudes move faster. Now, as far as the temperature of the medium is higher, the dissipation of the nonlinear waves is less. Also dissipation is found to be lesser when the value of  $q$  is higher. By putting  $\eta = 0$ , and  $\zeta = 0$ , in the solu-

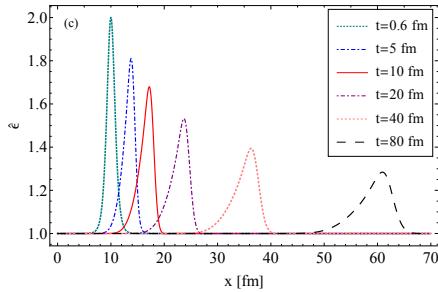


FIG. 1: (Color online)  $\hat{\epsilon}$  as a function of  $x$  at different time for  $T = 200\text{MeV}$  and  $q = 1.08$ .

tion, the equations of propagation of nonlinear waves are found to be Breaking-wave type. In Fig.2, it is seen that the nonlinear waves are not localised in space and tending to break. Since the EoS and transport coefficients con-

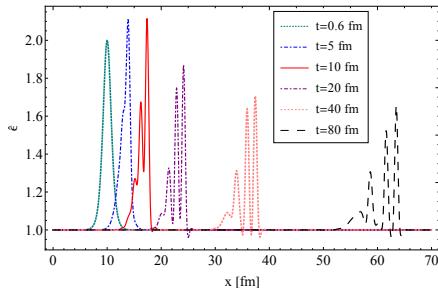


FIG. 2: (Color online)  $\hat{\epsilon}$  as a function of  $x$  at different time for  $t = 10$  fm and  $T = 200$  MeV.

trol the bulk evolution of a system, therefore, for a thorough treatment of the propagation of nonlinear waves in a non-extensive background( $q$ -background), one should consider the effects of non-extensivity, not only through the EoS( $q$ -EoS) but also through the transport coefficients( $q$ -viscosities). In Ref. [3]

without the presence of the dissipative terms, the  $q$  alone (through  $q$ -EoS only) can not produce such dissipative nature, *i.e.*,  $q$  can only have a role in dissipation in the presence of explicit dissipative terms. One may be tempted to think that, perhaps, less dissipation is the unique signature of presence of finite  $q$  (increasing  $q$ -values). However, it may be noted that the less dissipation may be accounted through the smaller values of the transport coefficients. So there may be an ambiguity behind the observed smaller dissipation in a system, whether it is non-extensive system with larger  $q$  or it is extensive with lower viscosity. With less dissipation in extensive background (*i.e.* without  $q$ -background) the nonlinear wave will not preserve the solitonic nature simultaneously due to breaking of the wave (see Fig. 2), whereas the  $q$  makes dissipation less with preserving the solitonic nature (see Fig. 3). Therefore, the causes for less dissipation with Tsallis background can be distinguished with the propagation of a nonlinear wave.

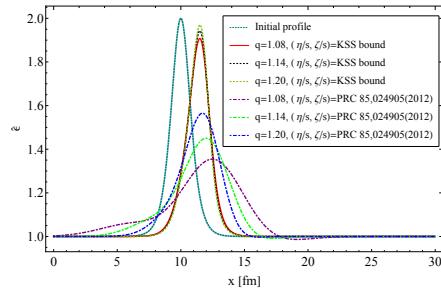


FIG. 3: (Color online) A comparison on propagation of nonlinear waves is shown for extensive vs non-extensive background .

## References

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