Isospin mixing within relativistic mean-field models including the delta meson

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Abstract. We investigate isospin mixing effects in the asymmetry as obtained in parity-violating electron scattering from ^{4}He, ^{12}C , ^{16}O, ^{40}Ca and ^{56}Ni. The scattering analysis is developed within plane (PWBA) and distorted wave (DWBA) Born approximations accounting for nucleon form factors, which are given by the Galster parametrization. We use Walecka's Model (QHD), including the σ, ω, ρ and δ mesons as well as the electromagnetic interaction. The δ meson effects are specially interesting once it should add a contribution for isospin mixing together with the electromagnetic and ρ meson fields. Our model includes lagrangians with non-linear terms as well as lagrangians including density dependent couplings. The model is solved in a Hartree approximation with spherical symmetry using a self-consistent calculation by means of an expansion of the nuclear wave functions and potentials in an harmonic oscillator basis. Results using four different parametrizations are obtained and compared with calculations using non-relativistic models.

1. Introduction

Although the weak interaction may be dwarfed by the electromagnetic interaction in an electron-nucleus scattering, the use of a polarized beam can make the effects of the first one stand out. This is possible since the electromagnetic interaction is independent of the polarization state of the incident particle. In this way, we can define the asymmetry $\mathcal{A}$ as the ratio between the difference and the sum of the scattering cross-sections for beams with positive and negative polarizations.

For nuclei with $N = Z$ in a *Plane Wave Born Approximation* (PWBA), the asymmetry has the form [1]

$$\mathcal{A} \propto \sin^2 \theta_W [1 + R_V^{T=0} + \Gamma(q)] + [1 + R_V^0]\frac{G_E^s}{G_E^0}. \tag{1}$$

The terms $R_V^{T=0}$ and R_V^0 are radiative corrections originated from higher order corrections in the Standard Model and θ_W is the Weinberg angle. The term $\Gamma(q)$ accounts for the isospin mixing, i. e., contributions related to differences between proton and neutron distributions in the nucleus. Finally, G_E^s/G_E^0 represents the ratio of the strangeness nucleon form factor to the nucleon isoescalar form factor.

While the nuclei properties for low momentum transfer are satisfactorily explained using u and d quarks, for higher momentum transfer, excitations like the creation of quark-antiquark pairs may happen. Those particle-antiparticle pairs may give rise to significant effects in the

interactions between the probe and the nucleons. The lower mass of the s quark, when compared with c, b and t quarks, should imply in the presence of $s\bar{s}$ pairs, which originates the strangeness form factor G^s. In an attempt to determine this form factor, measurements have been made using ^{1}H and ^{4}He targets [2, 3].

When interactions that differentiate protons from neutrons are not taken in to account and the radiative corrections as well as the strangeness content are disregarded, the asymmetry is directly related to $\sin^2\theta_W$ and can be used as a test of the Standard Model. Experiments using ^{12}C with this goal are already completed [4] and new ones using ^{1}H targets are in progress [5]. However, the presence of the electromagnetic interaction makes the isospin symmetry only an approximated one, and a good understanding of the isospin mixing $\Gamma(q)$ is very important in order to correctly analyze the asymmetry measurements.

We shall be concerned here on the effects of isospin mixing, based on an relativistic model for the nucleus, while ignoring any effects due to strangeness content or corrections originated from higher approximations in the Standard Model. Furthermore, only spherical nuclei with $N = Z$ in their ground states are considered.

2. Nuclear Model

The model chosen to describe the nuclear structure is based on Walecka's Model [6]. The interactions include the exchange of σ (scalar), ω (vector), ρ (isovector) and δ (isoscalar) mesons and the electromagnetic interaction. We start from two realistic lagrangians: one including non-linear terms (NL) and another one which contains density dependent couplings (DD). They are given by [7, 8]

$$
\begin{aligned}
\mathcal{L}_{NL} = &\; \overline{\Psi}\{i\gamma^\mu\partial_\mu - m\}\Psi + \frac{1}{2}\partial^\mu\sigma\partial_\mu\sigma \\
&- \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{3}g_2\sigma^3 - \frac{1}{4}g_3\sigma^4 - g_\sigma\overline{\Psi}\Psi\sigma \\
&+ \frac{1}{2}\partial_\mu\vec{\delta}\cdot\partial^\mu\vec{\delta} - \frac{1}{2}m_\delta^2\vec{\delta}^2 - g_\delta\overline{\Psi}\vec{\tau}\cdot\vec{\delta}\Psi \\
&- \frac{1}{4}\Omega^{\mu\nu}\Omega_{\mu\nu} + \frac{1}{2}m_\omega^2\omega^\mu\omega_\mu - g_\omega\overline{\Psi}\gamma^\mu\Psi\omega_\mu \\
&- \frac{1}{4}\vec{R}^{\mu\nu}\vec{R}_{\mu\nu} + \frac{1}{2}m_\rho^2\vec{b}^\mu\vec{b}_\mu - g_\rho\overline{\Psi}\gamma^\mu\vec{\tau}\cdot\vec{b}_\mu\Psi \\
&- \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - e\overline{\Psi}\gamma^\mu\frac{(1+\tau_3)}{2}\Psi A_\mu
\end{aligned}
\qquad
\begin{aligned}
\mathcal{L}_{DD} = &\; \overline{\Psi}\{i\gamma^\mu\partial_\mu - m\}\Psi \\
&+ \frac{1}{2}\partial^\mu\sigma\partial_\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 + \overline{\Psi}\Gamma_\sigma\sigma\Psi \\
&+ \frac{1}{2}\partial_\mu\vec{\delta}\cdot\partial^\mu\vec{\delta} - \frac{1}{2}m_\delta^2\vec{\delta}^2 + \overline{\Psi}\Gamma_\delta\vec{\tau}\cdot\vec{\delta}\Psi \\
&- \frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \overline{\Psi}\Gamma_\omega\gamma^\mu\omega_\mu\Psi \\
&- \frac{1}{4}\vec{R}_{\mu\nu}\vec{R}^{\mu\nu} + \frac{1}{2}m_\rho^2\vec{b}_\mu\cdot\vec{b}^\mu - \overline{\Psi}\Gamma_\rho\gamma^\mu\vec{\tau}\cdot\vec{b}_\mu\Psi \\
&- \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - e\overline{\Psi}\frac{1-\tau_3}{2}A^\mu\Psi,
\end{aligned}
$$

$$\tag{2}$$

with $\Omega_{\mu\nu} \equiv \partial_\mu\omega_\nu - \partial_\nu\omega_\mu$, $\vec{R}_{\mu\nu} \equiv \partial_\mu\vec{b}_\nu - \partial_\nu\vec{b}_\mu$, $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$, $\rho = \sqrt{J^\mu J_\mu}$ and $J^\mu = \overline{\Psi}\gamma^\mu\Psi$. The Dirac and Klein-Gordon equations of motion are then obtained from the Euler-Lagrange equation for nucleons and mesons, respectively. Furthermore, we use a mean-field approach, considering only the static, spherically symmetric case. After those approximations, we obtain for the barionic field

$$
[-i\vec{\alpha}\cdot\nabla + \gamma^0(m + g_\sigma\sigma(r) + g_\delta\tau_3\delta(r))]\Psi(\vec{r}) + \left[g_\omega\omega(r) + g_\rho\tau_3 b(r) + e\frac{(1+\tau_3)}{2}A(r)\right]\Psi(\vec{r}) = E\Psi(\vec{r}).
$$

$$\tag{3a}$$

As for mesonic fields, we obtain

$$[-\nabla^2 + m_\sigma^2]\sigma(r) = -g_\sigma \rho_s(\vec{r}) - g_2\sigma^2(r) - g_3\sigma^3(r) \tag{3b}$$

$$[-\nabla^2 + m_\delta^2]\delta(r) = -g_\delta \rho_3^s(r) \tag{3c}$$

$$[-\nabla^2 + m_\omega^2]\omega(r) = g_\omega \rho_B(\vec{r}) \tag{3d}$$

$$[-\nabla^2 + m_\rho^2]b_0(r) = g_\rho \rho_3(\vec{r}) \tag{3e}$$

$$-\nabla^2 A(r) = e\rho_B^p(\vec{r}), \tag{3f}$$

with

$$\rho_s(\vec{r}) \equiv \overline{\Psi}\Psi \tag{4a}$$

$$\rho_3^s(\vec{r}) \equiv \overline{\Psi}\tau_3\Psi \tag{4b}$$

$$\rho_B(\vec{r}) \equiv \overline{\Psi}\gamma^0\Psi \tag{4c}$$

$$\rho_3(\vec{r}) \equiv \overline{\Psi}\gamma^0\tau_3\Psi \tag{4d}$$

$$\rho_B^p(\vec{r}) \equiv \overline{\Psi}\gamma^0 \frac{1+\tau_3}{2}\Psi. \tag{4e}$$

For the DD lagrangean, we obtain

$$[-i\vec{\alpha}\cdot\nabla + \gamma^0(m + \Gamma_\sigma(\rho_b)\sigma(r) + \Gamma_\delta(\rho_b)\tau_3\delta(r))]\Psi(\vec{r})$$

$$+ \left[\Gamma_\omega(\rho_b)\omega(r) + \Gamma_\rho(\rho_b)\tau_3 b(r) + e\frac{(1+\tau_3)}{2}A(r)\right]\Psi(\vec{r})$$

$$+ \gamma^0\left[\frac{\partial\Gamma_\sigma(\rho_B)}{\partial\rho_B}\,\rho_s(\vec{r})\,\sigma(r) + \frac{\partial\Gamma_\delta(\rho_B)}{\partial\rho_B}\,\rho_3^s(\vec{r})\delta(r)\right. \tag{5a}$$

$$\left.+ \frac{\partial\Gamma_\omega(\rho_B)}{\partial\rho_B}\,\rho_B(\vec{r})\omega(r) + \frac{\partial\Gamma_\rho(\rho_B)}{\partial\rho_B}\,\rho_3(\vec{r})b(r)\right]\Psi(\vec{r}) = 0$$

and

$$-\nabla^2\sigma(r) + m_\sigma^2\sigma(r) = -\Gamma_\sigma(\rho_b)\,\rho_s(\vec{r}) \tag{5b}$$

$$-\nabla^2\delta(r) + m_\delta^2\delta(r) = -\Gamma_\delta(\rho_b)\,\rho_3^s(\vec{r}) \tag{5c}$$

$$-\nabla^2\omega(r) + m_\omega^2\omega(r) = \Gamma_\omega(\rho_b)\,\rho_B(\vec{r}) \tag{5d}$$

$$-\nabla^2 b(r) + m_\rho^2 b(r) = \Gamma_\rho(\rho_b)\,\rho_3(\vec{r}) \tag{5e}$$

$$-\nabla^2 A(r) = e\rho_B^p(\vec{r}). \tag{5f}$$

Note that the couplings $\Gamma_i(\rho_b)$ for the DD case are now functions of the barionic density and are parametrized according to [8].

As we use a Hartree method, the nucleus wave equation is given by a Slater determinant. We can, in this way, write the equation for the barionic field as an eigenvalue equation $h\psi_\alpha = E_\alpha\psi_\alpha$, with

$$\psi_\alpha(\vec{r}) = \begin{pmatrix} g_{n\kappa t}(r)\,\phi_{\kappa m}(\theta,\varphi) \\ if_{n\kappa t}(r)\,\phi_{-\kappa m}(\theta,\varphi) \end{pmatrix}\zeta_t. \tag{6}$$

Using an expansion of the wave functions $g_{n\kappa t}(r)$ and $f_{n\kappa t}(r)$ and of the mesonic fields in a spherically symmetric harmonic oscillator basis, we can solve the system self-consistently from an initial guess for the mesonic fields.

3. Asymmetry

The asymmetry can be defined in terms of the electron-nucleus cross-sections as

$$\mathcal{A} = \frac{d\sigma_+ - d\sigma_-}{d\sigma_+ + d\sigma_-}, \tag{7}$$

where $d\sigma_+$ and $d\sigma_-$ stands for positive and negative helicity cross-sections. For nuclei with $N = Z$ in PWBA, assuming that $\rho_B^n(q)/\rho_B^p(q) = N/Z$, this expression takes a remarkable simple form:

$$\mathcal{A}_0 = \frac{\sqrt{2}\,Gb}{4\pi e}\beta_v^0\, q^2. \tag{8}$$

Here G represents the Fermi constant for the weak force, $\beta_v^0 = -2\sin^2\theta_W$, b is a characteristic electron's weak interaction constant and q is the momentum transfer.

The Isospin Mixture function $\Gamma(q)$ can then be defined in such a way that

$$\mathcal{A} = \mathcal{A}_0[1 + \Gamma(q)] \qquad \text{or, equivalently,} \qquad \Gamma(q) = \frac{\mathcal{A}}{\mathcal{A}_0} - 1. \tag{9}$$

This definition is useful as it exposes the asymmetry dependence on the differences between neutron and proton distributions.

4. Results

Some numerical results obtained solving the above described model for ^{4}He, ^{12}C, ^{16}O, ^{40}O and ^{56}Ni are shown in Tables 1 and 2. The experimental energies and radii are taken from References [9, 10, 11]. The PWBA and DWBA asymmetries are displayed for q values from the first diffraction maximum region on Figures 1 and 2 and compared with non-relativistic results based on a Hartree-Fock Skyrme interaction calculation [12] for the ^{12}C nucleus. As can be noted, the first diffraction maximum occurs at lower momentum transfer for heavier nuclei. Results using NL models for ^{12}C are not displayed, once numerical stability was not achieved for those calculations and this particular nucleus.

Table 1. Results for binding energy per nucleon and charge radius for non-linear model without δ (NL3 [7]), with δ (NLδ [13]), density dependent without δ (TW [8]) and with δ (DDHδ [14]).

Nucleus	Energy (MeV)					Charge radius (fm)				
	NL3	NLδ	TW	DDHδ	Exp.	NL3	NLδ	TW	DDHδ	Exp.
^{4}He	-6.932	-6.934	-6.578	-6.578	-7.073	1.972	1.973	1.948	1.950	1.696
^{12}C	–	–	-7.025	-7.026	-7.680	–	–	2.417	2.420	2.471
^{16}O	-7.914	-7.927	-7.700	-7.701	-7.976	2.683	2.683	2.642	2.646	2.730
^{40}Ca	-8.491	-8.672	-8.344	-8.346	-8.551	3.454	3.436	3.402	3.408	3.485
^{56}Ni	-8.594	-8.729	-8.477	-8.480	-8.642	3.703	3.684	3.678	3.684	–

5. Conclusions

Asymmetry measurements are a way to obtain unique information about the difference between neutron and proton distributions in nuclei. As recent experimental results for the strangeness contributions are consistent with zero [3], the isospin mixture emerges as a very important contribution, determinant to the understanding of the asymmetry. The calculations of this

Figure 1. Isospin Mixing for ^{12}C in PWBA (left) and DWBA (right). Full line: Skyrme results from Reference [12].
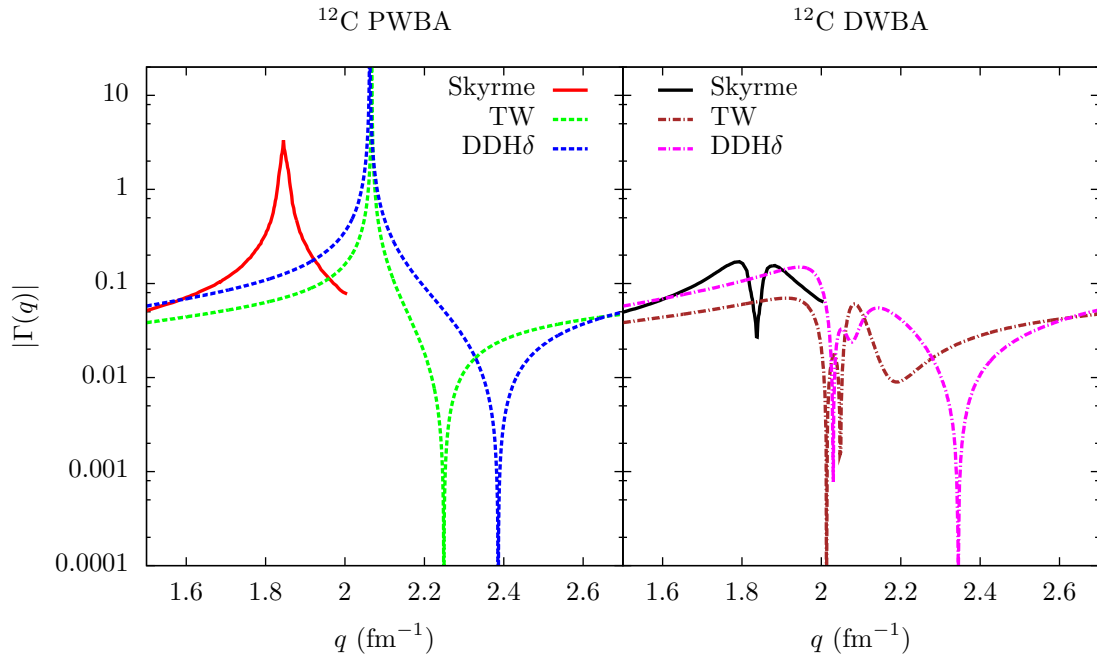

Figure 2. Isospin Mixing for ^{4}He and ^{16}O. Relativistic DWBA comparing NL (dashed lines) to DD (dashed-dotted lines) results.
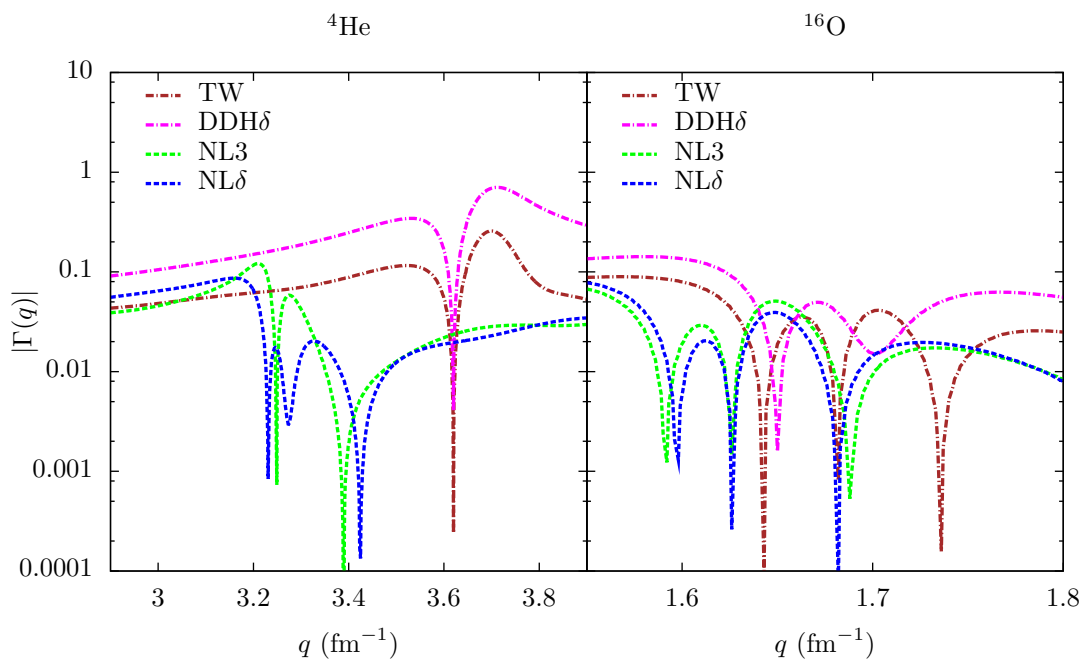
Table 2. Results for proton skin (difference between proton and neutron distributions radii).

Nucleus	Proton Skin (fm)			
	NL3	NLδ	TW	DDHδ
^{4}He	0.0140	0.0155	0.0141	0.0186
^{12}C	–	–	0.0253	0.0322
^{16}O	0.0276	0.0289	0.0293	0.0360
^{40}Ca	0.0485	0.0497	0.0513	0.0617
^{56}Ni	0.0491	0.0514	0.0517	0.0639

term shows small contributions in the region of low momentum transfer and should favor the measurement of θ_W as a test of the Standard Model. Furthermore, our results agree with the non-relativistic ones in magnitude but the position of the diffraction minima are substantially different. It should be noted, however, that the ^{12}C is an open-shell nuclei and the model herein used assumes a spherical, closed-shell nuclei. As can be seen from our results, the differences due to inclusion of the δ meson are more noticeable for the DD case. This can be attributed to the fact that in this case just minor modifications on the ρ parameters were made after the inclusion of the δ, while the parameters for the other meson couplings were maintained. For the NL case, a full re-parametrization was done when the new meson was added. Although the differences between the various parametrizations are small at low momentum transfer, as q increases they become very large. According to our theoretical analysis we conclude that the best q windows to measure those differences depend on the particular nucleus.

Acknowledgments

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