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Cosmological consequences of Brans–Dicke theory in 4*D* from 5*D* scalar-vacuum

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Abstract We study 5*D* Brans–Dicke theory in the framework of (gravitational) baryogenesis and primordial light element formation. Such a model is able to explain the present cosmic accelerated expansion without recurring to matter fields in 5*D* or dark energy in 4*D*. In fact, the 5*D* to 4*D* reduction arises a space-matter tensor of geometrical origin that plays the role of a new ingredient on-shell. For an isotropic and homogeneous Universe, the cosmological equations admit a power-law solution of the scale factor, $a(t) \sim t^{\alpha}$, we constrain the exponential factor α by using the present bounds on the matter–antimatter asymmetry in the Universe and Big Bang Nucleosynthesis. A possible connection with dark matter relic abundance is also discussed.

1 Introduction

Although general relativity (GR) is the best theory of gravitational interaction and its predictions have been tested with high precision [1, 2], there are still open questions (arising at short distances and small time scales) for which GR predictability gets lost. Without any doubt, the prediction of the existence of the gravitational waves, or the CMBR as well as the formation of primordial light elements (Big Bang Nucleosynthesis) represent significant successes of GR. However, Einstein's theory seems not to be completely satisfactory at the regime of the ultraviolet scale in which deviations from the Hilbert–Einstein action are needed. This leads to the introduction of new ingredients, such as dark matter and dark energy, necessary to fit the present picture of the Universe [3, 5–12]. A possibility that one may consider is the generalization of GR such that the gravitational action includes higher-order curvature invariants, $\mathcal{L} \sim f(R, R_{\mu\nu}R^{\mu\nu}, \Box^k R, \ldots)$ [13–26, 26–31]. The latter allows the inflationary behaviour of the early Universe to remove the primordial singularity and the explanation of the flatness and horizon problems [13, 14, 32–43]. Moreover, a high curvature regime requires the introduction of curvature invariants in order to build up self-consistent effective actions in curved spacetime[1] [44–46]. Possible candidates for dark energy are a time-dependent cosmological term [52, 53], quintessence [54–57], dissipative fluids [58], Chaplygin gas [59, 60], K-essence [61–64], scalar–tensor theories [65–71], and other more exotic models [72]. Among the other possibilities to explain the accelerated expansion of the Universe, it has been also explored the possibility related to Brans–Dicke (BD) theory in 5*D* (without recurring to matter fields in 5*D* or dark energy in 4*D*). More specifically, it was shown in [73] (see also [66, 67, 74–77]) that the vacuum BD field equations in 5*D* are equivalent, on every hypersurface orthogonal to the extra dimension, to a BD theory in 4*D* equipped with a new matter–energy source of geometrical origin. This model implies that in a FRW cosmology, the reduced BD theory yields the accelerated expansion of a matter-dominated universe. This is consistent with current observations and with a decelerating radiation-dominated epoch. An extension of this model to $f(R)$ gravity has been proposed in [78, 79]. This approach finds its bases in induced-matter theory. A formulation of 5*D* relativity, in which matter in 4*D* is counter-effect induced by geometry in 5*D*. In modified theories of gravity, such a scheme provides more degrees of freedom that can be exploited in order to build a coherent cosmological model according to observations.

In this work, we investigate how matter–antimatter asymmetry in the Universe and the formation of primordial light elements, that is the Big Bang Nucleosynthesis (BBN), can influence the framework descending from reduced BD 5*D*-cosmology to 4*D*. The origin of the baryon asymmetry in the early Universe is an unsolved problem of cosmology and particle physics. Observations show that the Universe is mostly made up of matter, contrary to what is expected from QFT in which matter and antimatter should be present in equal amounts. For such asymmetry to occur, the Sakharov conditions must be fulfilled [80]: (1) baryon number violation,

[1] It is worth mentioning that some models have been proposed in which deviations from GR are described by screening effects [47], that is by introducing an additional degree of freedom that obeys a nonlinear equation the couples to the environment. Screening mechanisms allow to circumvent solar system and laboratory tests by suppressing, in a dynamical way, deviations from GR (screening mechanisms studied in the literature are the chameleon [48, 49], symmetron [50], and Vainshtein [51]).

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(2) C and CP violation, and (3) deviation from thermal equilibrium. The CMB temperature anisotropies provide a strong probe of the baryon asymmetry. In fact, the observation of the acoustic peaks in CMB and the measurements of large-scale structures provide an estimation of the baryon asymmetry parameter η given by $\eta^{(CMB)} \sim (6.3 \pm 0.3) \times 10^{-10}$ [81]. Yet, measurement of η can be also carried out in the context of the BBN, leading to $\eta^{(BBN)} \sim (3.4 - 6.9) \times 10^{-10}$ [82]. These two values are compatible, although derived in two different eras of the Universe evolution.

Moreover, we also explore the implications of the formation of light elements in the early Universe on 5D-cosmology. The latter occurred in the early phases of the Universe evolution, between the first fractions of a second after the Big Bang ($\sim$ 0.01 s) and a few hundred seconds after it, when the Universe was hot and dense (indeed BBN, together with cosmic microwave background radiation, provides the strong evidence about the high temperatures characterizing the primordial Universe). It describes the sequence of nuclear reactions that yielded the synthesis of light elements [83–85], and therefore drive the observed Universe. In general, from BBN physics, one may infer stringent constraints on a given cosmological model. In particular, in the present paper, after we have found some analytical solutions of field equations, we will derive the physical constraints on the free parameter α of the model.

The paper is organized as follows. In section II, we recall the FRW cosmological equations inferred from the dimensional reduction of the scalar-vacuum BD field equations in 5D to 4D adopting the scheme of induced-matter theory. In section III, we study homogeneous and isotropic solutions of the vacuum BD field equations in 5D (focusing in particular to power-law solutions) applied to the baryon asymmetry in the Universe. We will discuss gravitational baryogenesis considering baryon asymmetry generated by the coupling of baryon current with the scalar Ricci curvature. In section IV, we study the consequences of 4D cosmological scenarios allowed by the 5D power-law solutions to the formation of light elements. In the last section, we draw our conclusions.

2 Dimensional reduction of Brans–Dicke theory in 5*D*

A Brans–Dicke (BD) theory of gravity in 5D is described by the action [73, 75]

$$S_{(5)} = \int d^5x\sqrt{|\gamma^{(5)}|}\left[\phi R^{(5)} - \frac{\omega}{\phi}\gamma^{AB}(\nabla_A\phi)(\nabla_B\phi)\right] + 16\pi\int d^5x\sqrt{|\gamma^{(5)}|}L_m^{(5)}, \tag{1}$$

where $R^{(5)}$ is the curvature scalar associated with the 5D metric γ_{AB}; $\gamma^{(5)}$ is the determinant of γ_{AB}; ϕ is a scalar field; ω is a dimensionless coupling constant; and $L_m^{(5)}$ represents the Lagrangian of the matter fields in 5D and does not depend on ϕ. The effective equations for gravity in 4D are given by[2]

$$G_{\mu\nu}^{(4)} = \frac{8\pi}{\phi}\left(S_{\mu\nu} + T_{\mu\nu}^{(BD)}\right) + \frac{\omega}{\phi^2}\left[\left(D_\mu\phi\right)\left(D_\nu\phi\right) - \frac{1}{2}g_{\mu\nu}(D_\alpha\phi)\left(D^\alpha\phi\right)\right] + \frac{1}{\phi}\left(D_\mu D_\nu\phi - g_{\mu\nu}D^2\phi\right) - g_{\mu\nu}\frac{V(\phi)}{2\phi} \tag{2}$$

where we have introduced the quantity $V(\phi)$, which (as we will see bellow) plays the role of an effective or induced scalar potential; $S_{\mu\nu}$ is the reduced energy-momentum tensor (EMT) of the matter fields in 5D

$$S_{\mu\nu} = T_{\mu\nu}^{(5)} - g_{\mu\nu}\left[\frac{(\omega+1)T^{(5)}}{4+3\omega} - \frac{\epsilon T_{44}^{(5)}}{\Phi^2}\right], \tag{3}$$

while $T_{\mu\nu}^{(BD)}$ can be interpreted as an induced energy-momentum tensor for an effective BD theory in 4D, given by [73]

$$8\pi T_{\mu\nu}^{(BD)} = 8\pi T_{\mu\nu}^{(STM)} + \frac{\epsilon\overset{*}{\phi}}{2\Phi^2}\left[\overset{*}{g}_{\mu\nu} + g_{\mu\nu}\left(\frac{\omega\overset{*}{\phi}}{\phi} - g^{\alpha\beta}\overset{*}{g}_{\alpha\beta}\right)\right] + \frac{1}{2}g_{\mu\nu}V, \tag{4}$$

with

$$\frac{8\pi}{\phi}T_{\mu\nu}^{(STM)} \equiv \frac{D_\mu D_\nu\Phi}{\Phi} - \frac{\epsilon}{2\Phi^2}\left\{\frac{\overset{*}{\Phi}\overset{*}{g}_{\mu\nu}}{\Phi} - \overset{**}{g}_{\mu\nu} + g^{\alpha\beta}\overset{*}{g}_{\mu\alpha}\overset{*}{g}_{\nu\beta} - \frac{g^{\alpha\beta}\overset{*}{g}_{\alpha\beta}\overset{*}{g}_{\mu\nu}}{2} + \frac{g_{\mu\nu}}{4}\left[\overset{*}{g}^{\alpha\beta}\overset{*}{g}_{\alpha\beta} + \left(g^{\alpha\beta}\overset{*}{g}_{\alpha\beta}\right)^2\right]\right\}. \tag{5}$$

Since in BD, ϕ acts as the inverse of the Newtonian gravitational constant G, (5) is identical to the induced EMT used in STM (space–time–matter theory) [86]. The second term in (4) depends on the first derivatives of ϕ with respect to the fifth coordinate and represents the effective EMT in 4D coming from the scalar field. Taking the trace of (2), we obtain a simple relation between $R^{(4)}$, $S = g^{\mu\nu}S_{\mu\nu}$ and $T^{(BD)} = g^{\mu\nu}T_{\mu\nu}^{(BD)}$, namely (we note that $g^{\mu\nu}T_{\mu\nu}^{(5)} = T^{(5)} - \epsilon T_{44}^{(4)}/\Phi^2$)

$$R^{(4)} = -\frac{8\pi}{\phi}\left(S + T^{(BD)}\right) + \frac{\omega(D_\alpha\phi)(D^\alpha\phi)}{\phi^2} + \frac{3D^2\phi}{\phi} + \frac{2V}{\phi}. \tag{6}$$

[2] Details are given in Appendix A: This equation can be obtained writing down Ricci tensor, that is first equation in (A9), and Ricci scalar (A11) by making explicit 4D quantities and then substituting these into (A8). Afterwards, one isolates $G_{\mu\nu}^{(4)} = R_{\mu\nu}^{(4)} - \frac{1}{2}g_{\mu\nu}R^{(4)}$.

Since we are interested in induced-matter solutions of $5D$ Brans–Dicke theory hereon, we will focus on a scalar-vacuum Brans–Dicke cosmology in $5D$. Actually, a general key point for the embedding of $4D$ Einstein theory into a $5D$ Kaluza–Klein induced-matter approach is provided by Campbell–Magaard (CM) theorem [87]. This theorem states that any analytic Riemannian space of N-dimension can be locally embedded in a Ricci-flat Riemannian space of $(N+1)$-dimension. As a consequence, the $5D$ metric (A5) is commonly taken, in cosmological applications, considering $4D$ spacetime as a hypersurface $y = y_0 = const$ orthogonal to the $5D$ extra-coordinate. Therefore one can consider the line element in the form:

$$ds^2 = n^2(t,y)dt^2 - a^2(t,y)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)\right] + \epsilon\Phi^2(t,y)dy^2, \tag{7}$$

where $k = 0, +1, -1$ and (t, r, θ, ϕ) are the usual coordinates for a spacetime with spherically symmetric spatial sections. If one considers the previous metric in field equations, the vacuum ($T^{(5)}_{AB} = 0$) Brans–Dicke cosmological equations (see (A1)) give: for the temporal component $A = B = 0$

$$3\frac{\dot a}{a}\left(\frac{\dot a}{a} + \frac{\dot\Phi}{\Phi}\right) + \frac{3kn^2}{a^2} + \frac{3\epsilon n^2}{\Phi^2}\left[\frac{\overset{**}{a}}{a} + \frac{\overset{*}{a}}{a}\left(\frac{\overset{*}{a}}{a} - \frac{\overset{*}{\Phi}}{\Phi}\right)\right] = \frac{1}{\phi}\left[\ddot\phi + \dot\phi\left(\frac{\omega\dot\phi}{2\phi} - \frac{\dot n}{n}\right)\right] + \frac{\epsilon n^2\overset{*}{\phi}}{\Phi^2\phi}\left(\frac{\overset{*}{n}}{n} - \frac{\omega\overset{*}{\phi}}{2\phi}\right) \tag{8}$$

while, for spatial components $A = B = 1, 2, 3$

$$\begin{aligned} &\frac{2\ddot a}{a} + \frac{\dot a}{a}\left(\frac{\dot a}{a} - \frac{2\dot n}{n}\right) + \frac{\ddot\Phi}{\Phi} + \frac{\dot\Phi}{\Phi}\left(\frac{2\dot a}{a} - \frac{\dot n}{n}\right) + \frac{kn^2}{a^2} + \frac{\epsilon n^2}{\Phi^2}\left[\frac{2\overset{**}{a}}{a} + \frac{\overset{*}{a}}{a}\left(\frac{\overset{*}{a}}{a} + \frac{2\overset{*}{n}}{n}\right) + \frac{\overset{**}{n}}{n} - \frac{\overset{*}{\Phi}}{\Phi}\left(\frac{2\overset{*}{a}}{a} + \frac{\overset{*}{n}}{n}\right)\right] \\ &= \frac{\dot\phi}{\phi}\left(\frac{\dot a}{a} - \frac{\omega\dot\phi}{2\phi}\right) + \frac{\epsilon n^2\overset{*}{\phi}}{\Phi^2\phi}\left(\frac{\overset{*}{a}}{a} - \frac{\omega\overset{*}{\phi}}{2\phi}\right), \end{aligned} \tag{9}$$

along the extra-coordinate $A = B = 4$

$$3\left[\frac{\ddot a}{a} + \frac{\dot a}{a}\left(\frac{\dot a}{a} - \frac{\dot n}{n}\right)\right] + \frac{3kn^2}{a^2} + \frac{3\epsilon n^2}{\Phi^2}\left(\frac{\overset{*}{a}}{a}\right)\left(\frac{\overset{*}{a}}{a} + \frac{\overset{*}{n}}{n}\right) = \frac{\dot\phi}{\phi}\left(\frac{\dot\Phi}{\Phi} - \frac{\omega\dot\phi}{2\phi}\right) + \frac{\epsilon n^2}{\Phi^2\phi}\left[\overset{**}{\phi} + \overset{*}{\phi}\left(\frac{\omega\overset{*}{\phi}}{2\phi} - \frac{\overset{*}{\Phi}}{\Phi}\right)\right], \tag{10}$$

and, finally, for the mixed component $A = 0,\ B = 4$

$$3\left(\frac{\overset{*}{n}\dot a}{na} + \frac{\dot\Phi\overset{*}{a}}{\Phi a} - \frac{\overset{*}{\dot a}}{a}\right) = \frac{\overset{*}{\dot\phi}}{\phi} - \frac{\overset{*}{n}\dot\phi}{n\phi} - \frac{\dot\Phi\overset{*}{\phi}}{\Phi\phi} + \frac{\omega}{\phi^2}\dot\phi\overset{*}{\phi}. \tag{11}$$

The wave equation (A4) reads

$$\nabla^2\phi = \frac{1}{n^2}\left[\ddot\phi + \dot\phi\left(\frac{3\dot a}{a} + \frac{\dot\Phi}{\Phi} - \frac{\dot n}{n}\right)\right] + \frac{\epsilon}{\Phi^2}\left[\overset{**}{\phi} + \overset{*}{\phi}\left(\frac{3\overset{*}{a}}{a} + \frac{\overset{*}{n}}{n} - \frac{\overset{*}{\Phi}}{\Phi}\right)\right] = 0. \tag{12}$$

A suitable solution set for these equations can be inferred by assuming that the metric coefficients are separable functions of their arguments [73]

$$n(t,y) = N(y), \quad a(t,y) = P(y)Q(t), \quad \Phi(t,y) = F(t), \quad \phi(t,y) = U(y)W(t). \tag{13}$$

In particular, in the hypothesis of a spatially-flat scenario, as it is suggested by observations, one may consider power-law solutions

$$n(t,y) = Ay^{\alpha}, \quad a(t,y) = By^{\beta}t^{l}, \quad \Phi(t,y) = Ct^{m}, \quad \phi(t,y) = Dy^{\gamma}t^{s}. \tag{14}$$

Here A, B, C, and D are some constants with the appropriate units; while α, β, γ while l, m, and s are parameters that will be constrained by field Eqs. (8)–(12). In particular, substituting (14) into (12), we obtain

$$s(s-1+3l+m)C^2t^{(s-2)}y^{(\gamma-2\alpha)} + \epsilon\gamma(\gamma-1+3\beta+\alpha)A^2t^{(s-2m)}y^{(\gamma-2)} = 0. \tag{15}$$

This equation can be satisfied with several sets of parameters (a detailed analysis of these solutions can be found in [73]):

- $s = 0,\ \gamma = 0$;
- $s = 0,\ \gamma = 1 - 3\beta - \alpha$;
- $\gamma = 0,\ s = 1 - 3l - m$;
- $s = 1 - 3l - m,\ \gamma = 1 - 3\beta - \alpha$;
- $m = 1,\ \alpha = 1, \quad s(s+3l)C^2 + \epsilon\gamma(\gamma+3\beta)A^2 = 0$.

Here, we focus on the case $V = 0$ (Sect. IV of [73]) that implies

$$\gamma = \frac{s(s+6l-2)(s+3l)}{(s+3l-1)\left[s^2+s(3l-2)+6l(l-1)\right]}. \tag{16}$$

$$\omega = \frac{6(2l-1)l}{s(s+6l-2)}, \quad p = n\rho, \quad n = \frac{2-s-3l}{3l}, \quad s = \frac{2(2l-1)}{(1-n)[2-3l(n+1)]}. \tag{17}$$

Consequently, a effective BD cosmology in $4D$ can be developed in terms of the only metric potentials $a(t)$ and $\phi(t)$ written as

$$a(t) = a_f\left(\frac{t}{t_f}\right)^{\alpha}, \quad \alpha = \frac{2[1+\omega(1-n)]}{\left[4+3\omega\left(1-n^2\right)\right]} \tag{18}$$

$$\phi(t) = \phi_f\left(\frac{t}{t_f}\right)^{\beta}, \quad \beta = \frac{2(1-3n)}{\left[4+3\omega\left(1-n^2\right)\right]}, \quad \phi_f = \frac{4\pi\rho_0(Ay_0t_f)^2\left[4+3\omega\left(1-n^2\right)\right]^2}{(3+2\omega)\left[4-6n+3\omega(n-1)^2\right]}. \tag{19}$$

where $a(t_f)$, ϕ_f, and t_f are constants. In this parameterization, the $n = 0$ models become identical to those presented in the original BD paper [88]. For $n \neq 0$, we recover the type A-I solutions discussed in [71], although in a slightly different notation. As shown in [73], the range of ω is $-2 < \omega < -3/2$ $(1 < l < 2)$ (within this interval, it is allowed a matter-dominated universe with accelerated expansion). For the radiation-dominated epoch, it is $\omega = -3/2$ $(l = 1/2)$. In our analysis, we relax such range being mainly interested to study the modifications of cosmology during the pre-BBN, keeping hence free all parameters of the $5D$ cosmological model.

3 Gravitational baryogenesis in modified $5D$ to $4D$-cosmology

A suitable scheme mechanism for generating baryon asymmetry during the expansion of the universe has been developed within supergravity theories [89, 90]. The interaction responsible for the CPT violation is given by a coupling between the derivative of the Ricci scalar curvature R and the baryon current J^μ [91]

$$\frac{1}{M_*^2}\int d^4x\sqrt{-g}\, J^\mu \partial_\mu R, \tag{20}$$

where M_* is the cutoff scale characterizing the effective theory (see Ref. [93–106] for further applications).

In order to get interactions that violate the baryon number B in thermal equilibrium (to satisfy the first Sakharov condition), it is required that a net baryon asymmetry should be generated and get frozen in below the decoupling temperature[3] T_D. From Eq. (20), one gets [91]

$$\frac{1}{M_*^2}J^\mu\partial_\mu R = \frac{1}{M_*^2}(n_B - n_{\bar{B}})\dot{R}, \tag{21}$$

where

$$\mu_B = -\mu_{\bar{B}} = -\frac{\dot{R}}{M_*^2}. \tag{22}$$

For relativistic particles, the net baryon number density of matter in the early Universe is given by [92] $n_B - n_{\bar{B}} = \frac{g_b}{6}\mu_B T^2$, where $g_b \sim \mathcal{O}(1)$ is the number of intrinsic degrees of freedom of baryons, so that the baryon asymmetry reads

$$\eta \equiv \frac{n_B - n_{\bar{B}}}{s} \approx \frac{n_B}{s} \simeq -\frac{15\, g_b}{4\pi^2 g_*}\frac{\dot{R}}{M_*^2 T}\bigg|_{T_D}, \tag{23}$$

where $s = \frac{2\pi^2 g_{*s}}{45}T^3$ is the entropy per unit volume, i.e. entropy density, in the radiation-dominated era, and $g_{*s} \sim 106$ is the number of degrees of freedom for particles which contribute to the entropy of the Universe [92]. As arises from (23), the parameter η is different from zero provided that $\dot{R} \neq 0$. In the radiation-dominated era, described by GR, the baryon asymmetry is zero ($\eta = 0$) because in that case $\dot{R} = 0$. $5D$-corrections modify $\dot{R}$, making $\eta \neq 0$. To obtain the $5D$-corrected derivative of the Ricci scalar $\dot{R}$, we need first to compute the trace of the Einstein equation $R = -8\pi G T_g = -8\pi G(\rho - 3p)$, where $T_g = \rho - 3p$ is the trace of the energy-momentum tensor of matter. Such a term disappears in GR since the radiation equation of state of relativistic particles is

[3] During the evolution of the Universe, the CPT violation generates the baryon asymmetry (B-asymmetry). This occurs when baryon (or lepton) violating interactions are still in thermal equilibrium. The asymmetry is frozen at the decoupling temperature T_D, when the baryon (or lepton) violation goes out of equilibrium. The temperature T_D is derived from the relation $\Gamma(T_D) \simeq H(T_D)$, where Γ is the interaction rate of processes, and H is the expansion rate of the Universe. More specifically, in the regime $\Gamma \gg H$, or $T > T_D$, the B-asymmetry is generated by B-violating processes at thermal equilibrium; at $T = T_D$, i.e. $\Gamma \simeq H$, the decoupling occurs, while when $\Gamma < H$, or $T < T_D$ the B-asymmetry gets frozen.

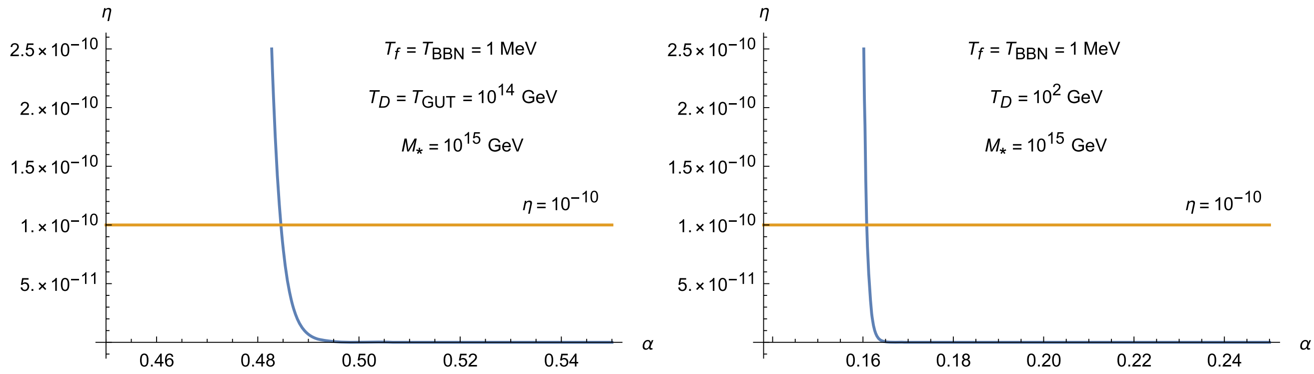
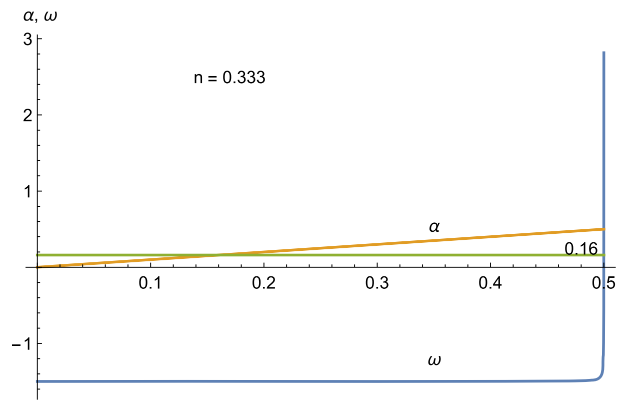


Fig. 1 *Left*—η vs α for fixed values $T_f = T_{BBN} = 1$MeV, $M_* = 10^{15}$GeV, and the decoupling is assumed to occur at GUT scales $T_D = 10^{14}$GeV. *Right*—η vs α for fixed values $T_f = T_{BBN} = 1$MeV, $M_* = 10^{15}$GeV, and the decoupling is assumed to occur at low scales $T_D = 10^2$GeV

Fig. 2 α and ω vs l, Eq. (18). Here, we fixed the adiabatic index to $n = 0.333$ (it is a tiny deviation from the adiabatic index in the era radiation dominated $n = 1/3$). As we can see, $\alpha \simeq 0.16$ for $l \simeq 0.16$, to which corresponds $\omega \simeq -1.5$. Notice that ω has an asymptotic at $l \simeq 0.5$ for $n \sim 1/3$



given by $p = \rho/3$. In a modified $5D$-cosmology, the total energy-momentum tensor contains both matter and scalar fields, so that the total trace does no vanish owing to the scalar field contribution, as we will see. By using (18), one gets

$$R = -6(\dot{H} + 2H^2) = \frac{6\alpha(2\alpha - 1)}{t^2}, \quad \alpha = \frac{2[1 + \omega(1 - n)]}{4 + 3\omega(1 - n^2)}. \tag{24}$$

where $H = \dot{a}/a$ is the expansion rate of the Universe. The time derivative of (24) gives

$$\dot{R} = 12\alpha(1 - 2\alpha)\left(\frac{16\pi^2 g_*}{45}\right)^{3/2}\left(\frac{T_f^2}{M_P}\right)^3\left(\frac{T}{T_f}\right)^{3/\alpha}. \tag{25}$$

where T_f (or t_f) is the temperature (times) in which the Universe starts to evolve according to GR. To obtain (25), we have assumed entropy conservation so that $Ta = T_0$ (we set $a_0 = 1$ and $T_0 \simeq 10^{-4}$eV). Such a transition occurs at BBN or well before that time (the so-called pre-BBN era). Substituting Eqs. (25) into (23), one obtains the expression for the baryon asymmetry

$$\eta = \frac{425 g_b}{\pi^2 g_*}\left(\frac{16\pi^2 g_*}{45}\right)^{3/2}\left(\frac{T_f}{M_P}\right)^{3(2\alpha-1)/\alpha}\left(\frac{M_P}{M_*}\right)^2\left(\frac{T_D}{M_P}\right)^{(3-\alpha)/\alpha}. \tag{26}$$

Figure 1 is reported η in (26) as function of α for two significant cases of the decoupling temperature, $T_D \sim T_{GUT} \sim 10^{15}$GeV and $T_D \sim 10^2$GeV. We have fixed $g_b = 2$, $g_* \sim 106$, $M_P \sim 10^{19}$GeV, and the *transition* temperature (from pre-BBN Universe described by modified $5D$-cosmology to the standard cosmology) to $T_f \sim T_{BBN} \sim 1$MeV. Let us discuss the results displayed in this picture from high temperature $T_D \sim T_{GUT} \sim 10^{15} GeV$ to $T_D \sim 10^2$GeV (below this temperature, the sphaleron effects are no more operative):

- $T_D \sim T_{GUT} \sim 10^{15} GeV$—From Fig. 1(*Left*), it follows that the exponential parameter α turns out to be of the order $\alpha \sim 0.49$, i.e. it corresponds to a tiny deviation from the standard cosmological evolution of the Universe for which $\alpha = 1/2$ (radiation dominated era).
- $T_D \sim 10^2$GeV—The observed baryon asymmetry follows for $\alpha \sim 0.161$, Fig. 1(*Right*). This requires $\omega < 0$. According to Eq. (18), that give α and ω vs l, negative values of ω are allowed by the cosmological model under consideration, as shown in Fig. 2.

4 Primordial light element $\{^4He, D, Li\}$ in modified 5D-cosmology

In this section, we analyse the effects of the Brans–Dicke induced-matter cosmology on the primordial light element formed in the early stage of the Universe. We consider the scale factor (18) where t_f (T_f) is interpreted as the instant (temperature) at which the Universe starts to evolve according to GR. We can write down the modified 4D-hypersurface expansion rate $H = \dot{a}/a$ in terms of the expansion rate H_{GR} of GR as

$$H(T) \equiv Z(T)\, H_{\rm GR}(T)\,. \tag{27}$$

Here $H_{GR} = \sqrt{\frac{8\pi}{3M_{Pl}}\rho(T)}$ is the usual expansion rate of the Universe in GR, with $\rho = \frac{\pi^2 g_*}{30} T^4$, while the factor $Z(T)$ is defined as

$$Z(T) \equiv \eta \left(\frac{T}{T_f}\right)^{\nu}, \quad \eta \equiv 2\alpha\,, \quad \nu \equiv \frac{1-2\alpha}{\alpha}\,. \tag{28}$$

For $\alpha = 1/2$, it follows $\nu = 0$, $\eta = 1$, so that $Z(T) = 1$ and GR is recovered. Owing to the modified expansion rate induced by the new sources of higher dimensional within the field equations, our goal is to infer the bounds on the parameter α that controls scale factor evolution from the primordial abundances of light elements, that is deuterium ^2H, helium ^4He, and litium ^7Li. To this aim, one replaces the Z-factor entering the expressions of the primordial light elements (related to the effective number of neutrinos species, $Z_\nu = \left[1 + \frac{7}{43}(N_\nu - 3)\right]^{1/2}$ [107]) with the $Z(T)$ factor entering (28) [108, 109]. Being interested in deviations from the standard cosmology, hereafter, we shall assume that the number of neutrino generations is $N_\nu = 3$. Following the approach given in [110], we can observe:

- ^4He abundance—The relevant reactions for the BBN processes giving ^4He are

$$n + p \to {}^2H + \gamma; \quad {}^2H + {}^2H \to {}^3He + n; \quad {}^2H + {}^2H \to {}^3H + p \tag{29}$$

  while the reactions

$${}^2H + {}^3H \to {}^4He + n \quad \text{and} \quad {}^2H + {}^3He \to {}^4He + p \tag{30}$$

  produce the helium ^4He. The best fit of the primordial ^4He abundance is given by [111, 112]

$$Y_p = 0.2485 \pm 0.0006 + 0.0016[(\eta_{10} - 6) + 100(Z - 1)]\,. \tag{31}$$

  Here Z is defined in (28), and η_{10} is given by [108, 109]

$$\eta_{10} \equiv 10^{10}\eta_B \equiv 10^{10}\frac{n_B}{n_\gamma}\,, \quad \eta_{10} \simeq 6\,. \tag{32}$$

  The quantity $\eta_B = n_B/n_\gamma$ is the baryon to photon ratio [113]. The value $Z = 1$ refers to the standard BBN results in GR for the ^4He-fraction, which means $(Y_p)|_{GR} = 0.2485 \pm 0.0006$. The observed data relative to the helium ^4He and the value $\eta_{10} = 6$ provide the abundance [114]

$$0.2561 \pm 0.0108 = 0.2485 \pm 0.0006 + 0.0016[100(Z - 1)]\,. \tag{33}$$

  These results imply the constrain

$$Z = 1.0475 \pm 0.105\,. \tag{34}$$

- ^2H abundance—The reaction producing deuterium ^2H is given by $n + p \to {}^2H + \gamma$. The best fit of deuterium abundance gives [108]

$$y_{Dp} = 2.6(1 \pm 0.06)\left(\frac{6}{\eta_{10} - 6(Z-1)}\right)^{1.6} \tag{35}$$

  The standard result in GR is $y_{Dp}|_{GR} = 2.6 \pm 0.16$, that follows by setting $Z = 1$ and $\eta_{10} = 6$. From observation with (35), one gets [114]

$$2.88 \pm 0.22 = 2.6(1 \pm 0.06)\left(\frac{6}{\eta_{10} - 6(Z-1)}\right)^{1.6}, \tag{36}$$

  which implies

$$Z = 1.062 \pm 0.444\,. \tag{37}$$

  Such a constraint partially overlaps with the helium abundance (34).

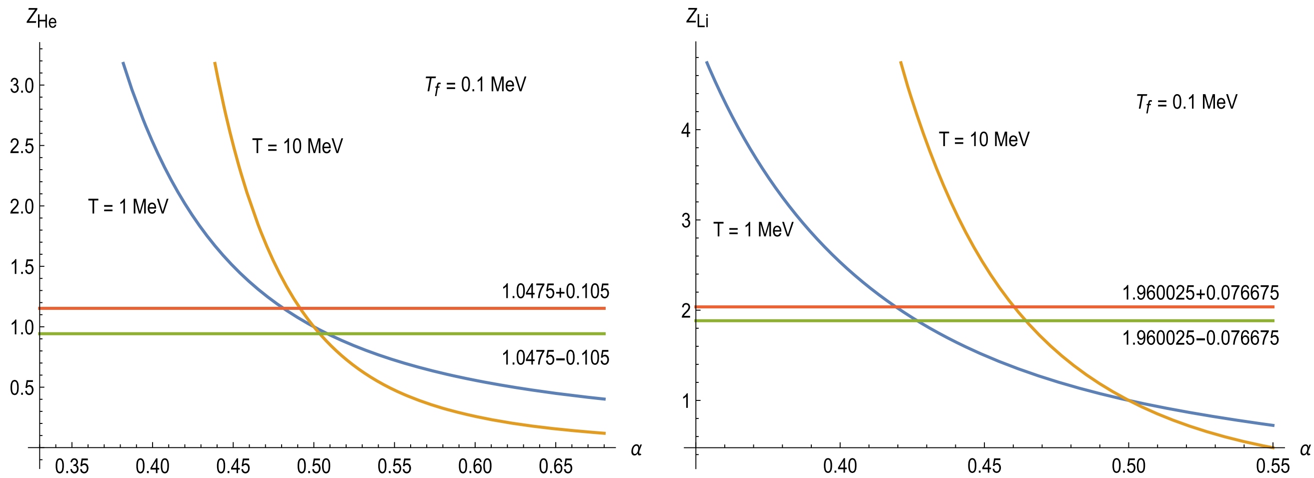


Fig. 3 *Left*—$Z_{4_{He}}$ vs α. The range (34) is reported. The baryon parameter is fixed to $\eta_{10} = 6$ $T_f = 0.1$MeV, and we have taken the temperature $T = \{1, 10\}$MeV. *Right*—Z_{Li} vs α. The experimental bounds (39) are reported. The other values are the same of *Left* panel

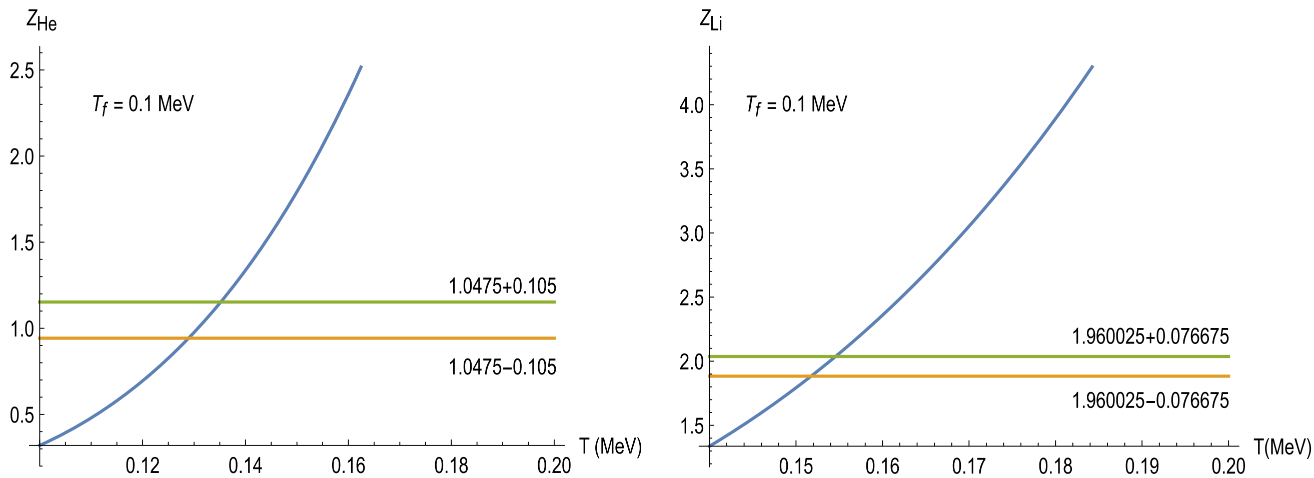


Fig. 4 *Left*—$Z_{4_{He}}$ vs T. The experimental bounds (34) are reported. Here $\alpha = 0.16$, the baryon parameter is fixed to $\eta_{10} = 6$, and $T_f = 0.1$ MeV. *Right*—Z_{Li} vs T. The experimental bounds (39) are reported. The other values are the same of *Left* panel

- ^7Li abundance—The ratio between the ^7Li abundance in GR and the observed is $\frac{Li|_{GR}}{Li|_{obs}} \in [2.4 - 4.3]$ [107], the best fit for ^7Li abundance is [108]

$$y_{Li} = 4.82(1 \pm 0.1)\left[\frac{\eta_{10} - 3(Z-1)}{6}\right]^2 . \tag{38}$$

The constraints on lithium abundance $y_{Li} = 1.6 \pm 0.3$ [114] imply

$$Z = 1.960025 \pm 0.076675 . \tag{39}$$

Notice that this value does not overlap with the constraints on ^2H and ^4He abundance given in Eqs. (37) and (34). Moreover, it must be mentioned that the 5D BD cosmological model does not allow to solve the *the lithium problem* [107], related to the fact that η_{10} given in (32) allows to successfully fit the abundances of D and ^4He, but does not fit the observations of ^7Li.

In Fig. 3—*Left*, we plot the factor $Z(T)$ given in (28) taking into account the constraints (34). We assume $\eta_{10} = 6$ and $T_f = 0.1$MeV. The plots show the behaviour of the factor Z_{He} for typical BBN temperatures, $T = 1$MeV and $T = 10$MeV. One gets that $Z_{He} \neq 1$ for α ranging in the intervals

$$\begin{aligned} 0.48 \lesssim \alpha \lesssim 0.51 \quad &\text{for } T = 1\text{MeV}, \\ 0.495 \lesssim \alpha \lesssim 0.501 \quad &\text{for } T = 10\text{MeV}, \end{aligned} \tag{40}$$

A similar result is inferred for the deuterium using the constraints on Z descending from data on this element Eq. (37).

In the case of lithium, the plot provided in Fig. 3—*Right*, using the constraints on Z given by Eq. (39), indicates that $Z_{Li} \neq 1$ for α in the range

$$
\begin{aligned}
&0.42 \lesssim \alpha \lesssim 0.45 \quad \text{for } T = 1\text{MeV}, \\
&0.46 \lesssim \alpha \lesssim 0.47 \quad \text{for } T = 10\text{MeV}.
\end{aligned} \tag{41}
$$

The results obtained in Eqs. (40) and (41) do not overlap. However, the difference between the different ranges is very tiny, suggesting that the lithium problem could be ameliorated in the modified cosmology framework.

We finally discuss the case in which the value of the parameter α is fixed to $\alpha = 0.16$, which is relevant for successful baryogenesis at a low-energy scale. Figure 4 is reported the behaviour of Z_{He} and Z_{Li} vs T, for $T_f = 0.1$MeV. As a result, $Z(T) \neq 1$ in temperature range of the order $T \in [0.130, 0.135]$MeV for *He* and $T \in [0.152, 0.156]$MeV for *Li*.

5 Conclusions

In this paper, we have considered a higher dimensions cosmological model, in particular 5*D* BD theory. In this context, it is possible to show that the BD field equations in 5*D* are equivalent to those of BD in 4*D* with some new ingredients in the sources side deriving from higher dimensional counter-terms. In the framework of FRW Universe, field equations lead to different classes of solutions. In particular, it is possible to obtain a power-law solution $a(t) \sim t^{\alpha}$ of the scale factor under specific conditions. Such a power-law solution has been used to investigate the baryon asymmetry in the Universe induced by a gravitational mechanism. The effective mechanism is the coupling of baryon currents to the Ricci scalar, which induces the CPT symmetry violation since the Ricci curvature depends on time also in the radiation regime. This class of solutions has been also studied for the formation of primordial light elements obtaining a constraint on the parameter α that controls the scale factor evolution.

The 5*D*-modified model opens interesting cosmological scenarios with relevant consequences in different contexts, such as, for example, the thermal dark matter (DM) freeze-out mechanism, which allows deriving a bound on α from the observed DM relic abundance (more precisely, the bound on α can be inferred by using the dark matter (DM) annihilation cross-section which enters the cold DM relic abundance Ω_{cdm}). By comparing the factor $Z(T)$ in (28) with that one of Ref. [115], $Z(T) \sim T^{\frac{2}{n}-2}$, one gets $\alpha = n/2$. For DM composed only by weakly interacting massive particles (WIMPs), the DM relic density in modified cosmology reads [115]

$$
\Omega_{cdm} h^2 \simeq 10^9 \frac{(\bar{l}+1) x_f^{(\bar{l}+1)} \text{GeV}^{-1}}{(h_*/g_{*s}^{1/2}) M_P \bar{\sigma}}, \tag{42}
$$

where $\bar{l} = l + (2\alpha - 1)$ (notice that l here is not related to the exponent appearing in (14) but it refers to angular momentum), $\bar{\sigma}$ is the WIMP cross-section, h_* is the number of relativistic degrees of freedom for entropy density, and $x_f \equiv m/T_F$ (T_F is the freeze-out temperature) [115, 116]

$$
x_f = \ln[0.038(\bar{l}+1)(g/g_*^{1/2}) M_p m \bar{\sigma}] - (\bar{l}+1) \ln[\ln[0.038(\bar{l}+1)(g/g_*^{1/2}) M_p m \bar{\sigma}]], \tag{43}
$$

with $g = 2$ the spin polarizations of the dark matter particle and m the mass of WIMPs particles. Here[4] $\bar{l} = l$ ($\alpha = 1/2$) for GR ($l = 0$, 1 corresponds to s-wave and p-wave polarizations). Since, cosmological data on cold dark matter density give $\Omega_{cdm} h^2 = 0.1198 \pm 0.0012$ [118, 119], one infers $|1 - 2\alpha| \lesssim 1.6 \times 10^{-4}$. Therefore, the DM relic density allows a tiny deviation from the standard cosmological model as a counter-effect of 5*D* Brans–Dicke cosmology. Interestingly, this bound is compatible with the allowed values for α obtained from *He* during the light element formation and partially from gravitational baryogenesis at GUT scales (for higher decoupling temperatures the parameter α approaches to 1/2).

In conclusion, modified 5*D* cosmological models offer interesting possibilities for studying their consequences in different scenarios. Since the strong analogy between Brans–Dicke models and higher-order theories of gravity, for example, the present analysis could represent a straightforward check also for such kinds of models. A general analysis from this point of view will be faced elsewhere.

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[4] We note that in [115], it has been used the parametrization $\langle \sigma v \rangle = \sigma_0 x^{-l}$, where $l = 0$ corresponds to s-wave annihilation, $l = 1$ to p-wave annihilation, and so on. The modification of standard cosmology induces the corrections to the parameter l via $\bar{l}$. When $\alpha = 1/2$, hence the evolution of the Universe is described by the standard cosmological model, one gets $\bar{l} = l$, reproducing the standard results.

Appendix A: 5*D* Brans–Dicke field equations

The equations for the gravitational field in $5D$ derived from (1) read

$$G_{AB}^{(5)} = R_{AB}^{(5)} - \frac{1}{2}\gamma_{AB}R^{(5)} = \frac{8\pi}{\phi}T_{AB}^{(5)} + \frac{\omega}{\phi^2}\left[(\nabla_A\phi)(\nabla_B\phi) - \frac{1}{2}\gamma_{AB}\left(\nabla^C\phi\right)(\nabla_C\phi)\right] + \frac{1}{\phi}\left(\nabla_A\nabla_B\phi - \gamma_{AB}\nabla^2\phi\right), \tag{A1}$$

where $\nabla^2 \equiv \nabla_A\nabla^A$ and $T_{AB}^{(5)}$ represent the energy-momentum tensor (EMT) of matter fields in $5D$ with trace $T^{(5)} = \gamma^{AB}T_{AB}^{(5)}$. For the sake of generality of the reduction procedure in this section $T_{AB}^{(5)} \neq 0$.

The field equation for the scalar field ϕ is determined by (1) as

$$\frac{2\omega}{\phi}\nabla^2\phi - \frac{\omega}{\phi^2}(\nabla_A\phi)\left(\nabla^A\phi\right) + R^{(5)} = 0. \tag{A2}$$

Taking the trace of (A1), we find

$$R^{(5)} = -\frac{16\pi}{3\phi}T^{(5)} + \frac{\omega}{\phi^2}(\nabla_A\phi)\left(\nabla^A\phi\right) + \frac{8}{3\phi}\nabla^2\phi. \tag{A3}$$

Combining the last two equations, we get

$$\nabla^2\phi = \frac{8\pi}{4+3\omega}T^{(5)}. \tag{A4}$$

In this work, we use coordinates where the metric in $5D$ can be written as[5]

$$dS^2 = \gamma_{AB}dx^A dx^B = g_{\mu\nu}(x,y)dx^\mu dx^\nu + \epsilon\Phi^2(x,y)dy^2\,. \tag{A5}$$

Conventional $4D$ spacetime is represented by the hypersurface $\Sigma_y : y = y_0 =$ constant, which is orthogonal, along the extra dimension, to the $5D$ unit vector

$$\hat{n}^A = \frac{\delta_4^A}{\Phi}, \quad n_A n^A = \epsilon, \tag{A6}$$

the tensor $g_{\mu\nu}$ represent ordinary $4D$ metric.

The effective field equations (FE) in $4D$ are obtained from the dimensional reduction of (A1) and (A4). In order to achieve this lower dimensional scheme, we notice that

$$\begin{aligned}
\nabla_\mu\nabla_\nu\phi =& D_\mu D_\nu\phi + \frac{\epsilon}{2\Phi^2}\overset{*}{g}_{\mu\nu}\overset{*}{\phi},\\
\nabla_4\nabla_4\phi =& \epsilon\Phi(D_\alpha\Phi)\left(D^\alpha\phi\right) + \overset{**}{\phi} - \frac{\overset{*}{\Phi}}{\Phi}\overset{*}{\phi},\\
\nabla^2\phi =& D^2\phi + \frac{(D_\alpha\Phi)(D^\alpha\phi)}{\Phi} + \frac{\epsilon}{\Phi^2}\left[\overset{**}{\phi} + \overset{*}{\phi}\left(\frac{g^{\mu\nu}\overset{*}{g}_{\mu\nu}}{2} - \frac{\overset{*}{\Phi}}{\Phi}\right)\right],
\end{aligned} \tag{A7}$$

here asterisk denotes partial derivative with respect to the extra coordinate (i.e. $\partial/\partial y = *$); D_α is the covariant derivative on Σ_y, which is calculated with $g_{\mu\nu}$, and $D^2 \equiv D^\alpha D_\alpha$.

Using these expressions, the $4D$ spacetime components ($A = \mu,\ B = \nu$) of the $5D$ field Eq. (A1) can be written as follows:

$$\begin{aligned}
G_{\mu\nu}^{(5)} =& \frac{8\pi}{\phi}T_{\mu\nu}^{(5)} + \frac{\omega}{\phi^2}\left[\left(D_\mu\phi\right)(D_\nu\phi) - \frac{1}{2}g_{\mu\nu}(D_\alpha\phi)\left(D^\alpha\phi\right)\right] + \frac{1}{\phi}\left(D_\mu D_\nu\phi - g_{\mu\nu}D^2\phi\right)\\
&- \frac{g_{\mu\nu}(D_\alpha\Phi)(D^\alpha\phi)}{\Phi\phi} - \frac{\epsilon g_{\mu\nu}}{2\Phi^2\phi}\left[2\overset{**}{\phi} + \overset{*}{\phi}\left(g^{\alpha\beta}\overset{*}{g}_{\alpha\beta} - 2\frac{\overset{*}{\Phi}}{\Phi} + \omega\frac{\overset{*}{\phi}}{\phi}\right)\right] + \frac{\epsilon\overset{*}{g}_{\mu\nu}\overset{*}{\phi}}{2\Phi^2\phi}.
\end{aligned} \tag{A8}$$

[5] Notation: $x^\mu = (x^0, x^1, x^2, x^3)$ are the coordinates in $4D$, and y is the coordinate along the extra dimension. We use spacetime signature $(+, -, -, -)$, while $\epsilon = \pm 1$ allows for spacelike or timelike extra dimension.

To construct the Einstein tensor in $4D$, we have to express $R^{(5)}_{\alpha\beta}$ and $R^{(5)}$ in terms of the corresponding $4D$ quantities. The Ricci tensor $R^{(4)}_{\mu\nu}$ of the metric $g_{\mu\nu}$ and the scalar field Φ is related to the Ricci tensor $R^{(5)}_{AB}$ of γ_{AB} by [86]

$$
\begin{aligned}
R^{(5)}_{\alpha\beta} =& R^{(4)}_{\alpha\beta} - \frac{D_\alpha D_\beta \Phi}{\Phi} + \frac{\epsilon}{2\Phi^2}\left(\frac{\overset{*}{\Phi}\overset{*}{g}_{\alpha\beta}}{\Phi} - \overset{**}{g}_{\alpha\beta} + g^{\lambda\mu}\overset{*}{g}_{\alpha\lambda}\overset{*}{g}_{\beta\mu} - \frac{g^{\mu\nu}\overset{*}{g}_{\mu\nu}\overset{*}{g}_{\alpha\beta}}{2}\right),\\
R^{(5)}_{44} =& -\epsilon\Phi D^2\Phi - \frac{\overset{*}{g}{}^{\lambda\beta}\overset{*}{g}_{\lambda\beta}}{4} - \frac{g^{\lambda\beta}\overset{**}{g}_{\lambda\beta}}{2} + \frac{\overset{*}{\Phi}g^{\lambda\beta}\overset{*}{g}_{\lambda\beta}}{2\Phi}.
\end{aligned}
\tag{A9}
$$

From (A1) to (A3) and the second equation in (A9), we obtain

$$
\begin{aligned}
\frac{D^2\Phi}{\Phi} =& -\frac{(D_\alpha\Phi)(D^\alpha\phi)}{\Phi\phi} - \frac{\epsilon}{2\Phi^2}\left[g^{\lambda\beta}\overset{**}{g}_{\lambda\beta} + \frac{\overset{*}{g}{}^{\lambda\beta}\overset{*}{g}_{\lambda\beta}}{2} - \frac{\overset{*}{\Phi}g^{\lambda\beta}\overset{*}{g}_{\lambda\beta}}{\Phi}\right] - \frac{\epsilon}{\Phi^2\phi}\left[\overset{**}{\phi} + \overset{*}{\phi}\left(\frac{\omega\overset{*}{\phi}}{\phi} - \frac{\overset{*}{\Phi}}{\Phi}\right)\right]\\
&+\frac{8\pi}{\phi}\left[\frac{(\omega+1)T^{(5)}}{4+3\omega} - \frac{\epsilon T^{(5)}_{44}}{\Phi^2}\right].
\end{aligned}
\tag{A10}
$$

Substituting this expression into $R^{(5)} = \gamma^{AB}R_{AB}$, we find

$$
\begin{aligned}
R^{(5)} =& R^{(4)} + \frac{2(D_\alpha\Phi)(D^\alpha\phi)}{\Phi\phi} - \frac{\epsilon}{4\Phi^2}\left[\overset{*}{g}{}^{\alpha\beta}\overset{*}{g}_{\alpha\beta} + \left(g^{\alpha\beta}\overset{*}{g}_{\alpha\beta}\right)^2\right] + \frac{2\epsilon}{\Phi^2\phi}\left[\overset{**}{\phi} + \overset{*}{\phi}\left(\frac{\omega\overset{*}{\phi}}{\phi} - \frac{\overset{*}{\Phi}}{\Phi}\right)\right]\\
&+\frac{16\pi}{\phi}\left[\frac{\epsilon T^{(5)}_{44}}{\Phi^2} - \frac{(\omega+1)T^{(5)}}{4+3\omega}\right],
\end{aligned}
\tag{A11}
$$

where $R^{(4)} = g^{\alpha\beta}R^{(4)}_{\alpha\beta}$ is the scalar curvature of the spacetime hypersurfaces Σ_y.

References

1. S.G. Turyshev, Usp. Fiz. Nauk **179**, 3034 (2009)
2. G.M. Tino, L. Cacciapuoti, S. Capozziello, G. Lambiase, F. Sorrentino, Prog. Part. Nucl. Phys. **112**, 10377 (2020)
3. J.L. Sievers et al., Astrophys. J. **591**, 599 (2003)
4. A.G. Riess et al., Supernova search team. Astron. J. **116**, 1009 (1998)
5. S. Perlmutter *et al.* [Supernova cosmology project collaboration]
6. S. Perlmutter et al., Astrophys. J. **517**, 565 (1999)
7. S. Cole *et al.* [2dFGRS Collaboration]
8. S. Cole et al., Mon. Not. Roy. Astron. Soc. **362**, 505 (2005)
9. G. Hinshaw *et al.*, [WMAP Collaboration]
10. G. Hinshaw et al., Astrophys. J. Suppl. **170**, 288 (2007)
11. S.M. Carroll, W.H. Press, E.L. Turner, Ann. Rev. Astron. Astrophys. **30**, 499 (1992)
12. V. Sahni, A.A. Starobinsky, Int. J. Mod. Phys. D **9**, 373 (2000)
13. A.A. Starobinski, Phys. Lett. B **91**, 99–102 (1980)
14. A.A. Starobinsky, Sov. Astron. Lett. **9**, 302 (1983)
15. C.G. Boehmer, T. Harko, F.S.N. Lobo, Astropart. Phys. **29**, 386 (2008)
16. C.G. Boehmer, T. Harko, F.S.N. Lobo, JCAP **0803**, 024 (2008)
17. T.P. Sotiriou, V. Faraoni, Rev. Mod. Phys. **82**, 451 (2010)
18. A. De Felice, S. Tsujikawa, Living Rev. Rel. **13**, 3 (2010)
19. S. Nojiri, S.D. Odintsov, V.K. Oikonomou, Phys. Rept. **692**, 1 (2017)
20. A. Silvestri, M. Trodden, Rep. Prog. Phys. **72**, 096901 (2009)
21. R. Durrer, R. Maartens, Gen. Relat. Grav. **40**, 301 (2008)
22. M. Sami, Lect. Notes Phys. **720**, 219 (2007)
23. E.J. Copeland, M. Sami, Sh. Tsujikawa, Int. J. Mod. Phys. D **15**, 1753 (2006)
24. L. Amendola and S. Tsujikawa, *Dark Energy: Theory and Observations*, Cambridge University Press, 2010
25. G.K. Chakravarty, S. Mohanty, G. Lambiase, Int. J. Mod. Phys. D **26**(13), 1730023 (2017)
26. S. Capozziello, G. Lambiase, M. Sakellariadou, An. Stabile, Phys. Rev. D **91**, 044012 (2015)
27. G. Lambiase, M. Sakellariadou, A. Stabile, A. Stabile, JCAP **1507**, 003 (2015)
28. G. Lambiase, M. Sakellariadou, A. Stabile, JCAP **1312**, 020 (2013)
29. T. Biswas, E. Gerwick, T. Koivisto, A. Mazumdar, Towards singularity and ghost free theories of gravity. Phys. Rev. Lett. **108**, 031101 (2012)
30. L. Buoninfante, A.S. Koshelev, G. Lambiase, A. Mazumdar, Classical properties of non-local, ghost- and singularity-free gravity. JCAP **09**, 034 (2018)
31. L. Buoninfante, A.S. Koshelev, G. Lambiase, J. Marto, A. Mazumdar, Conformally-flat, non-singular static metric in infinite derivative gravity. JCAP **06**, 014 (2018)
32. L. Pogosian, A. Silvestri, Phys. Rev. D **77**, 023503 (2008)
33. I. Sawicki, W. Hu, Phys. Rev. D **75**, 127502 (2007)
34. B. Li, J.D. Barrow, Phys. Rev. D **75**, 084010 (2007)

35. T. Clifton, Phys. Rev. D **78**, 083501 (2008)
36. S. Capozziello, G. Lambiase, H.J. Schmidt, Annalen Phys. **9**, 39 (2000)
37. G. Lambiase, S. Mohanty, A.R. Prasanna, Int. J. Mod. Phys. D **22**, 1330030 (2013)
38. B. Jain, V. Vikram, J. Sakstein, Astrophys. J. **779**, 39 (2013)
39. L. Lombriser, A. Slosar, U. Seljak, W. Hu, Phys. Rev. D **85**, 124038 (2012)
40. S. Derraro, F. Schmidt, W. Hu, Phys. Rev. D **83**, 063503 (2011)
41. L. Lombriser, F. Schmidt, T. Baldauf, R. Mandelbaum, U. Seljak, R.E. Smith, Phys. Rev. D **85**, 102001 (2012)
42. H. Motohashi, A.A. Starobinsky, J. Yakoyama, Phys. Rev. Lett. **110**, 121302 (2013)
43. S. Capozziello, G. Lambiase, Phys. Lett. B **750**, 344 (2015)
44. N.D. Birrell, P.C.W. Davies, *Quantum Fields in Curved Space* (Cambridge University Press, Cambridge, UK, 1982)
45. I.L. Buchbinder, S.D. Odintsov, I.L. Shapiro, *Effective Action in Quantum Gravity* (IOP, Bristol, UK, 1992)
46. N.H. Barth, S. Christensen, Phys. Rev. D **28**, 8 (1983)
47. A. Joyce, B. Jain, J. Khoury, M. Trodden, Phys. Rep. **568**, 1 (2015)
48. J. Khoury, A. Weltman, Phys. Rev. Lett. **93**, 171104 (2004)
49. J. Khoury, A. Weltman, Phys. Rev. D **69**, 044026 (2004)
50. K. Hinterbichler, J. Khoury, Phys. Rev. Lett. **104**, 231301 (2010)
51. A.I. Vainshtein, Phys. Lett. B **39**, 393 (1972)
52. P.J.E. Peebles, B. Ratra, The cosmological constant and dark energy. Rev. Mod. Phys. **75**, 559 (2003)
53. T. Padmanabhan, Phys. Rept. **380**, 235 (2003)
54. I. Zlatev, L. Wang, P.J. Steinhardt, Phys. Rev. Lett. **82**, 896 (1999)
55. C. Armendariz, V. Mukhanov, P.J. Steinhardt, Phys. Rev. Lett. **85**, 4438 (2000)
56. R.R. Caldwell, R. Dave, P.J. Steinhardt, Phys. Rev. Lett. **80**, 1582 (1998)
57. S.E. Deustua, R. Caldwell, P. Garnavich, L. Hui, A. Refregier, Cosmological Parameters, Dark Energy and Large Scale Structure [arXiv:astro-ph/0207293]
58. L.P. Chimento, A.S. Jakubi, D. Pavon, Phys. Rev. D **62**, 063508 (2000)
59. M.C. Bento, O. Bertolami, A.A. Sen, Phys. Rev. D **66**, 043507 (2002)
60. Zong-Kuan. Guo, Yuan-Zhong. Zhang, Phys. Lett. B **645**, 326 (2007)
61. C. Armendariz-Picon, V. Mukhanov, P.J. Steinhardt, Phys. Rev. Lett. **85**, 4483 (2000)
62. R. de Putter, E.V. Linder, Astropart. Phys. **28**, 263 (2007)
63. R.J. Scherrer, Phys. Rev. Lett. **93**, 011301 (2004)
64. C. Bonvin, C. Caprini, R. Durrer, Phys. Rev. Lett. **97**, 081303 (2006)
65. O. Bertolami, P.J. Martins, Phys. Rev. D **61**, 064007 (2000)
66. N. Banerjee, D. Pavon, Phys. Rev. D **63**, 043504 (2001)
67. S. Sen, A.A. Sen, Phys. Rev. D **63**, 124006 (2001)
68. H. Kim, Mon. Not. Roy. Astron. Soc. **364**, 813 (2005)
69. H. Kim, Phys. Lett. B **606**, 223 (2005)
70. S. Das, N. Banerjee, Gen. Rel. Grav. **38**, 785 (2006)
71. W. Chakraborty, U. Debnath, Int. J. Theor. Phys. **48**, 232 (2009)
72. L. Conversi, A. Melchiorri, L. Mersini, J. Silk, Astropart. Phys. **21**, 443 (2004)
73. J. Ponce de Leon, JCAP03 (2010)030
74. S. Das, N. Banerjee, Phys. Rev. D **78**, 043512 (2008)
75. Y. Li-E Qiang, M. Ma, DYu. Han, Phys. Rev. D **71**, 061501 (2005)
76. Li.-e Qiang, Yan Gong, Yongge Ma, Xuelei Chen, Phys. Lett. B **681**, 210 (2009)
77. J.E. Madriz Aguilar, C. Romero, A. Barros, Gen. Rel. Grav. **40**, 117 (2008)
78. A. Troisi, Eur. Phys. J. C **77**, 171 (2017)
79. S.G. Rubin, A. Troisi, Int. J. Modern Phys. D **27**, 1841007 (2018)
80. A.D. Sakharov, Pisma. Zh. Eksp. Teor. Fiz. **5**, 32 (1967)
81. J. Dunkley et al., WMAP. Astrophys. J. Suppl. **180**, 306 (2009)
82. W.M. Yao et al., Particle Data Group. J. Phys. G **33**, 1 (2006)
83. E.W. Kolb, M.S. Turner, *The Early Universe* , Addison Wesley Publishing Company, (1989)
84. J. Bernstein, L.S. Brown, G. Feinberg, Rev. Mod. Phys. **61**, 25 (1989)
85. K.A. Olive et al., Particle data groups. Ch. Phys. C **38**, 0900001 (2014)
86. P.S. Wesson, J. Ponce de Leon, J. Math. Phys. **33**, 3883 (1992)
87. C. Romero, R. Tavakol, R. Zalaletdinov, Gen. Rel. Grav. **283**, 365 (1996)
88. C. Brans, R.H. Dicke, Phys. Rev. **124**, 925 (1961)
89. T. Kugo, S. Uehara, Nucl. Phys. B **222**, 125 (1983)
90. T. Kugo, S. Uehara, Prog. Theor. Phys. **73**, 235 (1985)
91. H. Davoudiasl, R. Kitano, G.D. Kribs, H. Murayama, P.J. Steinhardt, Phys. Rev. Lett. **93**, 201301 (2004)
92. E.W. Kolb, M.S. Turner, The Early Universe. Front. Phys. **69**, 1–547 (1990)
93. N. Azhar, A. Jawad, S. Rani, Phys. Dark Univ. **32**, 100815 (2021)
94. H. Davoudiasl, Phys. Rev. D **88**, 095004 (2013)
95. S.D. Odintsov, V.K. Oikonomou, EPL **116**, 49001 (2016)
96. S.D. Odintsov, V.K. Oikonomou, Phys. Lett. B **760**, 259 (2016)
97. V.K. Oikonomou, E.N. Saridakis, Phys. Rev. D **94**, 124005 (2016)
98. S. Bhattacharjee, P.K. Sahoo, Eur. Phys. J. C **80**, 289 (2020)
99. E.H. Baffou, M.J.S. Houndjo, D.A. Kanfon, I.G. Salako, Eur. Phys. J. C **79**, 112 (2019)
100. M.P.L.P. Ramos, J. Páramos, Phys. Rev. D **96**, 104024 (2017)
101. M. Fukushima, S. Mizuno, Ki. Maeda, Phys. Rev. D **93**, 103513 (2016)
102. H.M. Sadjadi, Phys. Rev. D **76**, 123507 (2007)
103. M.P.L.P. Ramos, J. Páramos, Phys. Rev. D **96**, 104024 (2017)
104. G.G. Luciano, Eur. Phys. J. C **82**, 558 (2022)

105. G.G. Luciano, Phys. Lett. B **833**, 137352 (2022)
106. N. Azhar, A. Jawad, S. Rani, Phys. Dark Univ. **32**, 100815 (2021)
107. S. Boran, E.O. Kahya, Adv. High Energy Phys. **2014**, 282675 (2014)
108. G. Steigman, Adv. High Energy Phys. **2012**, 268321 (2012)
109. V. Simha, G. Steigman, JCAP **06**, 016 (2008)
110. S. Bhattacharjee, P.K. Sahoo, Eur. Phys. J. Plus **135**, 350 (2020)
111. J.P. Kneller, G. Steigman, New J. Phys. **6**, 117 (2004)
112. G. Steigman, Annu. Rev. Nucl. Part. Sci. **57**, 463 (2007)
113. WMAP Collaboration (E. Komatsu et al.), Astrophys. J. Suppl. **192**, 18 (2011)
114. B.D. Fields, JCAP **03**, 010 (2020)
115. J.U. Kang, G. Panotopoulos, Phys. Lett. B **677**, 6 (2009)
116. E.W. Kolb, M.S. Turner, *The Early Universe* (Addison-Wesley Publishing Company, Redwood City, California, 1989)
117. S. Nojiri, S.D. Odintsov, E.N. Saridakis, Eur. Phys. J. C **79**, 242 (2019)
118. Aghanim, N. et al. (Planck COll.), Astron. Astrophys **641**, A5 (2020)
119. D.N. Spergel et al., WMAP collaboration. Astrophys. J. Suppl. **170**, 377 (2007)