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MODELS FOR POWER COMBINING MAGNETRONS IN A MAGIC TEE*

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Abstract

Industrial accelerator applications require efficient, scalable, continuous wave (CW) microwave power systems. Magnetrons are inexpensive and efficient devices for converting electrical energy into microwave power; however, their power output is limited to approximately 100 kW. Cost effective power combining magnetron systems would serve the accelerator industry by providing practical and affordable RF power to accelerator applications.

In a magic tee configuration, two oscillators can be power combined and locked to a common frequency. Researchers at General Atomics in collaboration with Thomas Jefferson National Accelerator Facility have constructed an experiment to demonstrate the power combining of magnetrons in a such a configuration. An analytic model is presented describing the power combining efficiency of a 2-port magic tee, accounting for two magnetron output signals, an injection signal, and a reactive load. The Adler-Chen model is solved numerically using robust computational geometry techniques. Completed solutions provide insight to the phenomena of magnetron frequency locking and optimal combining efficiency.

DEMAND FOR SCALABLE RF SOURCES

Recent advancements in materials and closed-cycle refrigeration enable the use of superconducting radiofrequency (SRF) accelerator technology in cryogen free environments to generate megawatt class electron beams [1]. Examples such as municipal wastewater treatment plants will require continuous megawatt sources operating with high capacity factor and electrical efficiency [2].

The commercial utility of accelerators driven by large tubes such as klystrons is limited by their lower efficiency and single points of failure. Some accelerators, such as synchrotron SOLEIL's Superconducting RF cryomodule system is powered by combing thousands of solid state amplifiers [3]. This work proposes a numerical scheme for computing the combining efficiency of magnetrons phase locked using a magic tee to demonstrate scalable power combination for commercial linacs.

Magnetrons are inexpensive and efficient microwave sources. A single vacuum electronic device operating at 915 MHz can be capable of sourcing up to 125 kW RF. However, their applicability as RF sources for accelerators is limited by factors including manufacturing variances, spectral noise, frequency drift, and as oscillators random startup phasing. Power combining and phase locking to a fixed frequency of magnetrons would demonstrate a scalable efficient RF power source making commercial accelerators more economically viable.

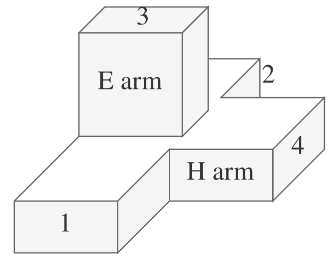

Figure 1: Schematic depiction of magic tee waveguide combiner.

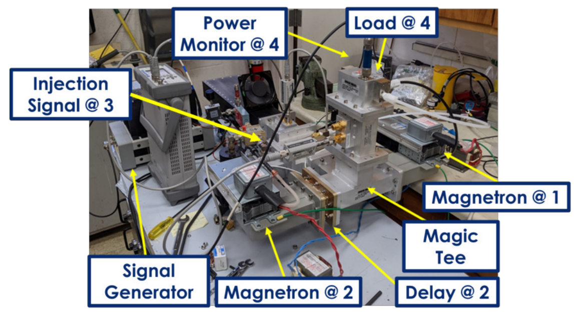

Figure 2: Layout of two-way combining experiment.

THE MAGIC TEE POWER COMBINER

The magic tee is a 4 port 3 dB coupler where ports 1 and 2 form E & H waveguide arms with ports 4 and 3 respectively (Fig. 1). Its scattering matrix is expressed in Eq. (1).

$$S = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \\ 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix} \tag{1}$$

Signals injected into ports 3 and 4 are split equally between ports 1 and 2. Signals combine at 3 in phase and at 4 180° out of phase.

General Atomics developed an experimental setup to demonstrate 2 way combining of magnetrons in a magic tee (Fig. 2). Two 2.45 GHz cooking magnetrons power ports 1 and 2 with 1 kW RF. A mechanical phase offset ψ is included at port 2. A low power amplifier signal is injected at port 3 and output power is combined and measured towards a load at port 4.

Each magnetron's natural frequency (f_1 , f_2) and start phasing (ϕ_1, ϕ_2) is nominally independent. Trim coils are installed to shift frequencies and as well can the injection signals of phase ϕ_3 and frequency f_3 be controlled and monitored with respect to the magnetron signals.

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MODELING COMBINING EFFICIENCY

The matching performance of a load has an effect on the phasing needed for optimal power combining and locking. Consider installing a load with impedance Z_L and reflection coefficient Γ. Applying this to port 4 of a magic tee configuration yields a 3-port system shown in Eq. (2).

$$S = \begin{bmatrix} s_{11} & -s_{11} & s_{13} \\ -s_{11} & s_{11} & s_{13} \\ s_{13} & s_{13} & 0 \end{bmatrix} \quad s_{11} = \frac{\Gamma}{2} \quad s_{13} = \frac{1}{\sqrt{2}} \tag{2}$$

Power injected with voltages $\mathbf{a} = |a_i|\exp(j\phi_i)$ for $i \in [1,2,3]$ will yield the resultant signal $\mathbf{b}$. Combining efficiency η can be inferred by subtracting the total resultant signal power $|\mathbf{b}|^2$ from injected power $|\mathbf{a}|^2$.

$$\mathbf{b} = S \times \mathbf{a} \quad \eta = \frac{\Sigma|\mathbf{a}|^2 - \Sigma|\mathbf{b}|^2}{\Sigma|\mathbf{a}|^2} \tag{3}$$

The effect of a straight waveguide section at port 2 with delay ψ is analytically represented by introducing a delay matrix.

$$S \rightarrow S_\psi \times S \times S_\psi \quad S_\psi = \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{-i\psi} & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{4}$$

Optimal efficiency can be analytically solved and occurs where $\phi_1 - \phi_2 + \psi = \pi$ is satisfied.

$$\eta = \frac{1 - \cos(\phi_1 - \phi_2 + \psi)}{2} \tag{5}$$

MODELING FREQUENCY LOCKING

The natural frequencies of magnetron's 1, 2, and injection frequency at port 3 are given by f_1, f_2 and f_3 respectively. Equation (6) describes how the mixed resultant frequency f' is driven by $\mathbf{b} = S \times \mathbf{a}$ and the pushing parameter α, where injection ratio and angle is given by $\rho = |\mathbf{b}|/|\mathbf{a}|$ and $\varphi = \angle\mathbf{b} - \angle\mathbf{a}$ respectively [4],[5]. The subscript i is uniformly implied to solve for each magnetron's driven frequency.

$$f' = f\left(1 - \frac{\rho}{Q}(\sin\varphi - \cos\varphi\tan\alpha)\right) \tag{6}$$

Partial frequency locking between differing ports i and j can be described by iso-surfaces $\Delta_{i,j}^{-1}(0)$ where Δ describes the frequency difference driven between ports and is a function of the phasing of the system. Complete frequency locking ϕ_L is given by the mutual intersection of these surfaces.

$$\Delta_{i,j}(\phi) = f_i' - f_j' \quad \phi_L = \bigcap_{i=1}^{3} \Delta_{i,i+1}^{-1}(0) \tag{7}$$

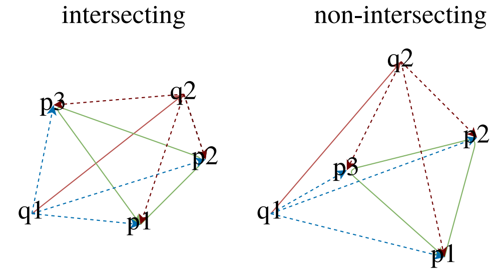

Figure 3: Identifying intersections of ray q through face p can be efficiently computed by comparing signed tetrehdra volumes formed by combinations of their vertices.

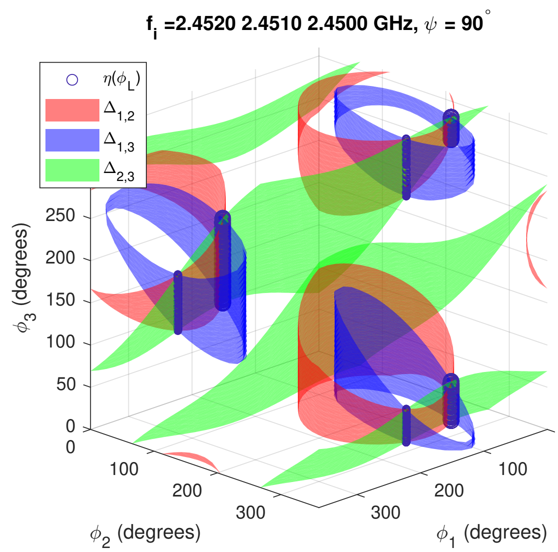

Figure 4: Locked combining efficiency is represented at the intersections proportional to marker size.

NUMERICAL METHODOLOGY

The isosurfaces satisfying $\Delta^{-1}(0)$ are computed numerically using a marching cubes algorithm and represented as a triangular mesh. The Moller-Trumbore ray intersection algorithm is used to efficiently compute the mesh-mesh intersections ϕ_L as a point cloud [6].

A coarse 3D meshgrid describing phase combinations of ϕ_1, ϕ_2, and ϕ_3 can return isosurfaces of $n = 10,000$ faces. To compute a mesh-mesh intersection, each ray of each triangle in one mesh must be tested for intersection with triangles from the other (Fig. 3), scaling with $O(n^2)$. The problem is therefore reduced by subdividing the space into m local groups using Octree Partitioning, reducing the computation size to $O(n^2/m^2)$.

The isosurfaces indicating frequency locking of two magnetrons to a 2.45 GHz injection signal with a -30 dB matched load and 90° delay at port 2 is depicted in Fig. 4.

BENCHMARKING NUMERICAL EFFORTS

The theoretical combining efficiency of two 75 kW 915 MHz magnetrons with Q=100 and pushing parameter $\alpha = 0.25$ was computed against a sweeping 500 W injection

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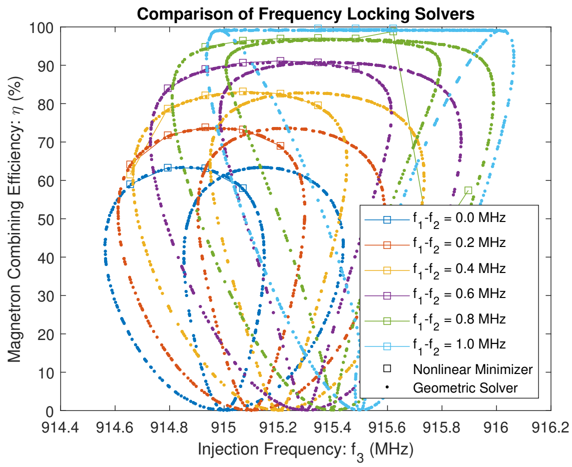

Figure 5: Current analysis benchmarked with previous work.

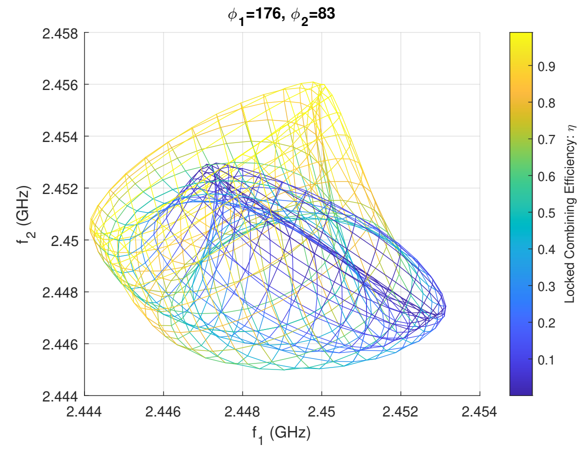

Figure 6: Top projection of surface in f_1, f_2, and $|\phi_3| < \pi$ showing locking to f_3, a 2.45 GHz injection signal.

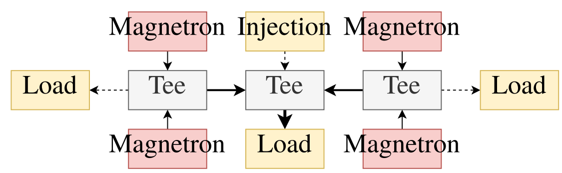

Figure 7: Layout of future 4-way combining experiment.

signal. A -24 dB reflective load is applied to port 4 and the frequency of magnetron 2 is swept incrementally from 0 to 1 MHz with respect to magnetron 1. Results from this work's geometric approach is compared to that of previous analysis in Fig. 5.

This work robustly solves for frequency locking and returns a complete solution reliably identifying locking conditions, even where combining can be inefficient. This technique is also computationally faster by an order of magnitude to that of traditional nonlinear iterative minimizing techniques.

REAL-TIME SOLUTIONS FOR CONTROLS

The free running magnetron frequency f_1 and f_2 can be monitored and tuned by trimming solenoid fields in experiment with respect to a controllable injection signal given by f_3 and ϕ_3. The relative phases difference of the magnetrons ϕ_1, ϕ_2 can be controlled by installing custom length rectangular waveguides and fine-tuned by individual waveguide bellows. Using this analysis, the control space given by f_1, f_2 and ϕ_3 can be solved for frequency locking, and inform the movement towards states of optimal combining efficiency. Locking can be maintained while moving towards higher combining efficiency by remaining on the control surface. As phasing drifts, the control surface will be recalculated, adjusting the controls response (Fig. 6).

FUTURE WORK

Controls software leveraging these numerical techniques will be developed by GA to demonstrate reliable and stable 2-way power combining of magnetrons. Further work will involve scaling this algorithm to controls software enabling the stable 4-way combining of magnetrons as in Fig. 7.

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