A SADDLE-POINT SOLUTION IN THE WEINBERG-SALAM THEORY*

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Summary

We briefly discuss some properties of a new saddle-point solution (energy $\sim$ 10 TeV) in the standard theory of the electroweak interactions.

Recently it was shown that the field configuration space of the classical Weinberg-Salam theory[1] without fermions has noncontractible loops passing through the vacuum configuration.[2] This makes it likely that there is a static classical solution of the field equations, which is a saddle-point of the energy functional, and therefore unstable. The solution would be the maximal energy configuration on some noncontractible loop, and all other loops homotopic to this one would pass through configurations of equal or greater energy. It would therefore be at the top of the energy barrier for going from the vacuum to the vacuum along a topologically nontrivial path. This energy barrier has a definite height because the Weinberg-Salam theory has a mass scale (the Higgs vacuum expectation value), in contrast to the case of a pure gauge theory such as Quantum Chromodynamics.

One believes that there is such a saddle-point solution by analogy with work of Taubes,[3] who has shown rigorously that in a slightly different context, namely the zero-monopole sector of the SO(3) gauge theory with an adjoint Higgs field and vanishing Higgs potential, one can apply Morse theory arguments to relate topological information about the space of field configurations to the existence of stationary points of the energy functional. Forgács and Horváth[4] have reviewed a number of other field theories in one, two, and three spatial dimensions, where explicitly known saddle-point solutions are related to the topology of the field configuration space.

We have coined the word "sphaleron"[5] to describe any classical solution of this type in a relativistic field theory. A sphaleron, being static and localized in space, is particle-like, but since it is unstable, we do not want to call it a soliton. Unlike a soliton, a sphaleron almost certainly does not correspond to a stable particle state in the quantum theory.

One can find a good approximation to a sphaleron in the Weinberg-Salam theory, which has gauge group $SU(2) \times U(1)$, by expanding to lowest nontrivial order in the weak mixing angle θ_W. In the limit that θ_W vanishes the U(1) field decouples and may consistently be set to zero in the field equations. There remains an SU(2) theory with a doublet Higgs field. Dashen, Hasslacher, and Neveu (DHN)[6] discovered some time ago a static solution in this theory, which was later rediscovered by Boguta.[7] The solution has finite energy, and the energy density is localized and spherically symmetric. The fields,[8] strictly speaking, are only axially symmetric. Burzlaff[8] proved recently that this solution rigorously exists, and also proved that it is unstable, by presenting a one-parameter family of field configurations, among which it is the configuration of maximal energy. In fact, there is a noncontractible loop in the configuration space passing through the vacuum and the DHN sphaleron, on which the sphaleron is the configuration of maximal energy.

We have estimated numerically the energy of the SU(2) sphaleron for the whole range of values of the quartic Higgs coupling λ (neither Dashen et al. nor Boguta had done this). We find that the sphaleron energy E_S increases from 7.6 TeV for $\lambda = 0$ to 13.5 TeV for $\lambda = \infty$ (see the Appendix for details).

When $\theta_W \neq 0$ the presence of the U(1) field makes it impossible for the solution to remain spherically symmetric in any sense. However, it is quite easy to find the changes in the sphaleron's properties to leading order in θ_W. To first order, the SU(2) gauge field and Higgs field remain unchanged, but they produce a U(1) current density which is axially symmetric and which acts as a source for the U(1) gauge field. By calculating the asymptotic form of this U(1) field, we find that the sphaleron has a magnetic dipole moment of strength $\sim 0.3\ \mathrm{GeV}^{-1}$. This is about 80 times larger than the magnetic moment of the W-boson, which is e/M_W.[9] The energy of the sphaleron decreases by $\sim$ 1% relative to the energy when $\theta_W = 0$.

Another interesting property of the sphaleron is that it has a baryon number and a lepton number of 1/2. This is a direct consequence of the anomalies in the fermionic currents of the Weinberg-Salam theory, whose significance was first discussed by 't Hooft.[10] These fractional charges are also related to the existence of a zero binding energy solution to the Dirac equation in the SU(2) sphaleron background. Consider for simplicity the SU(2) part of the Weinberg-Salam theory with only leptons, that is, with an SU(2) doublet $\Psi_L = (e_L^-, \nu_L)$ and with an SU(2) singlet e_R^-, and let these leptons be massless (no Yukawa term in the Lagrangian). In the sphaleron background there is a normalizable zero energy solution to the classical Dirac equations: Ψ_L as given in Ref. 11 and $e_R = 0$. Following Jackiw and Rebbi[12] this then implies that the sphaleron has lepton number 1/2, which agrees with the result derived from the anomaly equation. A similar analysis holds for the Weinberg-Salam theory with only quark fields. Thus in the full theory with one generation of quarks and leptons the sphaleron also has baryon number 1/2.

It has been claimed by 't Hooft[10] that tunneling between topologically distinct vacua is negligible in the weak interactions because the Euclidean action is so large, namely $> 8\pi^2/g^2$. But this argument only applies to a virtual quantum process. If there were sufficient real energy available, more than the sphaleron energy E_S, the tunneling process might be enhanced. There seem to be two situations, at least, where this could happen. The first is in high energy collisions of particles from a very powerful accelerator, and the signature of the process would be the violation of baryon and lepton number conservation. The other situation is for a system at very high temperature ($kT > E_S$). Thermal fluctuations might then produce the baryon number violating process via the sphaleron at a substantial rate. This could be important in the Universe at early times ($t \sim 10^{-15}$s, $dT \sim 10$ TeV), where these processes could be related to the baryon number as observed today, cf. Ref. 13. But, before one can address these problems, a better understanding of the role of the sphaleron and other solutions is required.

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In conclusion, it is clear that already the standard Weinberg-Salam theory for the electroweak interactions contains some interesting non-perturbative structure.

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Appendix

We will present here a table with numerical results for the energy of the SU(2) sphaleron (zeroth order in θ_W) and the magnetic dipole moment to order θ_W. Let us first establish our notation: v is the Higgs vacuum expectation value; λ is the quartic Higgs coupling; g and g' are the SU(2) and U(1) gauge coupling constants, respectively; the weak mixing angle θ_W is related to the electric charge e by $e = g \sin\theta_W = g' \cos\theta_W$. To be definite we take the following values: W-boson mass $M_W = \frac{1}{2} gv = 80$ GeV, $e^2/4\pi = 1/137$, and $\sin^2\theta_W = 0.23$.

The field configuration of the SU(2) sphaleron is determined by two radial functions. The differential equations for these functions, which follow from the field equations, can be integrated numerically. In the table below we give the resulting values for (i) the energy in units $4\pi v/g = 5.0$ TeV, and (ii) the magnetic dipole moment in units of $\frac{2\pi}{3}\frac{g'}{g^3 v} = 2.99\ e/M_W$. For comparison we also give the variational estimates of these quantities given in our original paper, to which we refer for further details.

λ/g^2	energy num.	energy var.	magnetic moment num.	magnetic moment var.
0	1.52	1.57	28	19.1
1	2.07	2.10	18	21.3
∞	2.70	2.72	16	19.5

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