COW test of the weak equivalence principle: A low-energy window to look into the noncommutative structure of space-time?

Anirban Saha

Department of Physics, West Bengal State University,Barasat, North 24 Paraganas, West Bengal, India

anirban@iucaa.ernet.in; COS.2, Oral, CICAHEP15.50.2

We construct the quantum mechanical model of an experiment by Colella, Overhauser and Werner (COW), where gravitationally induced quantum-mechanical phase shift in the interference between coherently split and separated neutron de Broglie waves are studied to demonstrated the validity of WEP, assuming that the underlying space time has a granular structure, described by a canonical noncommutative algebra of coordinates x^{μ}. The time-space sector of the algebra is shown to add a mass-dependent contribution to the gravitational acceleration felt by neutron de Broglie waves measured the experiment. This makes time-space noncommutativity a potential candidate which can cause a false-positive signature of the violation of WEP even if the ratio of the inertial mass m_i and gravitational mass m_g is a universal constant. We therefore argue that the COW-type experiments can be used as a probe for the evidence of NC structure of space-time.

1. Introduction

At the Planck scale the space-time is thought to have a granular structure that can be described by a noncommutative (NC) geometry with space-time coordinates x^{μ} satisfying the algebra

$$[x^{\mu}, x^{\nu}] = i\Theta^{\mu\nu}, \tag{1}$$

where $\Theta^{\mu\nu}$ is a constant anti-symmetric tensor. This idea of such NC space-time has gained interest in the recent past when it was commonly realized that the low energy effective theory of D-brane in the background of NS-NS B field lives on noncommutative manifold [1, 2]. Further, in the brane world scenario [3], our space-time may be the world volume of a D-brane, and thus can be described by noncommutative geometry (1). From the physical perspective as well, it has long been suggested that in the Gedanken experiment of localizing events in a space-time with Planck scale resolution, a sharp localization induces an uncertainty in the space-time coordinates which can be naturally described by the noncommutative geometry (1) [4]. Although effects of such a NC structure of space-time may only appear near the string/Planckian scale, we hope that some low energy relics of such effects may exist and their phenomenology can be explored at the level of quantum mechanics (QM) [5, 6, 7, 8, 9, 10].

2. The WEP and its experimental background

The structure of space-time may be best revealed through gravitational interaction. In fact, the central idea of Einstein's general theory of relativity (GTR) is based on the interpretation of gravity as a property of space-time, i.e. its curvature. This interpretation relies upon the Weak Equivalence Principle (WEP) which has its experimental foundation in the universality of free fall (UFF) that demands the following: That the ratio $\frac{m_i}{m_g} = \alpha$ between the inertial mass m_i and gravitational mass m_g, both appearing in the classical equation of motion

$$\ddot{x} = \frac{m_g}{m_i} g = \frac{g}{\alpha} = g' \tag{2}$$

of a freely falling "point like" particle immersed in the nearly homogeneous local gravitational field $g = \frac{GM_E}{{R_E}^2}$ caused by Earth's mass M_E is a universally constant. Here we have ignored the nominal height from ground level h with respect to the Earth's radius R_E. The effect of the Earth's gravitational field g is the gravitational acceleration of the particle $\ddot{x} = g' = \frac{g}{\alpha}$ which, if α indeed is a universal constant and does not vary from one particle species to another, is same for all kind of material particles.

As it happens, most theoretical attempts to connect GTR to standard model allow for violation of the WEP [11]. Naturally, the WEP has a long and persistent history of various kinds of experimental tests. Motivation for this stems from desire to gain insight into some alternative/modified version of GTR. In experimental tests of WEP with macroscopic objects we look for species-dependent value of the gravitational acceleration g' caused by change in the value of α for different particle species. In the Eötvös-type experiments possible violations are parametrised by the Eötvös factor, defined as

$$\eta\left(A,B\right)=\frac{\delta g'\left(A,B\right)}{g'_{average}\left(A,B\right)}=2\frac{\ddot{x}\left(A\right)-\ddot{x}\left(B\right)}{\ddot{x}\left(A\right)+\ddot{x}\left(B\right)} \tag{3}$$

for two macroscopic test masses made of materials A and B. Currently the lowest bound is reached for the elements Beryllium and Titanium, using rotating torsion balances [12], $\eta\left(\mathrm{Be},\mathrm{Ti}\right)<2.1\times10^{-13}$. Future tests like "MICRO-Satellite à traînée Compensée pour l'Observation du Principe d'Equivalence" (MICROSCOPE) [13], to be launched in 2016 aim at a lower bound of 10^{-15}. In the atomic/subatomic regime using improvements on the earlier Eötvös-type experiments, Dicke *et al* in 1961 concluded that neutrons and protons in nuclei experience the same gravitational acceleration g' within about $2\times10^{-9}g'$ [14] by comparing acceleration of PbC1 (neutron-proton ratio R = 1.45) and of Cu (R = 1.19). That a free neutron experiences the same g' it experiences within a nucleus was experimentally confirmed [15] in 1965 by measuring g' from the difference of fall of two well-collimated beams of high and low velocity neutrons while traversing a long evacuated horizontal flight path. A comparison of neutron scattering lengths, with measurement techniques both dependent[1] and independent[2] of gravity, also leads to a verification of the WEP [16] in 1976. However, these results, though obtained for free neutrons behaving as matter waves, are still a consequence of their classical parabolic path under gravity as required by the correspondence principle and hence no quantum features are involved.

The scenario changed during 1974 to 1979, when Colella, Overhauser and Werner (COW), in a series of experiments [17] demonstrated the validity of WEP using gravitationally induced quantum-mechanical phase shift in the interference between coherently split and separated neutron de Broglie waves at the 2 MW University of Michigan Reactor, the validity of equivalence principle in the so called "quantum limit" was claimed to have been examined. The verification was complimented in 1983 by repeating the experiment in an accelerated interferometer where gravitational effects are compensated [18]. This established that the Schrödinger equation in an accelerated frame predicts a phase shift which agrees with observation as assumed earlier by COW [19] for the validity of strong equivalence principle in the quantum limit. Since then, the WEP in the quantum domain has been verified, time and again, with ever increasing accuracy.

3. COW test and the NC space-time structure

Given the roll of WEP in attributing gravity as a property of the space-time, one may think of the COW experiments a test of the space-time property at the quantum level. Therefore, it will not be surprising if some trace of the granularity of the space-time structure that is believed to exist at the Plank scale resolution, manifest itself, even in the low energy regime where quantum mechanical tests of WEP are currently being performed. In this paper we therefore construct the quantum mechanical theory describing the COW experiment with the assumption that the underlying space-time that we live in has a NC structure described by the relation (1). Our motivation is to investigate if some manifestation of this NC geometry shows up in the observable results. Specifically, we work out the gravity-induced phase-shift which shows a leading order NC contribution. We shall argue that this NC term will lead to an apparent violation of WEP in COW-type test data. We also put forward a suggestion to trace this apparent violation of the WEP to its' NC origin if such COW-type experiments can be performed with different atomic/subatomic particles. That can serve as evidence of the NC structure of space-time.

4. Modeling the COW experiment in NC space-time

We start by discussing how to introduce the NC space-time structure in the system. Since in QM space and time could not be treated on an equal footing, we impose the geometry (1) at a field theoretic level and eventually reduce the theory to quantum mechanics[3]. This allows us to examine the effect of the whole sector of space-time

[1] Slow neutrons reflected from liquid mirrors after having fallen a hight.

[2] By transmission measurements on liquid prob.

[3] This is a reasonable starting point since single particle quantum mechanics can be viewed as the one-particle sector of quantum field theory in the very weakly coupled limit where the field equations are essentially obeyed by the Schrödinger wave function [6].

noncommutativity in an effective noncommutative quantum mechanical (NCQM) theory. Owing to the extreme smallness of the NC parameters the current/near future experiments can only hope to detect the first order NC effects. Since it has been demonstrated in various formulations of NC gravity [20] that the leading NC correction in the gravity sector is second order we can safely assume the Newtonian gravitational field g remains unaltered for all practical purpose.

The NC Schrödinger field theory describing cold neutron beams in Earth's gravitational field (along the x-axis) in a vertical xy $(i = 1, 2)$ plane is

$$\hat{S} = \int d^2xdt\ \hat{\psi}^\dagger \star \left[i\hbar\partial_0 + \frac{\hbar^2}{2m_i}\partial_i\partial_i - m_g g\hat{x}\right] \star \hat{\psi}. \tag{4}$$

Since there is no direct way to relate the physical observables to the NC operators in (4), we consider the NC fields $\hat{\psi}$ as functions in the deformed phase space where ordinary product is replaced by the star product [1, 6] which, for two fields $\hat{\phi}(x)$ and $\hat{\psi}(x)$, is given by

$$\hat{\phi}(x) \star \hat{\psi}(x) = \left(\hat{\phi} \star \hat{\psi}\right)(x) = e^{\frac{i}{2}\theta^{\alpha\beta}\partial_\alpha\partial'_\beta}\hat{\phi}(x)\hat{\psi}(x')\big|_{x'=x}. \tag{5}$$

Due to the linear form of the gravitational potential in action (4), expanding the star product and expressing everything in terms of commutative variables only gives corrections to first order in the NC parameters and all the higher order terms vanish. This leads to an equivalent commutative description of the original NC model in terms of the non-canonical action

$$\hat{S} = \int d^2xdt\psi^\dagger \left[i\hbar\left(1 - \frac{\eta}{2\hbar}m_g g\right)\partial_t + \frac{\hbar^2}{2m_i}{\partial_i}^2 - m_g gx - \frac{i}{2}m_g g\theta\partial_y\right]\psi, \tag{6}$$

where NC effect is manifest by the presence of NC parameter $\Theta^{10} = \eta$ among time and spatial directions. The term with spatial NC parameter $\Theta^{12} = \theta$ and first derivative ∂_y can be absorbed in the ${\partial_y}^2$ and is therefore inconsequential. We use a physically irrelevant rescaling[4] of the fields $\psi \mapsto \tilde{\psi} = \sqrt{\left(1 - \frac{\eta}{2\hbar}m_g g\right)}\psi$ to recast this non-canonical form of action with a conventionally normalized kinetic term such that the fields evolves in a canonical manner. This leads to

$$\hat{S} = \int d^2xdt\ \tilde{\psi}^\dagger \quad \left[i\hbar\partial_t + \frac{\hbar^2}{2m_i\left(1 - \frac{\eta}{2\hbar}m_g g\right)}{\partial_i}^2 - \frac{m_g gx}{\left(1 - \frac{\eta m_g g}{2\hbar}\right)}\right]\tilde{\psi}. \tag{7}$$

Comparing with the standard Schrödinger action we can immediately read off the observed inertial mass as $\tilde{m}_i = 2m_i\left(1 - \frac{\eta}{2\hbar}m_g g\right)$. Assuming the NC effect to be very small the interaction can be written in terms of this observed inertial mass $\tilde{m}_i$ as

$$\frac{m_g gx}{\left(1 - \frac{\eta m_g g}{2\hbar}\right)} = \tilde{m}_i g' x\left(1 + \frac{\eta\tilde{m}_i g'}{\hbar}\right), \tag{8}$$

where we have used equation (2) to replace $m_g g$ with $m_i g'$ [5]. The final form of the canonical action reads

$$\hat{S} = \int d^2xdt\ \tilde{\psi}^\dagger \quad \left[i\hbar\partial_t + \frac{\hbar^2}{2\tilde{m}_i}{\partial_i}^2 - \tilde{m}_i g' x\left(1 + \frac{\eta\tilde{m}_i g'}{\hbar}\right)\right]\tilde{\psi} \tag{9}$$

leading to the equation of motion

$$i\hbar\partial_t\tilde{\psi} = -\left[\frac{\hbar^2}{2\tilde{m}_i}{\partial_i}^2 + \tilde{m}_i g' x\left(1 + \frac{\eta\tilde{m}_i g'}{\hbar}\right)\right]\tilde{\psi} \tag{10}$$

that can be considered at the level of quantum mechanics with $\tilde{\psi}$ interpreted as the Schrödinger wave function.

[4] Since the experimental setup is confined to a small region of space-time where the local gravitational field g is essentially constant, this rescaling amounts to multiplying the field variable by a constant.

[5] Note that replacing $m_g g$ with $m_i g'$ follows from the definition of gravitational acceleration g' for an individual particle, as in (2), and not from the assumption of WEP. WEP is required when we assume that such accelerations for two separate particle species are identical for same gravitational field g.

Equation (10) describes the NCQM of a freely falling neutron in earth's gravity in terms of commutative variables. Thus equation (10) will serve as the desired theoretical model of the COW test constructed in a NC space-time. Note that since we have successfully expressed the equation (10) in terms of the commutative variables completely, bringing out the NC effect explicitly as an additional term in the process, so the principles of ordinary quantum mechanics readily apply to the equation. In the next section we use this equation in context of the COW experiment to calculate the quantum mechanical phase-shift induced by gravity.

5. Analysis of the model: NC effect in the COW phase-shift

We can readily derive the Ehrenfest relations

$$\frac{d}{dt} < x > = \frac{< p >}{\tilde{m}_i}, \tag{11}$$

$$\frac{d^2}{dt^2} < x > = g'\left(1 + \frac{\eta \tilde{m}_i g'}{\hbar}\right) = \tilde{g}' \tag{12}$$

for the average velocity and acceleration of the neutrons. Thus, though representing an NC system, this Schrödinger equation (10) behaves similar to that in ordinary/commutative space. However, the two crucial differences with the commutative result are

1. the appearance of observed inertial mass of the neutron $\tilde{m}_i$ in the average momentum (11) and
2. the observed gravitational acceleration $\tilde{g}'$ in (12) experienced by a quantum mechanically behaving system, namely the neutron, is now mass-dependent due to the NC structure of space-time.

Note that contrary to the common expectation that Ehrenfest theorem will lead to results mimicking classical behaviour i.e. a quantum mechanical wave packet will move, on an average, along a classical particle trajectory subject to the applied potential [21], here we have a observable quantum mechanical effect that is not washed out by the averaging process and shows up as a deviation from the classical trajectory. That this effect is of NCQM origin is established by the explicit appearance of the ratio $\frac{\eta}{\hbar}$.

In a COW-type experimental setting the gravitational potential is much smaller than the total energy of the neutrons and we can calculate the gravity induced phase-shift from (10) by the semi-classical prescription of matter-wave interferometry [22, 23]

$$\Delta\varphi_{grav} = -\frac{1}{\hbar}\tilde{m}_i \tilde{g}' \left(l_1 \sin\phi\right) \Delta t, \tag{13}$$

where ϕ is the tilt angle between the plane containing the coherently splitted neutron beams and the horizontal plane, giving rise to an effective height $l_1 \sin\phi$ of one of the neutron beam paths with respect to the other. Since the effective potential is time-independent here we can use the paraxial approximation to compute

$$\Delta t = l_2 / \frac{d}{dt} < x > = \frac{l_2 \tilde{m}_i \lambda_0}{h}, \tag{14}$$

where $\lambda_0 = h / < p >$ is the laboratory neutron de Broglie wavelength corresponding to the average neutron momentum $< p >$ in (11). Combining (13) and (14) we find

$$\Delta\varphi_{grav} = -\frac{A \sin\phi}{2\pi\hbar^2} \lambda_0 {\tilde{m}_i}^2 \tilde{g}', \tag{15}$$

where $A = l_1 l_2$ is the area enclosed by the interfering beams. This phase difference depends on the mass-dependent $\tilde{g}'$.

Comparing this theoretical prediction (15) with the experimentally measured gravity induced phase-shift one can obtain the *quantum mechanically observed* gravitational acceleration $\tilde{g}'\,(\mathrm{n})$ felt by a neutron. We intend to stress the quantum mechanical nature of the observation because phase-shift is a quantum phenomena and it is only in the quantum regime that any NC effect will be picked up. This data, when confronted with local classical gravitational acceleration g' measured with macroscopic bodies where no NC effect is possible, will exhibit a discrepancy given by

$$\frac{\delta g}{g_{\rm av}} = \frac{\tilde{g}'\left(\rm n\right) - g'}{g_{\rm av}} = \frac{g'\left(\rm n\right) - g'}{g_{\rm av}} + \frac{\eta \tilde{m}_i \left(g'\left(\rm n\right)\right)^2}{\hbar g_{\rm av}}. \tag{16}$$

Here $g'\left(\rm n\right) = \frac{g}{\alpha(\rm n)}$ is the acceleration the neutron would feel due to Earth's gravitational field g if our space-time followed the ordinary Hisenberg algebra instead of the NC algebra (1). The first term signifies the violation of the WEP, if any, caused by the non-universality of α, i.e. $\alpha\left(\rm n\right) \neq \alpha$ (macroscopic) and the second term arise as an effect of the NC structure of space-time showing an apparent violation even if α in equation (2) is a universal constant. This sets a limitation on the accuracy to which WEP can be verified at the quantum limit by COW experiments on ultra-cold neutrons.

6. Conclusion

In principle the apparent violation due to NC effect should be identifiable if the COW-type experiments can be performed with different atomic/subatomic particle species. With the first term vanishing/negligible in equation (16), the discrepancy for different species will vary linearly with their masses and the slope $\frac{\eta g'}{\hbar}$ will give the absolute value of the NC parameter. Such a linear variation of discrepancy with particle mass, if indeed observed, will serve to establish the granular structure of the space-time we live in. Of course this holds only if any true violation due to non-universality of α occurs beyond the accuracy level where the NC effect starts affecting the data. In the best case scenario the COW-type experiments and its other variants such as atom-interferometer based on fountain of laser-cooled atoms [24], may open a low-energy "window" to reveal the noncommutative structure of space-time.

7. Acknowledgement

The author would like to thank Patrick Das Gupta for enlightening comments and also the financial support of DST SERB under Grant No. SR/FTP/PS-208/2012.

References

[1] N. Seiberg, E. Witten, [JHEP **09** (1999) 032]; A.A. Bichl*et al*, [hep-th/0102103]
[2] See for example, R.J. Szabo, Phys. Rept. **378** (2003) 207 and the referances therein.
[3] I. Antoniadis, N. Arkani-Hamed, S Dimopoulos, G. Dvali, Phys. Lett. **B 436** 257-263,1998.
[4] S. Doplicher, K. Fredenhagen and J. E. Roberts, Phys. Lett. **B 331** 39 (1994); S. Doplicher, K. Fredenhagen and J. E. Roberts, Commun. Math. Phys. 172, 187 (1995); D.V. Ahluwalia, Phys. Lett. **B 339** 301-303, 1994.
[5] M. Chaichian, M.M. Sheikh-Jabbari, and A. Tureanu, Phys. Rev. Lett. **86**, 2716 (2001).
[6] B. Chakraborty, S. Gangopadhyay, A. Saha, Phys. Rev. **D 70** 107707 (2004).
[7] A. Saha, S. Gangopadhyay, Phys. Lett.**B 681** 96 (2009).
[8] A. Saha, S. Gangopadhyay, S. Saha, Phys. Rev.**D 83** (2011) 025004.
[9] O. Bertolami, J. G. Rosa, C. M. L. de Aragao, P. Castorina, D. Zappala, Phys. Rev. **D 72** (2005) 025010.
[10] A. Saha, Eur. Phys. J.**C 51** 199 (2007).
[11] T. D. Lee and C. N. Yang, Phys. Rev. **98** 1501 (1955); Y. Fujii, Nature (London), Phys. Sci. 234, 5 (1971); P. Fayet, Phys. Lett. **B 171** 261 (1986); T. R. Taylor and G. Veneziano, Phys. Lett. **B 213** 450 (1988); T. Damour and A. M. Polyakov, Nucl. Phys. **B 423** 532 (1994); S. Dimopoulos and G. F. Giudice, Phys. Lett. **B 379** 105 (1996); T. Damour, Class. Quantum Grav. 13 A33 (1996).
[12] S. Schlamminger *et al*, Phys. Rev. Lett. **100** 041101 (2008).
[13] http://microscope.onera.fr/
[14] R. H. Dicke, Sci. Am. 205, 84 (1961); R. H. Dicke, in Relativity, Groups and Topology, edited by C. DeWitt and B. DeWitt (Gordon and Breach Science Publishers, Inc., New York, 1964).
[15] J. W. T. Dabbs, J. A. Harvey, D. Paya, and H. Horstmann, Phys. Rev. **139** B756 (1965).

[16] L. Koester, Phys. Rev. **D 14** 907 (1976).
[17] A. W. Overhauser and B.Colella, Phys. Rev. Lett. 33, 1237 (1974); R.Collela*et al*, Phys. Rev. Lett. 34, 1472 (1975); S. A. Werner *et al*, Phys. Rev. Lett. **42** 1103 (1979).
[18] U. Bonse and T. Wroblewski, Phys. Rev. Lett. **51** 1401, (1983).
[19] J.-L. Staudenmann and S. A. Werner, R. Colella and A.W. Overhouser, Phys. Rev. **A 21** 1419 (1983).
[20] P. Mukherjee, A. Saha, Phys. Rev. **D 74** (2006) 027702 and the references therein.
[21] J.J. Sakurai, *Modern Quantum Mechanics*, Addison Wesley, 2000.
[22] Paul R. Berman, *Atom Interferometry*, Academic Press, 1997.
[23] A. D. Cronin, J. Schmiedmayer and D. E. Pritchard, Rev. Mod. Phys. **81** 1051 (2009).
[24] A. Peters, K. Y. Chung and S. Chu, Nature **400** 849 (1999).