BUMPY INFLATION: SUBLEADING EFFECTS IN AXION INFLATION

Ivonne Zavala
Department of Physics, Swansea University, Swansea, SA2 8PP, UK

An attractive inflaton candidate is an axion slowly rolling down a flat potential protected by a perturbative shift symmetry. However, realisations of axion inflation within natural and monomial inflation have been disfavoured by observations and are difficult to embed in string theory. We show that subleading, but significant non-perturbative corrections to the axion potential can superimpose sharp cliffs and gentle plateaus into the potential, whose overall effect is to enhance the number of e-folds of inflation. Sufficient inflation is achieved for smaller field ranges compared to the potential without such corrections and the tensor-to-scalar ratio results un-observably small, but there is a large negative running of the spectral index. This brings both single-field chaotic and natural inflation in UV complete theories like string theory, back into the favoured region of current observations with distinctive signatures.

1 Introduction

The latest results from Planck and BICEP2/Keck [1] agree with the simplest inflationary scenario where a period of quasi exponential acceleration is driven by a single scalar field, slowly rolling down a very flat potential, whose quantum fluctuations seeded the large scale structures observed today. The conditions on the inflaton potential for a sufficiently long epoch of slow-roll inflation are:

$$\epsilon_V \equiv \frac{M_{Pl}^2}{2}\left(\frac{V'}{V}\right)^2 \ll 1, \qquad |\eta_V| \equiv M_{Pl}^2 \frac{|V''|}{V} \ll 1\,. \tag{1}$$

The challenge in slow-roll inflation is that higher order corrections, encoded in higher dimensional operators, $\mathcal{O}_d = c_d V(\phi)(\phi/M_{Pl})^{d-4}$ generically steepen the potential and spoil the slow-roll conditions (1). Moreover, for even moderately large $\phi \gtrsim M_{Pl}$, Planck suppressed operators are dangerously large, and can prevent inflation from occurring at all.

Invoking a symmetry is an attractive possibility to prevent large quantum corrections. Such a symmetry is provided, for example, if the inflaton is an axion with a shift symmetry. When the symmetry is broken non-perturbatively to a discrete symmetry, the potential is sufficiently flat for slow-roll *natural inflation* [7], for large values of the symmetry breaking scale, or axion decay constant, $f \gtrsim 4M_{Pl}$, and requires super-Planckian field displacements. When the symmetry is broken, e.g. spontaneously, one has an axionic realisation of monomial inflation, whose attempted string theoretic embeddings are known as *axion monodromy* [8].

Axion models give vanilla natural or monomial inflation only to leading order. Whilst the effective potential is protected from dangerous perturbative corrections, non-perturbative effects like instantons give further contributions of the form $\sum_n \Lambda_n^4 \cos(n\phi/f)$, where Λ_n are mass scales. Then large and frequent modulations are introduced into the potential, trapping the inflaton in a local minimum before it reaches the global one, and obstructing large numbers of e-folds of slow-roll natural inflation [9]. On the other hand, when non-perturbative corrections correspond to tiny, frequent superimposed features in the slow-roll potential, their impact on the background trajectory of the inflaton is negligible, whilst leaving only small imprints on the CMB such as a large, possibly oscillating running of the scalar spectral index [10].

Assuming slow-roll from the time the observed perturbations in the CMB exited the horizon up to the end of inflation, the amplitude of the tensor modes, encoded in the tensor-to-scalar ration, r, can be related to the inflaton field excursion via the Lyth bound [2] and the inflationary scale, giving

$$\frac{\Delta\phi}{M_{Pl}} \gtrsim 2 \times \left(\frac{r}{0.01}\right)^{1/2}, \qquad V_{inf}^{1/4} \simeq \left(\frac{r}{0.1}\right)^{1/4} \times 1.8 \times 10^{16}\ \text{GeV}\,. \tag{2}$$

Therefore, an observation of primordial gravitational waves with $r \sim 10^{-1} - 10^{-2}$ would fix the scale of inflation to be around the GUT scale and the inflaton field range to be super-Planckian implying that inflation is highly sensitive to quantum gravity effects. UV completions of high scale/large field inflation via string theory are difficult, due to the proximity of the inflationary scale (2) and the string scale, which is typically $M_s \lesssim 10^{17}$ GeV for perturbative string theory. This proximity puts under pressure the validity of 4D effective field theory during inflation, where, in order to be able to neglect massive string excitations and Kaluza-Klein modes, a hierarchy $V_{inf}^{1/4} \lesssim M_{kk} \lesssim M_s \lesssim M_{Pl}$ is required [4,6], M_{kk} being the compactification scale.

Here we show that non-perturbative corrections to axion models of inflation not only allow inflation, but can even help it with important implications on the field ranges, the decay constant and cosmological predictions. We quote here the relevant measurements and bounds on the CMB observables from Planck 2015/BICEP2-Keck [1]. For the scalar perturbations, the Planck analysis gives (including the tensor perturbations at the pivot scale $k_* = 0.05\text{Mpc}^{-1}$ and 68%CL for Planck TT+lowP): $n_s = 0.9667 \pm 0.0066\,, \alpha_s \equiv dn_s/d\ln k = -0.0126^{+0.0098}_{-0.0087}\,, r < 0.07$ (at 95% CL).

2 Bumpy Chaotic Inflation

We consider the following axionic monomial potential with non-perturbative corrections [11,12]

$$V(\phi) = A + \frac{1}{2}m^2\phi^2 + \lambda\phi\cos\left(\frac{\phi}{f}\right)\,. \tag{3}$$

For $\lambda = A = 0$, we recover ϕ^2 inflation, which gives a field range of $\Delta\phi \sim 15\, M_{Pl}$, with a tensor to scalar ratio $r \sim 0.12$ and an inflationary scale of around the GUT scale (2). While it gives a consistent value for the spectral index $n_s \sim 0.966$, this model is basically excluded by the latest results on r.

We consider the interesting case $\lambda/f < m^2$. Taking e.g. $m^2/d^4 = 10\, M_{Pl}^2$, $f = 1/3\, M_{Pl}$, and $\lambda/d^4 = 3.3\, M_{Pl}^3$ (where we have scaled the parameters by $d^2 = 9.3 \times 10^{-8}$ to match the normalisation of scale perturbations $P_s \simeq 2.1 \times 10^{-9}$ and for convenience with the numerics, also $t \to t/d^2$). We draw the potential in Fig. 1, together with the corresponding smooth ϕ^2 model with $\lambda = 0$. One can now easily compute the CMB observables from the slow-roll parameters at horizon crossing for the pivot scale: $n_s = 1 - 4\epsilon + 4\delta\,, \alpha_s = -8\xi + 20\epsilon\delta - 8\epsilon^2\,, r = 16\epsilon\,,$ where $\epsilon \equiv 2\frac{H'^2}{H^2}$, $\delta \equiv \frac{H''}{H}$, $\xi\frac{H'''H'}{H^2}$, H is the Hubble parameter and a prime denotes derivative with respect to the inflaton.

Assuming the pivot scale crossed the horizon at around $N_e \sim 55$ e-folds before the end of inflation yield the following values for the inflationary observables

$$n_s = 0.9667\,, \qquad r = 3.1 \times 10^{-5}\,, \qquad \alpha_s = -0.015\,, \tag{4}$$

in the ballpark of the Planck measurements and constraints given by Planck15. The field range for the bumpy model from horizon crossing to the end of inflation is $\Delta\phi \sim 3.2\, M_{Pl}$, and the scale of inflation at horizon crossing is $V_{inf}^{1/4} = 9.9 \times 10^{-4} M_{Pl}$. Thus, non-perturbative corrections can modify a large field monomial model of inflation to one with intermediate field range and inflationary scale, making a consistent perturbative string theory realisation of the model possible. The distinctive signature of such a scenario is a suppressed tensor mode and a large negative running of the spectral index.

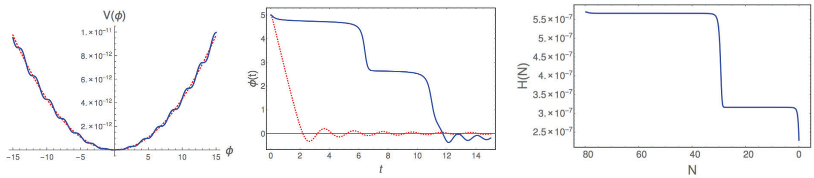

Figure 1 – Scalar potential (in Planck units) and inflaton's evolution in time for the smooth and bumpy chaotic model and Hubble parameter's evolution as a function of the number of e-folds N for the bumpy chaotic model.

3 Bumpy Natural Inflation

We now consider a single field modulated natural potential, which is generically predicted by string theory models:

$$V(\phi) = A + \Lambda^4 \left(1 + \cos\left(\frac{\phi}{f}\right)\right) + \tilde{\Lambda}^4 \left(1 + \cos\left(\frac{\phi}{\tilde{f}}\right)\right) \tag{5}$$

where $\tilde{\Lambda} < \Lambda$ and $\tilde{f} < f$ parameterise the bumps.

For example, choosing $\Lambda^4/d^4 = 1\,M_{Pl}^4$, $f = 1\,M_{Pl}$ and $\tilde{f} = 1/3\,M_{Pl}$, we tune $\tilde{\Lambda}^4/d^4 = 0.3329\,M_{Pl}^4$ (where now $d^2 = 9.1\times10^{-8}$) to ensure the turns in the potential are close to stationary points. The bumpy model again has a step-like shape, with steep regions connected by a plateau. The potential and scalar field's evolution for the smooth and bumpy models are given in Fig. 2 as well as the Hubble parameter's evolution as a function of the number of e-folds. Almost all the inflation proceeds in a slow-roll fashion, although there are large fluctuations in the slow-roll parameters when rolling down the steep slopes of the steps. Taking the pivot scale to cross the horizon at $N_e \sim 54$ e-folds before the end of inflation yield the following values for the cosmological observables:

$$n_s = 0.9677\,, \quad r = 3.5 \times 10^{-7}\,, \quad \alpha_s = -0.0025\,, \tag{6}$$

in agreement with the Planck measurements. As in the monomial model, tensor modes are undetectably small, whereas there is a large negative running of the spectral index. The field range from horizon crossing to the end of inflation is $\Delta\phi = 1.0\,M_{Pl}$ and the scale of inflation at horizon crossing is $V_{inf}^{1/4} = 3.2 \times 10^{-4} M_{Pl}$. We reiterate that non-perturbative corrections have made it possible to achieve single field, natural inflation with a Planckian axion decay constant, $f \sim M_{Pl}$, moderate field range and intermediate inflationary energy scale. Thus a consistent embedding in perturbative string theory becomes possible.

4 Conclusions

Axions provide an interesting class of inflationary models which are protected from dangerous perturbative corrections to the effective potential. Non-perturbative corrections typically introduce modulations into monomial and natural inflaton potentials. Whereas it was previously assumed that such bumps would either spoil slow-roll inflation [9] or produce negligible corrections to the slow-roll dynamics [8,10,11], we have seen that when the modulations take the form a series of steep cliffs and gentle plateaus, the large Hubble friction (or drag) and the sharp reduction

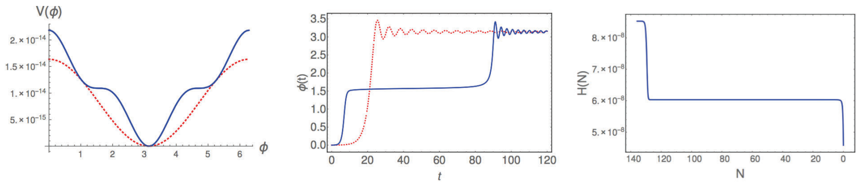

Figure 2 – Scalar potential and field evolution for the smooth and bumpy natural models and Hubble's parameter evolution as a function of the number of e-folds for the bumpy model. We use Planck units.

in acceleration at the end of the cliffs, cause the inflaton to roll slowly whenever it reaches a plateau. Hence, although slow-roll parameters have large fluctuations through the sharp cliffs, the plateaus provide regions of slow-roll that produce many e-folds of inflation.

Consequently, both single field monomial and natural inflation models can give sufficient e-folds for sub-Planckian axion decay constants, moderate field ranges and inflationary scales, when non-perturbative effects are included. This puts them back into the favoured region of current observations and the weakly coupled, supergravity limit of string theory. Such a scenario has distinctive signatures. In particular, the benchmark models considered here predict tensor modes 2-4 orders of magnitude below the projected bounds of future observations, $r \lesssim 10^{-3}$. Moreover, bumpy models also predict large, negative values for the running of the scalar spectral index over the scales probed by the CMB ($\alpha_s \approx -10^{-2}, -10^{-3}$, respectively), and small running of the running ($\beta_s \approx -10^{-4}, -10^{-5}$, respectively).

We considered simple potentials within effective field theory, and it would be important to understand whether the potentials and parameters emerging from string compactifications can fulfil the requisite tunings, such as for example in the models discussed in [12]. Also, a more detailed study of the inflationary observables would be very interesting, for example consequences of the running spectral index on all the scales probed by the CMB, and implications of the bumps for non-Gaussianities.

Acknowledgments.

I thank my collaborators on the topics discussed here, K. Kooner, N. Cabo-Bizet, O. Loaiza-Brito, S. Parameswaran and G. Tasinato.

References

1. **Planck Collaboration** Collaboration, P. Ade *et al.*, `arXiv:1502.02114 [astro-ph.CO]`. **BICEP2, Keck Array** Collaboration, P. Ade *et al.*, `arXiv:1510.09217 [astro-ph.CO]`.
2. D. H. Lyth, *Phys.Rev.Lett.* **78** (1997) 1861–1863, `arXiv:hep-ph/9606387 [hep-ph]`.
3. N. Arkani-Hamed et al. *JHEP* **06** (2007) 060, `arXiv:hep-th/0601001 [hep-th]`.
4. K. Kooner, S. Parameswaran, and I. Zavala, `arXiv:1509.07049 [hep-th]`.
5. S. Parameswaran et al. JCAP **1604** (2016) no.04, 008, `arXiv:1602.02812`.
6. S. Parameswaran and I. Zavala, to appear.
7. K. Freese, J. A. Frieman, and A. V. Olinto, *Phys. Rev. Lett.* **65** (1990) 3233–3236.
8. E. Silverstein et al. *Phys. Rev.* **D78** (2008) 106003, `arXiv:0803.3085 [hep-th]`. L. McAllister et al. *Phys. Rev.* **D82** (2010) 046003, `arXiv:0808.0706 [hep-th]`.
9. T. Banks et al. *JCAP* **0306** (2003) 001, `arXiv:hep-th/0303252 [hep-th]`.
10. T. Kobayashi et al. *JCAP* **1101** (2011) 026, `arXiv:1011.3988 [astro-ph.CO]`.
11. R. Flauger et al. `1412.1814 [hep-th]`; T. Kobayashi et al. `1510.08768 [hep-ph]`.
12. N.G. Cabo-Bizet, O. Loaiza-Brito and I. Zavala, to appear.