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A Generic Analysis of Nucleon Decay Branching Fractions in Flipped SU(5) Grand Unification

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Abstract: In flipped SU(5) grand unified theories, the partial decay lifetimes of certain nucleon decay channels depend generically on an unknown unitary matrix, which arises when left-handed lepton fields are embedded into anti-fundamental representations of SU(5). This dependency is particularly relevant when the neutrino mass matrix has a generic structure, introducing uncertainty in the prediction of nucleon decay branching fractions within flipped SU(5). In this paper, we demonstrate that this uncertainty can be parametrized using two parameters, which can be determined by measuring the partial lifetimes of $p \rightarrow \pi^0 e^+$, $p \rightarrow \pi^0 \mu^+$, and $n \rightarrow \pi^0 \bar{\nu}$. In addition, we establish upper limits on the ratios of the decay widths of these channels, offering a potential method to test flipped SU(5) in future nucleon decay experiments.

Keywords: grand unified theories; nucleon decay; flipped SU(5); supersymmetry



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1. Introduction

Flipped SU(5) \times U(1) grand unified theories (GUTs) [1–4] offer a promising framework for constructing GUT models. This framework includes right-handed neutrinos as an essential component and naturally explains small neutrino masses via the seesaw mechanism [5–9], in contrast to the standard SU(5) [10], where right-handed neutrinos are not required, as they are singlets of SU(5) and do not complete the SU(5) representations. In addition, flipped SU(5) incorporates the missing-partner mechanism [3,11,12] to naturally solve the doublet-triplet splitting problem.

Flipped SU(5) predicts a distinctive pattern of nucleon decay branching fractions [13–23]. The missing-partner mechanism suppresses the contribution of color-triplet Higgs exchange [24] to dimension-five operators [25,26], which contrasts sharply with the minimal supersymmetric SU(5) [27,28], where dimension-five proton decay operators could be problematic [29,30] unless the supersymmetric particle mass scale is very high [31–45]. Its contribution to dimension-six operators is also small [24] if the color-triplet Higgs lies around the GUT scale. Consequently, the SU(5) gauge boson exchange predominantly drives nucleon decays in flipped SU(5). Since this process is induced by gauge interactions, it allows for relatively robust predictions of certain nucleon decay branching fractions. For instance, the decay width of $p \rightarrow K^+ \bar{\nu}$ is predicted to vanish due to the unitarity of the Cabibbo–Kobayashi–Maskawa (CKM) matrix [13]. Ratios of decay widths, such as $\Gamma(p \rightarrow K^0 e^+)/\Gamma(p \rightarrow \pi^0 e^+)$ and $\Gamma(p \rightarrow K^0 \mu^+)/\Gamma(p \rightarrow \pi^0 \mu^+)$, are determined by the CKM matrix elements [17], along with the relevant hadron masses and matrix elements.

However, some important decay channels suffer from uncertainties in their decay branching fraction calculations, even though these processes are induced by gauge interactions. This uncertainty arises from an unknown unitary matrix U_ℓ , introduced when embedding left-handed lepton fields into anti-fundamental representations of SU(5) [13].

As discussed in Ref. [17], in some specific scenarios as in Refs. [46–48], this unitary matrix corresponds to the Pontecorvo–Maki–Nakagawa–Sakata (PMNS) matrix, which allows us to make a robust prediction. Generically speaking, however, unknown matrix elements, $(U_\ell)_{11}$ and $(U_\ell)_{12}$, appear in the expressions for the decay widths, causing uncertainty in the calculation. It is therefore crucial to study the impact of this uncertainty on the predicted nucleon decay branching fractions and to evaluate whether flipped SU(5) can still be tested in upcoming nucleon decay experiments [49], such as the Hyper–Kamiokande experiment [50].

In this paper, we demonstrate that this uncertainty can be parametrized using only two parameters and these parameters can be determined by measuring the lifetimes of the three nucleon decay channels, $p \rightarrow \pi^0 e^+$, $p \rightarrow \pi^0 \mu^+$, and $n \rightarrow \pi^0 \bar{\nu}$. We also derive upper limits on the ratios between the decay widths of these channels, providing a means to test flipped SU(5) in future experiments.

2. Nucleon Decay in Flipped SU(5)

We first describe the flipped SU(5) model considered in this paper. This model is a supersymmetric (SUSY) $SU(5) \times U(1)$ gauge theory, where the three generations of matter fields in the minimal supersymmetric Standard Model (MSSM), together with three right-handed neutrinos, are embedded into **10**, **5**, and **1** representations of $SU(5)$, which we denote by F_i , \bar{f}_i and ℓ_i^c , respectively, with $i = 1, 2, 3$ the generation index. Their $U(1)$ charges in units of $1/\sqrt{40}$ are $+1$, -3 , and $+5$, respectively. We also add singlet fields ϕ_i to give masses to the right-handed neutrinos after the $SU(5) \times U(1)$ gauge symmetry is spontaneously broken. This symmetry breaking is triggered by the vacuum expectation values (VEVs) of a pair of **10** and **10** Higgs fields, H and \bar{H} , whose $U(1)$ charges are $+1$ and -1 , respectively. The MSSM Higgs doublet fields, H_u and H_d are embedded into anti-fundamental and fundamental representations of $SU(5)$ with $U(1)$ charges $+2$ and -2 ; these representations are denoted by \bar{h} and h , respectively. We further assume that this model possesses two \mathbb{Z}_2 symmetries. One is the standard R -parity, under which F_i , \bar{f}_i , ℓ_i^c , and ϕ_i are odd and H , \bar{H} , h , and \bar{h} are even. Another one is to remove the mass term for H and \bar{H} , where only the H field is odd and the other fields are even¹; the absence of this mass term is advantageous for suppressing the dimension-five nucleon-decay operators induced by the color-triplet Higgs exchange. We summarize the field content and charge assignments in Table 1.

Table 1. The field content and the charge assignments in the flipped SU(5) model. The $U(1)$ charges are given in units of $1/\sqrt{40}$.

Fields	Components	SU(5)	U(1)	\mathbb{Z}_2	R -parity
F_i	d_i^c, Q_i, v_i^c	10	+1	+	–
\bar{f}_i	u_i^c, L_i	5	–3	+	–
ℓ_i^c	e_i^c	1	+5	+	–
ϕ_i	ϕ_i	1	0	+	–
H	d_H^c, Q_H, v_H^c	10	+1	–	+
\bar{H}	$d_{\bar{H}}^c, Q_{\bar{H}}, v_{\bar{H}}^c$	10	–1	+	+
h	D, H_d	5	–2	+	+
\bar{h}	\bar{D}, H_u	5	+2	+	+

The renormalizable superpotential in this model is given by

$$W = \lambda_1^{ij} F_i F_j h + \lambda_2^{ij} F_i \bar{f}_j \bar{h} + \lambda_3^{ij} \bar{f}_i \ell_j^c h + \lambda_4 H H h + \lambda_5 \bar{H} \bar{H} \bar{h} + \lambda_6^{ij} F_i \bar{H} \phi_j + \mu^{ij} \phi_i \phi_j, \quad (1)$$

where we assume the μ -term for h and \bar{h} to be absent² so that the doublet-triplet splitting problem can be solved by the missing-partner mechanism.

The scalar potential of this model has an F - and D -flat direction along $|\nu_H^c| = |\nu_{\bar{H}}^c|$ and we assume that these fields develop VEVs in this direction to break the $SU(5) \times U(1)$ gauge symmetry—the origin of this potential is destabilized by a soft SUSY-breaking mass term and the flat direction at large field values is lifted by a non-renormalizable superpotential term. See Refs. [51,52] for more detailed discussions on this symmetry breaking. After H and \bar{H} acquire VEVs, the color components in these fields, d_H^c and $d_{\bar{H}}^c$, form vector-like mass terms with those in h and \bar{h} , D and \bar{D} , via the couplings λ_4 and λ_5 in Equation (1), while the MSSM doublet Higgs fields remain massless—i.e., the doublet-triplet splitting is realized. Q_H , $Q_{\bar{H}}$, and a linear combination of ν_H^c and $\nu_{\bar{H}}^c$ are eaten by the gauge fields corresponding to the broken symmetries, giving them masses of the order of the GUT scale. The other combination of ν_H^c and $\nu_{\bar{H}}^c$ acquires a mass of the order of the soft SUSY-breaking scale. The VEVs of H and \bar{H} also generate mass terms for the right-handed neutrinos ν_i^c and the singlet fields ϕ_i via the couplings λ_6^{ij} . The integration of these fields then gives rise to the dimension-five Weinberg operators [53], which generate neutrino masses after H_u acquires a VEV.

To see the flavor structure of this model, we take a flavor basis where λ_2^{ij} is real and diagonal without loss of generality. In this case, we find that the MSSM matter fields and right-handed neutrinos are embedded into F_i , \bar{f}_i , and ℓ_i^c as

$$\begin{aligned} F_i &= \left\{ Q_i, V_{ij} e^{-i\varphi_j} d_j^c, (U_{\nu^c})_{ij} \nu_j^c \right\}, \\ \bar{f}_i &= \left\{ u_i^c, (U_\ell^*)_{ij} L_j \right\}, \\ \ell_i^c &= (U_{\ell^c})_{ij} e_j^c, \end{aligned} \quad (2)$$

where V is the CKM matrix, U_{ν^c} , U_ℓ , and U_{ℓ^c} are unitary matrices, and φ_i are real phase parameters satisfying the condition $\sum_i \varphi_i = 0$. For the detailed procedure to obtain this result, see Refs. [13,18]. The doublet quark and lepton fields are expressed in terms of the mass eigenstates as

$$Q_i = \begin{pmatrix} u_i \\ V_{ij} d_j \end{pmatrix}, \quad L_i = \begin{pmatrix} U_{ij} \nu_j \\ e_i \end{pmatrix}, \quad (3)$$

where U is the PMNS matrix. We also need another unitary matrix U_ν to diagonalize the mass matrix of light neutrinos m_ν :

$$m_\nu^D = U_\nu^T m_\nu U_\nu. \quad (4)$$

We find that the PMNS matrix is related to the unitary matrices U_ℓ and U_ν as

$$U = U_\ell^T U_\nu. \quad (5)$$

Notice that if the neutrino mass matrix m_ν is predicted, as in the case of the scenario considered in Refs. [46–48], U_ν is obtained from Equation (4) and then U_ℓ is given by Equation (5) as a function of the PMNS matrix. Otherwise, we have ambiguity in the determination of U_ℓ and U_ν .

Now we discuss the nucleon decay in this model. The exchange of the $SU(5)$ gauge bosons generates the effective operators of the form

$$\mathcal{L}_{\text{eff}} = C^{ijkl} \mathcal{O}_{ijkl} + \text{h.c.}, \quad (6)$$

where

$$\mathcal{O}_{ijkl} \equiv \int d^2\theta d^2\bar{\theta} \epsilon_{abc} \epsilon_{\alpha\beta} (u_i^{c\dagger})^a (d_j^{c\dagger})^b e^{-\frac{2}{3}g'B} (e^{2g_3G} Q_k^\alpha)^c L_l^\beta, \quad (7)$$

with G and B the $SU(3)_C$ and $U(1)_Y$ gauge vector superfields, respectively, and g_3 and g' the corresponding gauge couplings. Notice that another type of the dimension-six

operators allowed by the Standard Model gauge symmetry is not induced in flipped SU(5), as the right-handed charged lepton fields e_i^c are singlets of SU(5). We also note that as the interaction (6) is induced by the gauge interaction, the following analysis is applicable not only to the SUSY flipped SU(5) model but also to non-SUSY ones. The Wilson coefficients C^{ijkl} are given by

$$C^{ijkl} = \frac{g_5^2}{M_X^2} (U_\ell^*)_{il} V_{kj}^* e^{i\varphi_j}, \quad (8)$$

where M_X is the mass of the SU(5) gauge bosons and g_5 is the SU(5) gauge coupling.

By using the effective interactions in Equation (6), we compute the nucleon decay widths, with taking the renormalization-group effect into account. For the detailed discussion on this prescription, see Ref. [17]. In what follows, we summarize the resultant expressions given in Ref. [17].

Let us begin with the $p \rightarrow \pi^0 \ell_i^+$ decay channels, where ℓ_1^+ and ℓ_2^+ denote positron and anti-muon, respectively. The decay widths of these channels are expressed as

$$\Gamma(p \rightarrow \pi^0 \ell_i^+) = \frac{g_5^4 m_p |V_{ud}|^2 |(U_\ell)_{1i}|^2}{32\pi M_X^4} \left(1 - \frac{m_\pi^2}{m_p^2}\right)^2 A^2 \left(\langle \pi^0 | (ud)_R u_L | p \rangle_{\ell_i} \right)^2, \quad (9)$$

where m_p and m_π denote the masses of proton and pion, respectively; A is a renormalization factor³; the quantity in the last parenthesis is the hadron matrix element, for which we use the results obtained from the QCD lattice simulation in Ref. [54]:

$$\begin{aligned} \langle \pi^0 | (ud)_R u_L | p \rangle_e &= \frac{1}{\sqrt{2}} \langle \pi^+ | (ud)_R d_L | p \rangle_e = -0.112 \text{ GeV}^2, \\ \langle \pi^0 | (ud)_R u_L | p \rangle_\mu &= \frac{1}{\sqrt{2}} \langle \pi^+ | (ud)_R d_L | p \rangle_\mu = -0.114 \text{ GeV}^2. \end{aligned} \quad (10)$$

These matrix elements are obtained at the scale of 2 GeV in the $\overline{\text{MS}}$ scheme. Generically, the current precision of the calculation of the hadron matrix elements is $\mathcal{O}(10)\%$ [54]. As we see, the decay widths (9) depend on the matrix elements $(U_\ell)_{1i}$.

There are other decay channels which are related to the above channels. For example, the neutron decay channels $n \rightarrow \pi^- \ell_i^+$ are simply related to the above channels via SU(2) isospin relations as

$$\Gamma(n \rightarrow \pi^- \ell_i^+) = 2\Gamma(p \rightarrow \pi^0 \ell_i^+). \quad (11)$$

We note that the current limits on the $n \rightarrow \pi^- \ell_i^+$ decay lifetimes are [55]

$$\tau(n \rightarrow \pi^- e^+) > 5.3 \times 10^{33} \text{ years}, \quad (12)$$

$$\tau(n \rightarrow \pi^- \mu^+) > 3.5 \times 10^{33} \text{ years}, \quad (13)$$

which are weaker than the corresponding proton decay channels by more than a factor of two [56]:

$$\tau(p \rightarrow \pi^0 e^+) > 2.4 \times 10^{34} \text{ years}, \quad (14)$$

$$\tau(p \rightarrow \pi^0 \mu^+) > 1.6 \times 10^{34} \text{ years}. \quad (15)$$

Namely, the $p \rightarrow \pi^0 \ell_i^+$ channels impose stronger constraints than the $n \rightarrow \pi^- \ell_i^+$ channels.

The decay channels containing π^0 in the final state is related to those with η as

$$\frac{\Gamma(p \rightarrow \eta \ell_i^+)}{\Gamma(p \rightarrow \pi^0 \ell_i^+)} = \frac{(1 - m_\eta^2/m_p^2)^2}{(1 - m_\pi^2/m_p^2)^2} \frac{\left(\langle \eta | (ud)_R u_L | p \rangle_{\ell_i} \right)^2}{\left(\langle \pi^0 | (ud)_R u_L | p \rangle_{\ell_i} \right)^2}. \quad (16)$$

Notice that these ratios are independent of unknown parameters such as M_X , g_5 , A , and $(U_\ell)_{1i}$. These ratios are found to be numerically small. For example, for the positron mode⁴, the hadron matrix element is computed as [57]

$$\langle \eta | (ud)_R u_L | p \rangle_e = 0.006 \text{ GeV}^2, \quad (17)$$

with which the ratio is estimated to be as small as 1.3×10^{-3} . The current limits on the η channels are [55]

$$\tau(p \rightarrow \eta e^+) > 1.0 \times 10^{34} \text{ years}, \quad (18)$$

$$\tau(p \rightarrow \eta \mu^+) > 4.7 \times 10^{33} \text{ years}, \quad (19)$$

and thus the η channels give much weaker constraints on flipped SU(5) than the pion channels.

For the proton decay channels including a neutral kaon in the final state, their decay widths are given by

$$\Gamma(p \rightarrow K^0 \ell_i^+) = \frac{g_5^4 m_p |V_{us}|^2 |(U_\ell)_{1i}|^2}{32\pi M_X^4} \left(1 - \frac{m_K^2}{m_p^2}\right)^2 A^2 \left(\langle K^0 | (us)_R u_L | p \rangle_{\ell_i}\right)^2, \quad (20)$$

where m_K is the kaon mass and [54]

$$\begin{aligned} \langle K^0 | (us)_R u_L | p \rangle_e &= 0.0854 \text{ GeV}^2, \\ \langle K^0 | (us)_R u_L | p \rangle_\mu &= 0.0860 \text{ GeV}^2. \end{aligned} \quad (21)$$

Again, we can remove the dependence on unknown parameters by taking the ratios,

$$\frac{\Gamma(p \rightarrow K^0 \ell_i^+)}{\Gamma(p \rightarrow \pi^0 \ell_i^+)} = \frac{|V_{us}|^2 (1 - m_K^2/m_p^2)^2}{|V_{ud}|^2 (1 - m_\pi^2/m_p^2)^2} \frac{(\langle K^0 | (us)_R u_L | p \rangle_{\ell_i})^2}{(\langle \pi^0 | (ud)_R u_L | p \rangle_{\ell_i})^2} \simeq 0.02. \quad (22)$$

The current experimental limits on these channels are [58,59]

$$\tau(p \rightarrow K^0 e^+) > 1.0 \times 10^{33} \text{ years}, \quad (23)$$

$$\tau(p \rightarrow K^0 \mu^+) > 3.6 \times 10^{33} \text{ years}, \quad (24)$$

which are much weaker than the limits on the π^0 channels. We note in passing that the effective Lagrangian in Equation (6) does not induce $n \rightarrow K^- \ell_i^+$.

All in all, for the ℓ_i^+ channels, $p \rightarrow \pi^0 e^+$ and $p \rightarrow \pi^0 \mu^+$ provide most sensitive probes of flipped SU(5). The decay widths of these channels depend on the unknown matrix elements $(U_\ell)_{11}$ and $(U_\ell)_{12}$, respectively. The decay rates of the other ℓ_i^+ channels are related to these two channels through the multiplication of constant factors, as we described above.

Next, we discuss the anti-neutrino channels. As nucleon-decay experiments are unable to detect the anti-neutrino in the final state, we take the sum over all the neutrino generations when we calculate the decay widths. For the $p \rightarrow \pi^+ \bar{\nu}$ channel, we have

$$\Gamma(p \rightarrow \pi^+ \bar{\nu}) \equiv \sum_i \Gamma(p \rightarrow \pi^+ \bar{\nu}_i) = \frac{g_5^4 m_p}{32\pi M_X^4} \left(1 - \frac{m_\pi^2}{m_p^2}\right)^2 A^2 \left(\langle \pi^+ | (ud)_R d_L | p \rangle_e\right)^2. \quad (25)$$

As we see, this expression does not depend on U_ℓ ; this follows from the unitarity condition $\sum_i |(U_\ell)_{1i}|^2 = 1$. The $n \rightarrow \pi^0 \bar{\nu}$ channel is related to this channel by isospin and we find

$$\Gamma(n \rightarrow \pi^0 \bar{\nu}) = \frac{1}{2} \Gamma(p \rightarrow \pi^+ \bar{\nu}). \quad (26)$$

The current experimental constraints on these channels are [60]

$$\tau(p \rightarrow \pi^+ \bar{\nu}) > 3.9 \times 10^{32} \text{ years}, \quad (27)$$

$$\tau(n \rightarrow \pi^0 \bar{\nu}) > 1.1 \times 10^{33} \text{ years}. \quad (28)$$

We see that in this case the limit on the neutron decay channel is better than that on the proton decay channel by more than a factor of two. We thus consider $n \rightarrow \pi^0 \bar{\nu}$, rather than $p \rightarrow \pi^+ \bar{\nu}$, in the following analysis.

Finally, it follows from the unitarity of the CKM matrix that the kaon channels are not induced by the interaction in Equation (6) [13]:

$$\Gamma(p \rightarrow K^+ \bar{\nu}) = \Gamma(n \rightarrow K^0 \bar{\nu}) = 0. \quad (29)$$

This is a characteristic prediction of flipped SU(5), and, in particular, the detection of these modes excludes flipped SU(5). This is in stark contrast to the case of the minimal SUSY SU(5), where $p \rightarrow K^+ \bar{\nu}$ induced by the exchange of the color-triplet Higgs tends to be the dominant decay channel [61,62].

In conclusion, to examine the prediction of flipped SU(5) in future nucleon-decay experiments, the $p \rightarrow \pi^0 e^+$, $p \rightarrow \pi^0 \mu^+$, and $n \rightarrow \pi^0 \bar{\nu}$ channels are most useful. We will discuss in the subsequent section that we can determine the unknown matrix elements $|(U_\ell)_{11}|$ and $|(U_\ell)_{12}|$ by measuring the partial decay widths of these channels.

3. Results

In what follows, we consider the ratios

$$R_{\ell_i} \equiv \frac{\Gamma(p \rightarrow \pi^0 \ell_i^+)}{\Gamma(n \rightarrow \pi^0 \bar{\nu})} = 2|V_{ud}|^2 |(U_\ell)_{1i}|^2 \frac{(\langle \pi^0 | (ud)_R u_L | p \rangle_{\ell_i})^2}{(\langle \pi^+ | (ud)_R d_L | p \rangle_e)^2} = |V_{ud}|^2 |(U_\ell)_{1i}|^2, \quad (30)$$

where in the last equation we use Equation (10) and neglect the small difference in the values of $\langle \pi^0 | (ud)_R u_L | p \rangle_e$ and $\langle \pi^0 | (ud)_R u_L | p \rangle_\mu$. This expression clearly shows that we can determine $|(U_\ell)_{1i}|$ by measuring the ratio R_{ℓ_i} . By using the unitarity of U_ℓ , we also obtain the following inequalities:

$$R_{\ell_i} \leq |V_{ud}|^2, \quad (31)$$

$$R_e + R_\mu \leq |V_{ud}|^2, \quad (32)$$

with $|V_{ud}|^2 \simeq 0.95$ [63]. These inequalities are useful to discriminate the GUT models. For example, if the nucleon decay operators are induced predominantly by the SU(5) gauge boson exchange in the standard SU(5), R_e is predicted to be [17]⁵

$$R_e|_{\text{SU}(5)} = \frac{1 + (1 + |V_{ud}|^2)^2}{|V_{ud}|^2} \simeq 5, \quad (33)$$

which clearly violates the above inequalities. This observation indicates that the measurement of the lifetimes of only two decay channels, $p \rightarrow \pi^0 e^+$ and $n \rightarrow \pi^0 \bar{\nu}$, is already capable of distinguishing this scenario from flipped SU(5). Let us also show R_μ in this case just for completeness [17]:

$$R_\mu|_{\text{SU}(5)} = |V_{us}|^2 \simeq 0.05, \quad (34)$$

which is consistent with the condition (31).

As discussed in Ref. [17], in the flipped SU(5) scenario considered in Refs. [46–48], the matrix elements $(U_\ell)_{11}$ and $(U_\ell)_{12}$ are given by the PMNS matrix elements as

$$(U_\ell)_{11} = \begin{cases} U_{11} & \text{for normal order} \\ U_{13} & \text{for inverted order} \end{cases}$$

$$(U_\ell)_{12} = \begin{cases} U_{21} & \text{for normal order} \\ U_{23} & \text{for inverted order} \end{cases} . \quad (35)$$

These relations hold also in the case where the neutrino mass matrix m_ν in Equation (4) is almost diagonal. We can test this prediction through the measurement of R_{ℓ_i} .

To see the dependence of R_{ℓ_i} on the unknown matrix elements $(U_\ell)_{11}$ and $(U_\ell)_{12}$, it is convenient to parametrize them by means of two real parameters, ξ and ϕ . The unitarity condition,

$$|(U_\ell)_{11}|^2 + |(U_\ell)_{12}|^2 + |(U_\ell)_{13}|^2 = 1 , \quad (36)$$

indicates that it is possible to parametrize these matrix elements as⁶

$$|(U_\ell)_{11}| = \cos \phi \cos \xi , \quad |(U_\ell)_{12}| = \sin \phi \cos \xi , \quad |(U_\ell)_{13}| = \sin \xi , \quad (37)$$

with $0 \leq \phi, \xi \leq \pi/2$.

In Figure 1, we show the contour plots of (Figure 1a) R_e and ((Figure 1b) R_μ in the ϕ - ξ plane. The blue and red crosses correspond to the choice in Equation (35) with the normal and inverted orderings, respectively, which is the case for the scenario considered in Refs. [46–48]. We use NuFIT 5.3 [65] without the inclusion of the tabulated Super-Kamiokande atmospheric data to compute the PMNS matrix elements⁷.

$$R_e = \begin{cases} 0.64 & \text{for normal order} \\ 0.021 & \text{for inverted order} \end{cases}$$

$$R_\mu = \begin{cases} 0.072 & \text{for normal order} \\ 0.54 & \text{for inverted order} \end{cases} . \quad (38)$$

As we see, R_e and R_μ play complementary roles in probing the parameter space. It is also found that both R_e and R_μ are suppressed for $\xi \simeq \pi/2$; in this case, $p \rightarrow \pi^+ \bar{\nu}$ and $n \rightarrow \pi^0 \bar{\nu}$ become the dominant channel for proton and neutron decays, respectively. This pattern of nucleon decay is distinctive. In the minimal SUSY SU(5), for instance, the decay rate of $p \rightarrow \pi^+ \bar{\nu}$ can be much larger than those of $p \rightarrow \pi^0 \ell^+$ if the contribution of the dimension-five operators dominates the dimension-six one (see, e.g., Refs. [34,42]). However, in this case, the rate of the $p \rightarrow K^+ \bar{\nu}$ decay channel also increases, whilst this does not occur in flipped SU(5). We can thus distinguish flipped SU(5) from the standard SU(5) even for small ξ , by looking into the $p \rightarrow \pi^+ \bar{\nu}$ (or $n \rightarrow \pi^0 \bar{\nu}$) and $p \rightarrow K^+ \bar{\nu}$ channels.

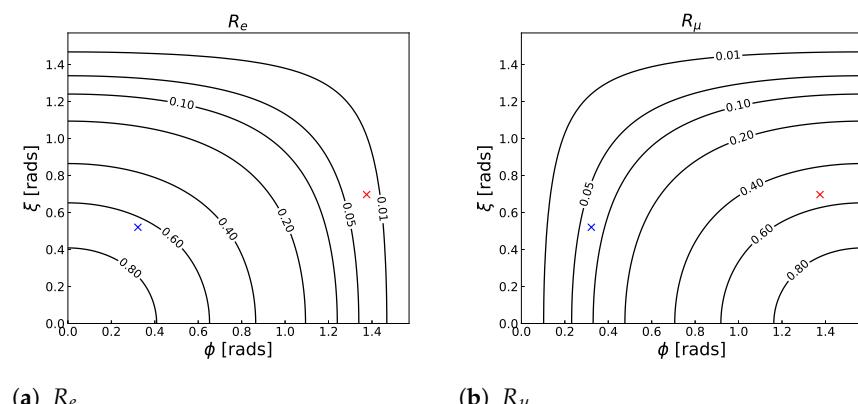


Figure 1. Contour plots of (a) R_e and (b) R_μ in the ϕ - ξ plane. The blue and red crosses correspond to the choice in Equation (35) with the normal and inverted orderings, respectively.

4. Conclusions and Discussion

We have examined the pattern of nucleon decay branching fractions in flipped SU(5). Since nucleon decay is induced by the gauge interactions in flipped SU(5), we can make relatively robust predictions for certain branching fractions, such as $\Gamma(p \rightarrow K^0 e^+)/\Gamma(p \rightarrow \pi^0 e^+)$ and $\Gamma(p \rightarrow K^0 \mu^+)/\Gamma(p \rightarrow \pi^0 \mu^+)$. Another notable prediction is the suppression of the $p \rightarrow K^+ \bar{\nu}$ channel. However, the calculation of some decay channels is affected by uncertainties due to an unknown unitary matrix U_ℓ . We have shown that this uncertainty can be parametrized using two real parameters, ϕ and ξ . These parameters can be determined by measuring the ratios R_e and R_μ , as illustrated in Figure 1. Additionally, we have derived upper limits on these ratios, which can be useful for testing flipped SU(5) in future nucleon decay experiments.

Extended scenarios for flipped SU(5), as discussed in Refs. [18–20,23], could present different patterns of nucleon decay branching fractions than those considered in this paper. The model dependence is encapsulated in the Wilson coefficients of the nucleon-decay effective operators at low energies, with their chirality and flavor structures influencing the nucleon decay channels. By measuring nucleon decays, we can examine these structures, allowing us to distinguish between different GUT scenarios. A detailed and comprehensive study on this topic will be presented in future work.

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Notes

- 1 We assume that this symmetry is violated by some Planck-scale suppressed operators in order to prevent the formation of stable domain walls.
- 2 It is possible to construct a flipped SU(5) model where this μ -term is forbidden by an R -symmetry [18].
- 3 Since only one type of the effective operator (7) is induced in the present case, all of the decay widths in question have the same renormalization factor, A . This is contrary to the case of the standard SU(5), where two types of the effective operators are induced by the gauge-boson exchange which are renormalized differently. For more detail, see Ref. [17].
- 4 We do not show the value for the μ^+ channel, since, as stated in Ref. [54], the treatment of the matrix elements for μ^+ in Ref. [57] is inappropriate. In any case, we expect that the difference from the e^+ case is $\mathcal{O}(m_\mu/m_p)$, with m_μ the muon mass, and thus it does not affect our conclusion.
- 5 The gauge boson exchange in the standard SU(5) induces two-types of dimension-six effective operators. The renormalization factors for these operators are slightly different; in Equation (33), we neglect this difference, which turns out to be a good approximation for a typical low-energy mass spectrum [17].
- 6 It is possible to extend this parametrization to the whole matrix elements with additional seven real parameters. In general, any 3×3 unitary matrix can be parameterized by three mixing angles and six phases, and we can identify two of the three mixing angles as ϕ and ξ . For a concrete expression, see, e.g., Ref. [64].
- 7 It is found that $|U_\ell|_{1i}$ is independent of the unknown Majorana phases in the PMNS matrix.

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