CP-odd and CP-even weak-basis invariants in the presence of vector-like quarks

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Abstract

We propose a minimal set of weak-basis invariants in an extension of the standard model (SM) where one up-type isosinglet vector-like quark is introduced, which allows us to obtain all the physical content of the CKM matrix. We present CP-odd invariants of lower order in mass than the one in the SM, which may have important consequences for Baryogenesis. We study the ECL, where the two lightest generations have vanishing mass, showing that in this extension, contrary to the SM, CP violation can be observed in collisions much above the electroweak scale.

Keywords: vector-like quarks, weak-basis invariants, CP violation, baryogenesis, extreme chiral limit

1. Introduction

The addition of vector like quarks (VLQ) to the spectrum of the standard model (SM) is one of the simplest extensions of the SM and VLQs may populate the desert between the SM and the Planck scale, without worsening the hierarchy problem. Among the best motivated VLQ-models with either one down-type [1] or one up-type [2, 3] isosinglet VLQ are those that

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attempt at solving the CKM unitarity problem [4–12]. There are also models which feature both up and down-type isosinglet VLQs [13, 14] and models which feature VLQs in different SU(2) representations [15].

The study of weak-basis invariants (WBIs) has been carried out both in the quark sector [16–18] and the lepton sector [19] (see [20] for a more recent analysis). In this paper, we study WBIs in an extension of the SM where an up-type SU(2) singlet VLQ is introduced to the SM. In particular, we are interested in identifying a set of WBIs which enables one to reconstruct the full 4×3 quark mixing matrix which arises in this extension of the SM. Let us recall that it has been shown [21] that, in the SM with 3 generations and squared mass matrices $h_i = m_i m_i^\dagger$, $i = u, d$, for the up and down quarks, the WBIs $\mathrm{tr}(h_u h_d)$, $\mathrm{tr}(h_u^2 h_d)$, $\mathrm{tr}(h_u h_d^2)$ and $\mathrm{tr}(h_u^2 h_d^2)$, enable one to obtain the four independent moduli of the CKM matrix. With the knowledge of these moduli, one can reconstruct the full CKM, including the strength of CP violation. There is only a two-fold ambiguity in the sign of CP violation which can be lifted by the evaluation of the CP-odd WBI $\mathrm{tr}[h_u, h_d]^3$ [16].

Here, we analyse the analogous question in the context of an extension of the SM with an up-type VLQ. In this case, there are nine independent moduli and we identify the nine WBIs which enable one to reconstruct the full quark mixing matrix. Special attention is given to the extreme chiral limit (ECL) where the first two generations may be considered massless. In this limit, there is no CP violation in the SM. We will show that in the VLQ extension there is a CP violation, even in the ECL, and identify CP-odd WBI which do not vanish in this limit.

One of the reasons why in the SM it is not possible to generate a baryon asymmetry of the Universe (BAU) consistent with observation, has to do with the fact that CP-violation in the SM is too small. A rough estimate is

$$\mathcal{I}_{\mathrm{CP}} = \mathrm{tr}[y_u y_u^\dagger, y_d y_d^\dagger]^3 \sim \frac{\mathrm{tr}[h_u, h_d]^3}{v^{12}} \sim 10^{-25}, \tag{1}$$

where y_u and y_d are the Yukawa-coupling matrices, which are related to the quark mass matrices as $y_u = \frac{1}{v} m_u$, $y_d = \frac{1}{v} m_d$ and v is the electro-weak scale. The SM-CP-odd WBI, $\mathrm{tr}[h_u, h_d]^3$ has a mass order of M^{12}. However, in our extension of the SM with the up-VLQ there is a CP-odd WBI of much lower order in mass, namely, of order M^8. Therefore, one expects that the contribution to BAU in the framework of models with VLQs to be much larger than in the SM.

The paper is organised as follows: in section 2 we present a set of WBIs which allow us to obtain all the information contained in the CKM matrix. In section 3, we study CP violation (in this extension of the SM) and show that we get a much bigger contribution to CP violation than in the SM due to lower order in mass CP-odd invariants. In section 4, we study the ECL, where at extremely high energies it is considered that $m_u = m_c = 0$ and $m_d = m_s = 0$, present a parametrization for the CKM matrix in this limit and show that we have CP violation, even at these energies. In the Appendices, we give a general setup of invariants for more than one VLQ and construct a simplified set of invariants completely characterizing all parameters for the case of one up-VLQ. Finally, in section 5, we present our conclusions.

2. Weak-basis invariants

We consider a model with 3 families of SM-like quarks and one additional up-type isosinglet VLQ (1-VLQ). In this model, there are 16 physical parameters in the quark sector: 4 related to the masses of the up-type quarks, 3 related to the masses of the down-type quarks and 9 related to the mixing. Following [2, 3], after spontaneous symmetry breaking, we write the

quark mass Lagrangian as

$$\mathcal{L}_M = -\bar{u}_{Li}(m_u)_{i\alpha}u_{R\alpha} - \overline{U}_{L1}(M_u)_{1\alpha}u_{R\alpha} - \bar{d}_{Li}(m_d)_{ij}d_{Rj}, \qquad (2)$$

where the indices i and j run from 1 to 3 and the index α runs from 1 to 4, u_{Li} denotes the 3 left-handed up-type SM quarks, $u_{R\alpha}$ are the 4 right-handed up-type quarks (the right-handed up-type SM quarks and the right-handed VLQ are indistinguishable as they have the same quantum numbers), U_{L1} is the 1 left-handed up-type VLQ, d_{Li} is the 3 left-handed down-type SM quarks and d_{Rj} is the 3 right-handed down-type SM quarks. The matrix m_u is a 3×4 matrix, the matrix M_u is a 1×4 matrix (i.e. it is a row vector) and the matrix m_d is a 3×3 matrix. The 4×4 up-type quark mass matrix is given by

$$\mathcal{M}_u = \begin{pmatrix} m_u \\ M_u \end{pmatrix}. \qquad (3)$$

2.1. Weak-basis transformations

In order to construct the WB invariants, we have to specify the WB tranformations. Under a general weak-basis transformation, the quark fields transform as

$$\begin{aligned} u_L &\to V\ u_L & d_L &\to V\ d_L \\ u_R &\to \mathcal{W}_u\ u_R & d_R &\to W_d\ d_R, \\ U_L &\to Z\ U_L \end{aligned} \qquad (4)$$

where the unitary V, W_d are 3×3 matrices, while $\mathcal{W}_u$ is a 4×4 unitary matrix. Notice that V appears simultaneously in the left-handed fields of the up and down sector, thus the charged current Lagrangian term is also left invariant. For one up-VLQ, Z is just some complex phase factor $e^{i\alpha}$, however here, we maintain a matrix-form formulation for Z, which can be easily generalized to a matrix in the case of more than one VLQ's.

These transformation properties for the quark fields imply that the mass matrices of the quarks transform under a weak-basis transformation as

$$m_u \to V^\dagger m_u \mathcal{W}_u, \quad M_u \to Z^\dagger M_u \mathcal{W}_u, \quad m_d \to V^\dagger m_d W_d. \qquad (5)$$

Hence, we have

$$\begin{aligned} m_u m_u^\dagger &\to V^\dagger m_u m_u^\dagger V & m_d m_d^\dagger &\to V^\dagger m_d m_d^\dagger V \\ m_u M_u^\dagger M_u m_u^\dagger &\to V^\dagger m_u M_u^\dagger M_u m_u^\dagger V & m_d^\dagger m_d &\to W_d^\dagger m_d^\dagger m_d W_d \\ M_u M_u^\dagger &\to Z^\dagger M_u M_u^\dagger Z \\ m_u^\dagger m_u &\to \mathcal{W}_u^\dagger m_u^\dagger m_u \mathcal{W}_u, & M_u^\dagger M_u &\to \mathcal{W}_u^\dagger M_u^\dagger M_u \mathcal{W}_u \end{aligned} \qquad (6)$$

Taking traces of appropriate combinations of the matrices in equation (6), one obtains different weak-basis invariant quantities. We must now find a minimal (and comprehensive) set of these quantities, such these can be easily related to the elements of the CKM matrix and to the quark masses, and thus obtain adequate information which relates both.

2.2. Constructing the invariants

The up-quark mass matrix is diagonalized by a bi-unitary transformation of the fields such that

$$\begin{pmatrix} u \\ U \end{pmatrix}_L \to \mathcal{V}\, u_L', \quad u_R \to \mathcal{W}\, u_R' \qquad (7)$$

where $\mathcal{V}$, $\mathcal{W}$ are unitary 4×4 matrices and

$$\mathcal{V}^\dagger \mathcal{M}_u \mathcal{W} = \mathcal{D}_u. \tag{8}$$

Defining for the up-quarks, $\mathcal{H}_u = \mathcal{M}_u \mathcal{M}_u^\dagger$, we have four invariants related to the masses

$$\text{tr}(\mathcal{H}_u) = m_u^2 + m_c^2 + m_t^2 + m_T^2,$$

$$\chi_1(\mathcal{H}_u) \equiv \frac{1}{2} \frac{\partial^2}{\partial x^2} \det(\mathcal{H}_u - x \mathbb{1}_{4 \times 4})|_{x=0}$$

$$= m_u^2 m_c^2 + m_u^2 m_t^2 + m_u^2 m_T^2 + m_c^2 m_t^2 + m_c^2 m_T^2 + m_t^2 m_T^2,$$

$$\chi_2(\mathcal{H}_u) \equiv -\frac{\partial}{\partial x} \det(H_u - x \mathbb{1}_{4 \times 4})|_{x=0}$$

$$= m_u^2 m_c^2 m_t^2 + m_u^2 m_c^2 m_T^2 + m_u^2 m_t^2 m_T^2 + m_c^2 m_t^2 m_T^2,$$

$$\det(\mathcal{H}_u) = m_u^2 m_c^2 m_t^2 m_T^2, \tag{9}$$

where m_u, m_c and m_t are the masses of the up-type SM quark and m_T is the (heavy) mass of the up-type VLQ. A similar set of three invariants defines the down-quark masses, where instead of χ_1 and χ_2 we just have

$$\chi(h_d) \equiv -\frac{\partial}{\partial x} \det(h_d - x \mathbb{1}_{3 \times 3})|_{x=0} = m_d^2 m_s^2 + m_d^2 m_b^2 + m_s^2 m_b^2. \tag{10}$$

where h_d is the equivalent 3×3 squared down-quark matrix.

We write the unitary matrix which diagonalizes the up-quark matrix (on the left) $\mathcal{V}$ as

$$\mathcal{V} = \begin{pmatrix} A \\ B \end{pmatrix}, \tag{11}$$

where A and B are 3×4 and 1×4 matrices, respectively. With these, we obtain (in the WB where the down-quark matrix is diagonal) the charged current Lagrangian

$$\mathcal{L}_W = \frac{g}{\sqrt{2}} W_\mu^+ \overline{u'}_L A^\dagger \gamma^\mu d'_L + \text{h.c.} \tag{12}$$

Thus we find that (in this model), the CKM matrix is given by the 4×3 non-unitary matrix

$$V_{\text{CKM}} = A^\dagger. \tag{13}$$

Furthermore, we obtain for the neutral current Lagrangian

$$\mathcal{L}_Z = \frac{g}{c_W} Z_\mu \left[\frac{1}{2} (\overline{u'}_L A^\dagger A \ \gamma^\mu u'_L - \overline{d'}_L \gamma^\mu d'_L) - s_W^2 \left(\frac{2}{3} \overline{u'} \gamma^\mu u' - \frac{1}{3} \overline{d'} \gamma^\mu d' \right) \right], \tag{14}$$

which means that, in the charge $2/3$ up-quark sector, we will have flavour-changing neutral currents (FCNCs) at tree-level. We define the FCNC-matrix

$$F^u \equiv A^\dagger A = V_{\text{CKM}} V_{\text{CKM}}^\dagger, \tag{15}$$

which, as known, in the SM is equal to the identity. However, here and in general for VLQ-models, it is not a diagonal matrix, thus leading to the FCNCs.

Due to the unitarity of $\mathcal{V}$ we have

$$AA^\dagger = \mathbb{1}_{3 \times 3}, A^\dagger A + B^\dagger B = \mathbb{1}_{4 \times 4},$$

$$BB^\dagger = 1, \quad AB^\dagger = 0 \Leftrightarrow BA^\dagger = 0. \tag{16}$$

Note that, although the CKM matrix is no longer unitary (in fact, it is not even a square matrix), we do still have $V_{\text{CKM}}^{\dagger} V_{\text{CKM}} = \mathbb{1}_{3 \times 3}$.

From equations (8, 11), the diagonalized up-type quark mass matrix $\mathcal{D}_u$ is given by

$$\mathcal{D}_u = A^{\dagger} m_u \mathcal{W} + B^{\dagger} M_u \mathcal{W}. \tag{17}$$

By multiplying equation (17) on the left by A or B and on the right by $\mathcal{W}^{\dagger}$, and using the identities from equation (16), we obtain

$$m_u = A \, \mathcal{D}_u \mathcal{W}^{\dagger}, \quad M_u = B \, \mathcal{D}_u \mathcal{W}^{\dagger}. \tag{18}$$

2.2.1. A set of Hermitian matrices relating physical parameters.   Now, consider the following Hermitian matrices

$$H_u^{(r)} \equiv m_u (m_u^{\dagger} m_u + M_u^{\dagger} M_u)^{r-1} m_u^{\dagger},$$
$$h_d \equiv m_d m_d^{\dagger} \tag{19}$$

All these Hermitian matrices transform under weak-basis transformations (WBT) in (4) as

$$h \rightarrow V^{\dagger} h \, V. \tag{20}$$

From equations (16, 18), in the WB where the down sector mass matrix is diagonal, these verify

$$H_u^{(r)} = A (\mathcal{D}_u^2)^r A^{\dagger},$$
$$h_d = \text{diag}(m_d^2, m_s^2, m_b^2) \tag{21}$$

and, with $A^{\dagger} = V_{\text{CKM}}$, we find the following relations between the WBIs and the CKM matrix moduli and the quark masses

$$\text{tr}(H_u^{(r)} h_d^s) = \sum_{\alpha=1}^{4} \sum_{i=1}^{3} (m_{u_\alpha}^2)^r (m_{d_i}^2)^s |V_{\alpha i}|^2, \tag{22}$$

where s and r are integers.

Henceforth we will use the simplified notation[1] $h_u \equiv H_u^{(1)} = m_u m_u^{\dagger}$.

Finally, we compute 9 invariants obtained from equation (22). To do this, we choose the pair (r, s) to be equal to (1, 0), (1, 1), (1, 2), (1, 3), (2, 0), (2, 1), (2, 2), (3, 0) and (3, 1). Knowing the masses of the quarks through the WBIs in equation (9), we obtain 9 independent equations of order equal or lower than M^8 for the 9 independent moduli of the 4×3 CKM matrix elements. From the knowledge of the 9 independent moduli, it is possible to know all the information contained in the CKM matrix up to a discrete ambiguity[2], as it was done in [22].

Thus, up to a discrete ambiguity, we obtain all the information contained in the CKM matrix by computing these 9 invariants.

As an example, in appendix B, we have explicitly constructed a comprehensive set of simplified invariants (which can be easily verified) and which completely characterizes all parameters of this extension of the SM with one up-VLQ. For a general setup concerning invariants for more than one VLQ, we refer to appendix A.

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[1] Note that $H_u^{(r)}$ corresponds to the upper-left 3×3 block of $\mathcal{H}_u^r$. The analogous quantity in the SM would be $h_u^r = (m_u m_u^{\dagger})^r$, but for these type of extensions, for $r \neq 1$, these are strictly different, i.e. $h_u^r \neq H_u^{(r)}$, even though both transform in the same way under WBTs.

[2] This discrete ambiguity is similar to what happens in the SM, where, when computing the four WBIs which determine the four independent moduli of the CKM matrix, we still have to fix the ambiguity of the sign of CP violation.

3. CP violation and consequences for baryogenesis

The argument we used in the last section to show that one can determine all parameters in an extension with a VLQ isosinglet in terms of WBIs was solely dependent on real quantities. However, the existence of CP violation (CPV) hinges on the existence of CP-odd WBIs.

In the SM the simplest (i.e. lowest order) quantity with such a property is [16]

$$
\begin{aligned}
\mathrm{tr}[h_u, h_d]^3 = {} & 6i(m_t^2 - m_c^2)(m_t^2 - m_u^2)(m_c^2 - m_u^2) \\
& \times (m_b^2 - m_s^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2)I_{\mathrm{CP}},
\end{aligned}
\tag{23}
$$

where

$$
I_{\mathrm{CP}} \equiv \mathrm{Im}\,(V_{\alpha i} V_{\beta j} V_{\alpha j}^* V_{\beta i}^*)
\tag{24}
$$

and $V_{\alpha i} V_{\beta j} V_{\alpha j}^* V_{\beta i}^*$ correspond to any invariant quartet ($i \neq j$, $\alpha \neq \beta$), which in the SM all have the same imaginary part up to a sign. Usually, in the literature, the quantity I_{CP} with $\alpha = u$, $\beta = c$, $i = s$ and $j = b$, is called Jarlskog invariant.[3]

When VLQs are introduced to the theory, in general more physical phases are present. For instance, in the case of the SM extended with an up-type VLQ isosinglet ($N = 1$) there exist three physical phases and therefore extra sources of CP violation. This then results in a larger set of necessary and sufficient conditions for CP invariance.

From [24], a minimal set of seven CP-odd WBI is:

$$
\begin{aligned}
\mathrm{tr}([h_u, h_d^s]H_u^{(2)}) &= 2im_{d_i}^{2s} m_{u_\alpha}^4 m_{u_\beta}^2 \,\mathrm{Im}\,(F_{\alpha\beta}^u V_{\alpha i}^* V_{\beta i}), \\
\mathrm{tr}([h_u^2, h_d^s]H_u^{(2)}) &= 2im_{d_i}^{2s} m_{u_\alpha}^4 m_{u_\beta}^2 m_{u_\rho}^2 \,\mathrm{Im}\,(F_{\alpha\rho}^u F_{\rho\beta}^u V_{\alpha i}^* V_{\beta i}), \\
\mathrm{tr}(\{h_u, h_d\}[h_d^2, H_u^{(2)}]) &= 2im_{d_i}^4 m_{u_\alpha}^2 m_{u_\beta}^4 \,\mathrm{Im}\,[V_{\alpha i} V_{\beta i}^* (m_{d_j}^2 V_{\alpha j}^* V_{\beta j} - m_{d_i}^2 F_{\beta\alpha}^u)], \\
\mathrm{tr}(\{H_u^{(2)}, h_d^2\}[h_d, H_u^{(2)}]) &= 2im_{d_i}^4 m_{u_\alpha}^2 m_{u_\beta}^2 m_{u_\rho}^4 \,\mathrm{Im}\,[V_{\alpha i} V_{\rho i}^* F_{\beta\alpha}^u (m_{d_j}^2 V_{\beta j}^* V_{\rho j} - m_{d_i}^2 F_{\rho\beta}^u)], \\
\mathrm{tr}[h_u, h_d]^3 &= 6im_{d_i}^2 m_{d_j}^4 m_{u_\alpha}^2 m_{u_\beta}^2 m_{u_\rho}^2 \,\mathrm{Im}\,(V_{\alpha i} V_{\beta j} V_{\alpha j}^* V_{\rho i}^* F_{\rho\beta}^u),
\end{aligned}
\tag{25}
$$

for $s = 1,2$ and with an implicit sum over the quark indices (with α, β, $\rho = 1, 2, 3, 4$ and i, $j = 1, 2, 3$). The vanishing of this set of CP-odd invariants is a necessary and sufficient condition for CP invariance.

The fact that CP violation now depends on a variety of invariants instead of a single one could mean that in the presence of a VLQ there exists an enhancement of CP violation, since even if the standard CP-odd invariant vanished, i.e. $\mathrm{tr}[h_u, h_d]^3 = 0$, there could exist CP violation arising from other sources. Not only that, but in principle these new sources could lead to a much larger CP violation than the one predicted in the SM. This is because now there are several CP-odd WBIs of dimension lower than the SM invariant in equation (23), of dimension M^{12}. In fact, the invariant of lowest dimension is $\mathrm{tr}([h_u, h_d]H_u^{(2)})$, of dimension M^8 which is necessarily[4] the lowest order for CP-odd invariants in these type of theory.

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[3] Jarlskog stated that $\det[h_u, h_d] = 0$ was a necessary and sufficient condition for CP invariance [23]. However, this is only valid for a model with 3 quark generations, as, in this case, we have $\mathrm{tr}[h_u, h_d]^3 = 3\det[h_u, h_d]$. The condition $\mathrm{tr}[h_u, h_d]^3 = 0$ is a necessary condition for CP invariance for any number of quark generations. Thus, this condition (derived in [16]) is more general.

[4] In this extension, when building WBIs using Hermitian matrices, as for instance h_u and h_d, as building blocks, all resulting invariants have even mass dimension M^{2n}. Now, any invariant of dimension M^2 is simply the trace of an Hermitian matrix, therefore real and providing no information regarding CPV. For dimension M^4 invariants the problem persists, because the product of two Hermitian matrices still has real trace. Similarly, dimension M^6 invariants can, in this case, also only be written as products of two Hermitian matrices, e.g. $\mathrm{tr}(h_d^2 h_u)$ or $\mathrm{tr}(h_d H_u^{(2)})$, leaving us with M^8 as the lowest possible dimension for CP-odd invariants.

Similarly to the SM quantity in equation (1), we expect that for a model with a VLQ, the size of CP violation is dominated by a dimensionless quantity such as

$$\mathcal{I}_{\mathrm{CP}} = \frac{\mathrm{tr}([h_u, h_d]H_u^{(2)})}{V^8}, \tag{26}$$

In the evaluation of this WBI and since we are considering a model beyond the SM, we also could expect to have some energy scale V different from v of the SM. Reasonably, one may choose this scale $V \sim m_T$, the mass of the VLQ, or attending to the constituent matrix elements of $\mathrm{tr}([h_u, h_d]H_u^{(2)})$ in equation (19), one may also consider $V^8 \sim v^6 m_T^2$. In each case, our new quantity in equation (26) has the potential to be much larger than the SM quantity. This could provide a significant enhancement of CP violation in the quark sector, which in turn could be useful in explaining the observed BAU[5]. A similar argument could even be extended to the next-lowest dimension invariants, those of dimension M^{10}, such as $\mathrm{tr}([h_u, h_d^2]H_u^{(2)})$ and $\mathrm{tr}([h_u^2, h_d]H_u^{(2)})$.

3.1. The size of CPV with one VLQ: a numerical example

Next, we provide a concrete numerical example of the size of CPV with one VLQ. We consider case II of appendix B in [3]. This example is relevant because it is associated to an extension of the SM with an up-type VLQ isosinglet, which addresses the CKM unitarity problem and accommodates the various tests posed by the electro-weak measurements, particularly newly derived stringent constraints set by the CPV parameter ε_K. We recall the up-quark mass matrix of the case, (given in the WB where the down-sector mass matrix is diagonal),

$$\mathcal{M}_u = \begin{pmatrix} 0 & 0 & 0 & 65.033 \\ 0 & 0 & 7.12124 & 15.8436e^{1.92462i} \\ 0 & 19.3672 & 172.73 & 4.21828e^{-1.56762i} \\ 0.0397187 & 1.63403 & 32.7938e^{-1.51551i} & 1475.32 \end{pmatrix} \mathrm{GeV}, \tag{27}$$

at the M_Z scale, with leads to the quark masses (in GeV)

$$m_u = 0.0018, m_c = 0.77, m_t = 174, m_T = 1477. \tag{28}$$

and the (full) quark mixing matrix

$$|\mathcal{V}^\dagger| \simeq \begin{pmatrix} 0.973609 & 0.223644 & 0.00382359 & 0.0453188 \\ 0.223785 & 0.973837 & 0.0395133 & 0.000740405 \\ 0.00824668 & 0.0388324 & 0.999212 & 0.000234355 \\ 0.0440136 & 0.010821 & 0.000312045 & 0.998972 \end{pmatrix}, \tag{29}$$

with $I_{\mathrm{CP}} \simeq 3.070 \times 10^{-5}$.

Using the down-quark (in GeV)

$$m_d = 0.003, \qquad m_s = 0.066, \qquad m_b = 2.90, \tag{30}$$

we obtain

$$|\mathcal{I}_{\mathrm{CP}}| = \left| \frac{\mathrm{tr}([h_u, h_d]H_u^{(2)})}{m_T^2 v^6} \right| \simeq 2.02 \times 10^{-10}, \tag{31}$$

which is to be compared with the SM estimate in equation (1).

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[5] Note that this enhancement of CP violation, possible in this type of extension, only fulfills one of the necessary conditions for Baryogensesis. There is also the requirement of a strong first-order phase transition, however, this topic is outside the scope of this work centered around the study of WBIs.

Furthermore, if we consider one other of the invariants of dimension M^{10} we find

$$|\mathcal{I}'_{\text{CP}}| \equiv \left| \frac{\text{tr}([h_u^2, h_d] H_u^{(2)})}{m_T^2 v^8} \right| \simeq 1.16 \times 10^{-10}, \tag{32}$$

which may also be relevant. The same is not true for the remaining invariants in equation (25).

4. The extreme chiral limit

An important and interesting result with regard to CP violation and VLQ's comes from the study of physical processes at very high energies. At these energies, the masses of the first two light quark generations are somewhat irrelevant and we may consider them insignificant or even to vanish. We refer to this situation as the ECL. In this limit, there is only one independent CP-odd WBI [24]. The vanishing of this invariant implies CP invariance.

Let us consider in this limit the following WB, where without loss of generality, the quark mass matrices assume the following form for

$$m_d = \text{diag}(0, 0, m_b), \qquad \mathcal{M}'_u = \mathcal{V} \cdot \text{diag}(0, 0, m_t, m_T), \tag{33}$$

$\mathcal{V}$ is a 4×4 unitary matrix and the CKM mixing is the 4×3 non-unitary left part of $\mathcal{V}^\dagger$.

In this limit and specific WB, one has from equation (33), that

$$\mathcal{M}'_u = \begin{pmatrix} 0 & 0 & m_{13}^c & m_{14}^c \\ 0 & 0 & m_{23}^c & m_{24}^c \\ 0 & 0 & m_{33}^c & m_{34}^c \\ 0 & 0 & m_{43}^c & m_{44}^c \end{pmatrix}, \tag{34}$$

where, in general, the m_c's are complex matrix elements.

Furthermore, one can always make a unitary transformation of the right-handed quark fields, e.g. in the (3, 4)—plane, $\mathcal{W}_{34}^R$, since any of these unitary transformation is unphysical. Additionally, because of the vanishing of quark masses for the first two generations, one may also redefine the left-handed quark fields through an arbitrary unitary transformation in the (1, 2)—plane, $\mathcal{U}_{12}^L$. Apart from these transformations, and given the fact that the down-sector is already diagonal, one has also the freedom of rephasing, not only all the RH quark fields, but also the LH quark fields.

Under all these transformations $\mathcal{M}'_u$ becomes

$$\mathcal{M}'_u \rightarrow \mathcal{M}_u = (\mathcal{K}_L \mathcal{U}_{12}^L) \cdot \mathcal{M}'_u \cdot (\mathcal{W}_{34}^R \mathcal{K}_R), \tag{35}$$

where the $\mathcal{K}_{L,R}$ are diagonal matrices of phases accounting for the rephasings.

The freedom in choosing $\mathcal{W}_{34}^R$ and $\mathcal{U}_{12}^L$ allows one to impose $m_{43} = m_{13} = 0$, meaning that, in this limit, there are only 6 independent mass matrix entries, and one may choose the (only) complex phase of $\mathcal{M}_u$ such that

$$\mathcal{M}_u = \begin{pmatrix} 0 & 0 & 0 & m_{14} \\ 0 & 0 & m_{23} & m_{24}e^{i\varphi} \\ 0 & 0 & m_{33} & m_{34} \\ 0 & 0 & 0 & m_{44} \end{pmatrix}, \tag{36}$$

with all the m_{ij} real.

With this minimal form in mind, it is straightforward to check that (after factoring out all unnecessary and unphysical phase factors), one obtains for unitary matrix $\mathcal{V}^{\dagger}$ which diagonalizes $\mathcal{M}_u$ as in equation (8),

$$
\begin{aligned}
\mathcal{V}^{\dagger} ={}& \mathcal{O}_{34}\mathcal{K}_{\delta_{\mathrm{CL}}}\mathcal{O}_{24}\mathcal{O}_{14}\mathcal{O}_{23} \\
={}& \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & c_{34} & s_{34} \\ 0 & 0 & -s_{34} & c_{34} \end{pmatrix}
\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & e^{i\delta_{\mathrm{CL}}} \end{pmatrix}
\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{24} & 0 & s_{24} \\ 0 & 0 & 1 & 0 \\ 0 & -s_{24} & 0 & c_{24} \end{pmatrix} \\
& \cdot \begin{pmatrix} c_{14} & 0 & 0 & s_{14} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s_{14} & 0 & 0 & c_{14} \end{pmatrix}
\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & c_{23} & s_{23} \\ 0 & 0 & -s_{23} & c_{23} \end{pmatrix},
\end{aligned}
\tag{37}
$$

where δ_{CL} is a function of the phase φ and the real parameters in equation (36).

Finally, one finds the CKM matrix for the ECL, corresponding to the first three columns of $\mathcal{V}^{\dagger}$, i.e.

$$
V_{\mathrm{CKM}} = \begin{pmatrix}
c_{14} & 0 & 0 \\
-s_{14}s_{24} & c_{23}c_{24} & s_{23}c_{24} \\
-s_{14}c_{24}s_{34}e^{i\delta_{\mathrm{CL}}} & -s_{23}c_{34} - c_{23}s_{24}s_{34}e^{i\delta_{\mathrm{CL}}} & c_{23}c_{34} - s_{23}s_{24}s_{34}e^{i\delta_{\mathrm{CL}}} \\
-s_{14}c_{24}c_{34}e^{i\delta_{\mathrm{CL}}} & s_{23}s_{34} - c_{23}s_{24}c_{34}e^{i\delta_{\mathrm{CL}}} & -c_{23}s_{34} - s_{23}s_{24}c_{34}e^{i\delta_{\mathrm{CL}}}
\end{pmatrix},
\tag{38}
$$

with $c_{ij} \equiv \cos(\theta_{ij})$, $s_{ij} \equiv \sin(\theta_{ij}) \in [0, \pi/2]$ and $\delta_{\mathrm{CL}} \in [0, 2\pi]$.

In the ECL, there exists only one independent CP-odd invariant, as already was implied in [24], and we may now write all of the invariants in equation (25) in terms of the invariant of lowest dimension,

$$
\mathrm{tr}([h_u, h_d]H_u^{(2)}) = 2im_b^2 m_t^2 m_T^2 (m_T^2 - m_t^2)I_{\mathrm{CL}},
\tag{39}
$$

where the ECL rephasing invariant yields

$$
I_{\mathrm{CL}} = \mathrm{Im}\,(F_{34}^u V_{33}^* V_{43}) = c_{24}c_{14}^2 c_{24}^2 c_{34} s_{23} s_{24} s_{34} \sin\delta_{\mathrm{CL}}.
\tag{40}
$$

We point out that even at very high energies there might exist new important sources of CPV. This contrasts with the SM, where there is no CP violation in the ECL, as can be straightforwardly verified with the SM-CP-odd invariant in equation (23), proportional to $m_c^2 - m_u^2$ and $m_s^2 - m_d^2$, and which vanishes in this limit.

A rough estimate of the size of CPV at very high energies leads to the upper-bound

$$
\mathcal{I}_{\mathrm{CL}} = \left[\frac{\mathrm{tr}([h_u, h_d]H_u^{(2)})}{m_T^2 v^6}\right]_{\mathrm{ECL}} \lesssim \frac{m_b^2 m_T^2}{m_t^4}|s_{23}s_{24}s_{34}|.
\tag{41}
$$

for small mixing angles.

Once more, these results demonstrate that the existence of VLQs could have very important implications for CPV. Nonetheless, the experimental measurement of CPV at extremely high energies is a notably challenging task [25, 26].

5. Conclusions

We have studied CP-even and CP-odd WB invariants in the context of extensions of the SM where vector like quarks are added to the spectrum of the SM.

WBIs are very useful for studying some of the physical implications of VLQ's. For example, it is well known that in high energy collisions one cannot distinguish a *d*-quark jet from a *s*-quark jet. Therefore, at high energy collisions (i.e. energies much higher than *v*) in the context of the SM, CP violation effects vanish. However, by constructing CP-odd invariants which, in the presence of VLQ do not vanish, one can show that the CP violating effects may be present in this limit, or even in the limit where $m_d = m_s$.

CP-even WBIs can also be very useful in the study of the flavour structure of models with VLQs. In the SM, it has been shown that the knowledge of four specific CP-even WBIs allows one to reconstruct the full V_{CKM} matrix with only a two-fold ambiguity in the sign of CP violation. In the specific case of an extension with a heavy up-type VLQ, the quark mixing matrix appearing in the weak charged currents is a 4×3 matrix. We have identified the nine CP-even WBIs which determine this quark mixing matrix.

We have pointed out that in the presence of a VLQ, the lowest order CP-odd WBI is of mass order M^8, much lower than the order of the CP-odd WBI in the SM (which is of order M^{12}). As a result, the strength of CP violation relevant for baryogenesis is much larger in models with VLQs than the corresponding one in the SM.

Acknowledgments

Data availability statement

No new data were created or analysed in this study.

Appendix A. Constructing WBIs for a general extension with VLQ isosinglets

Although in this work we focused specifically on extensions with one up-type VLQ isosinglet, it is straightforward to understand that all the results presented are applicable to extensions with one down-type VLQ isosinglet just by interchanging the up and down sectors.

However, the formalism we presented in section 2 is also applicable to a general extension with n_u up-type and n_d down-type VLQ isosinglets. In this appendix we briefly demonstrate that.

In the general case, WBTs are defined in an analogous way to before. They correspond to arbitrary 3×3 unitary transformations of the standard left-handed quark fields u_L and d_L, to $n \times n$ (with $n = n_u$ or $n = n_d$, depending on the specific sector) unitary transformations of the left-handed VLQ fields U_L or D_L, as well as $(3 + n) \times (3 + n)$ unitary transformations of the right-handed VLQ fields u_R or d_R.

The matrices relating the flavor eigenstates with the mass eigenstates u' and d', are the $(3 + n) \times (3 \times n)$ unitary matrices defined as

$$\mathcal{V} = \begin{pmatrix} A \\ B \end{pmatrix}, \tag{42}$$

for each sector, where A is $3 \times (3 \times n)$ and B is $n \times (3 \times n)$. The mass matrices $\mathcal{M}_u$ and $\mathcal{M}_d$ are diagonalized as

$$\mathcal{V}^\dagger \mathcal{M} \mathcal{W} = \mathcal{D} \tag{43}$$

with $\mathcal{D}$ being the diagonal matrix containing the quark masses of a given sector and $\mathcal{W}$ having the same size as $\mathcal{V}$. The unitarity of each $\mathcal{V}$ means that the relations in equation (16) are still true for each sector individually.

Moreover, for each sector, the $3 + n$ quark masses are determined by a set of $3 + n$ invariants constructed from $\mathcal{H}_{u,d} = \mathcal{M}_{u,d} \mathcal{M}_{u,d}^\dagger$. These can be defined as

$$\chi_\sigma(\mathcal{H}) = \frac{1}{\sigma!} \frac{\partial^\sigma}{\partial x^\sigma} \det(\mathcal{H} + x \mathbb{1}_{n \times n}) \bigg|_{x=0}, \tag{44}$$

where σ runs from 0 to $2 + n$, with $\chi_0(\mathcal{H}) = \det(\mathcal{H})$ and $\chi_{2+n}(\mathcal{H}) = \operatorname{tr}(\mathcal{H})$.

The Lagrangian describing the charged currents is now given by

$$\mathcal{L}_W = \frac{g}{\sqrt{2}} W_\mu^+ \overline{u'}_L A_u^\dagger A_d \gamma^\mu d_L' + \text{h.c.}, \tag{45}$$

so that now we define the $(3 + n_u) \times (3 + n_d)$ mixing matrix as $V_{\text{CKM}} \equiv A_u^\dagger A_d$.

The neutral current Lagrangian is given by

$$\mathcal{L}_Z = \frac{g}{c_W} Z_\mu \left[\frac{1}{2} (\overline{u'}_L A_u^\dagger A_u \gamma^\mu u_L' - \overline{d'}_L A_d^\dagger A_d \gamma^\mu d_L') - s_W^2 \left(\frac{2}{3} \overline{u'} \gamma^\mu u' - \frac{1}{3} \overline{d'} \gamma^\mu d' \right) \right], \tag{46}$$

so that now there exist FCNCs on both sectors, which are controlled by the matrices

$$\begin{aligned} F^u &= A_u^\dagger A_u = (A_u^\dagger A_d)(A_d^\dagger A_u) = V_{\text{CKM}} V_{\text{CKM}}^\dagger, \\ F^d &= A_d^\dagger A_d = (A_u^\dagger A_d)(A_d^\dagger A_u) = V_{\text{CKM}}^\dagger V_{\text{CKM}}, \end{aligned} \tag{47}$$

where we used the relation $AA^\dagger = \mathbb{1}_{3 \times 3}$ coming from the unitarity of $\mathcal{V}$.

Now, to construct WBIs that relate to the mixing, as done before, it is useful to write for both sectors as (using equation (43))

$$\mathcal{M} = \begin{pmatrix} m \\ M \end{pmatrix} = \begin{pmatrix} A \\ B \end{pmatrix} \mathcal{D} \, \mathcal{W}^\dagger, \tag{48}$$

From this we can define various Hermitian matrices which transform as in equation (20) under WBTs. Keeping the analogy with the one VLQ case, we can construct the set of matrices

$$H^{(r)} \equiv m(m^\dagger m + M^\dagger M)^{r-1} m^\dagger = A \mathcal{D}^{2r} A^\dagger, \tag{49}$$

which then can be used, just as in equation (22), to determine the moduli of all the entries of V_{CKM} from

$$\begin{aligned} \operatorname{tr}(H_u^{(r)} H_d^{(s)}) &= \operatorname{tr}(A_u \mathcal{D}_u^{2r} A_u^\dagger A_d \mathcal{D}_d^{2s} A_d^\dagger) = \operatorname{tr}(V_{\text{CKM}}^\dagger \mathcal{D}_u^{2r} V_{\text{CKM}} \mathcal{D}_u^{2s}) \\ &= \sum_{\alpha=1}^{3+n_u} \sum_{\beta=1}^{3+n_d} (m_{u_\alpha}^2)^r (m_{d_\beta}^2)^s |V_{\alpha\beta}|^2. \end{aligned} \tag{50}$$

The CP-odd WBI of dimension M^8 in equation (25) can now be written as

$$\text{tr}([h_u, h_d]H_u^{(2)}) = 2i \ \text{Im}\,[\text{tr}(A_u \mathcal{D}_u^2 A_u^\dagger A_d \mathcal{D}_d^2 A_d^\dagger A_u \mathcal{D}_u^4 A_u^\dagger)]$$

$$= 2i \ \text{Im}\,[\text{tr}(\mathcal{D}_u^2 \ V_{\text{CKM}} \mathcal{D}_d^2 \ V_{\text{CKM}}^\dagger \mathcal{D}_u^4 F^u)]$$

$$= 2i \sum_{\alpha,\rho=1}^{3+n_u} \sum_{\beta=1}^{3+n_d} m_{u_\alpha}^2 m_{d_\beta}^2 m_{u_\rho}^4 \ \text{Im}\,(F_{\rho\alpha}^u V_{\rho\beta}^* V_{\alpha\beta}), \tag{51}$$

It is important to note that now, in the general case, there is another distinct WBI of the same dimension. This can simply be obtained by interchanging the sector indices, i.e. $u \leftrightarrow d$, leading to

$$\text{tr}([h_d, h_u]H_d^{(2)}) = 2i \ \text{Im}\,[\text{tr}(\mathcal{D}_d^2 \ V_{\text{CKM}}^\dagger \mathcal{D}_u^2 \ V_{\text{CKM}} \mathcal{D}_d^4 F^d)]$$

$$= 2i \sum_{\alpha=1}^{3+n_u} \sum_{\beta,\rho=1}^{3+n_d} m_{u_\alpha}^2 m_{d_\beta}^2 m_{d_\rho}^4 \ \text{Im}\,(F_{\rho\beta}^d V_{\alpha\rho} V_{\alpha\beta}^*). \tag{52}$$

Appendix B. Simplified minimal set of Invariants for one up-VLQ

In this appendix, we consider a WB and a simplified minimal set of invariants. Our aim is to exemplify, in an alternative but comprehensive way, how this minimal set can be used to determine all physical parameters.

Consider the following Hermitian matrix

$$\overline{h}_u = m_u M_u^\dagger (M_u M_u^\dagger)^{-1} M_u m_u^\dagger, \tag{53}$$

as well as the combination

$$h_{\text{eff}} = h_u - \overline{h}_u, \tag{54}$$

which is the effective (squared) mass matrix that gives (in most cases) a very good approximation of the three light up-quarks masses and mixings. As can be checked, under WBTs, the Hermitian matrices in equations (53, 54) transform as the matrices $h_d = m_d m_d^\dagger$ and $h_u = m_u m_u^\dagger$ defined in equation (19), i.e. $h \to V^\dagger h V$.

We use this minimal set of Hermitian matrices to construct simple WB invariants. To do this, consider the WB where the up and down-quark mass matrices are given by

$$\mathcal{M}_u = \begin{pmatrix} m_1 & 0 & 0 & r_1 \\ 0 & m_2 & 0 & r_2 \\ 0 & 0 & m_3 & r_3 \\ 0 & 0 & 0 & m_4 \end{pmatrix}, \quad m_d = UD_d, \tag{55}$$

where all entries of $\mathcal{M}_u$ are real. This form is always possible with adequate unitary transformations of the quark fields.

The down-sector mass matrix contains the diagonal matrix D_d with the three down-quark masses and a 3×3 unitary matrix U, which one can write as

$$U = K_{\alpha'\beta'}O_{23}K_{\delta'}O_{13}O_{12} \tag{56}$$

where $K_{\alpha'\beta'} = \text{diag}(1, e^{i\alpha'}, e^{i\beta'})$, $K_{\delta'} = \text{diag}(1, 1, e^{i\delta'})$ and the O_{ij} are basic real orthogonal transformations in the ij-plane. In this WB, the down sector has a total of 9 physical parameters: 3 mixing angles, 3 quark masses and 3 being CP violating phases. The unitary matrix U is, for all physical relevant cases, very near to the 3×3 part of the CKM mixing matrix.

With respect to the up-quark mass matrix (and again for the physical relevant cases), the m_1, m_2, m_3 are very good approximations of the lightest quark masses m_u, m_c, m_t.

In this WB, one obtains very simple forms for the Hermitian matrices in equations (53), (54), (55),

$$h_d = U \, \mathrm{diag}(m_d^2, \, m_s^2, \, m_b^2) \, U^\dagger. \tag{57}$$

and

$$h_{\mathrm{eff}} = \mathrm{diag}(m_1^2, \, m_2^2, \, m_3^2),$$
$$\bar{h}_u = \mathrm{diag}(r_1, \, r_2, \, r_3) \cdot \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \cdot \mathrm{diag}(r_1, \, r_2, \, r_3). \tag{58}$$

Thus, $h_u = h_{\mathrm{eff}} + \bar{h}_u$ has also a very manageable form.

Alternatively, and in order to provide a more simplified description, we may switch from the parametrization of h_d in equation (57) to

$$h_d = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i\beta} \end{pmatrix} \begin{pmatrix} p & a & be^{i\delta} \\ a & q & c \\ be^{-i\delta} & c & r \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-i\alpha} & 0 \\ 0 & 0 & e^{-i\beta} \end{pmatrix}, \tag{59}$$

where a, b, c, p, q, r are all real parameters and α, β, $\delta \in [0, 2\pi]$. All these new parameters are evident functions of the nine physical parameters in equation (57). Notice that we have the same number of real parameters and complex phases as before.

One may now relate all parameters in the mass matrices with WBIs constructed from h_u, $\bar{h}_u$, h_{eff} and h_d. For instance, the quantities m_1, m_2, m_3 can be obtained by using $\mathrm{tr}(h_{\mathrm{eff}})$, $\chi(h_{\mathrm{eff}})$ and $\det(h_{\mathrm{eff}})$, while m_4 is obtained from $m_4^2 = \mathrm{tr}(\mathcal{H}_u) - \mathrm{tr}(h_u)$, where $\mathcal{H}_u = \mathcal{M}_u \mathcal{M}_u^\dagger$.

Now, using this knowledge, one then proceeds to determine r_1, r_2, r_3 from

$$\mathrm{tr}(\bar{h}_u h_{\mathrm{eff}}) = m_1^2 r_1^2 + m_2^2 r_2^2 + m_3^2 r_3^2,$$
$$\mathrm{tr}(\bar{h}_u h_{\mathrm{eff}}^2) = m_1^4 r_1^2 + m_2^4 r_2^2 + m_3^4 r_3^2,$$
$$\mathrm{tr}(\bar{h}_u h_{\mathrm{eff}}^3) = m_1^6 r_1^2 + m_2^6 r_2^2 + m_3^6 r_3^2, \tag{60}$$

or from three other invariants of $\mathcal{H}_u$: $\mathrm{tr}(\mathcal{H}_u)$, $\chi_1(\mathcal{H}_u)$ and $\chi_2(\mathcal{H}_u)$.

Subsequently, the parameters p, q and r can be determined from

$$\mathrm{tr}(h_d) = p + q + r,$$
$$\mathrm{tr}(h_d h_{\mathrm{eff}}) = m_1^2 p + m_2^2 q + m_3^2 r,$$
$$\mathrm{tr}(h_d h_{\mathrm{eff}}^2) = m_1^4 p + m_2^4 q + m_3^4 r, \tag{61}$$

and after that a, b and c are obtained with

$$\mathrm{tr}(h_d^2) = p^2 + q^2 + r^2 + 2(a^2 + b^2 + c^2),$$
$$\mathrm{tr}(h_d h_{\mathrm{eff}}) = m_1^2 p^2 + m_2^2 q^2 + m_3^2 r^2$$
$$\qquad + (m_1^2 + m_2^2)a^2 + (m_2^2 + m_3^2)b^2 + (m_1^2 + m_3^2)c^2,$$
$$\mathrm{tr}(h_d h_{\mathrm{eff}}^2) = m_1^4 p^2 + m_2^4 q^2 + m_3^4 r^2$$
$$\qquad + (m_1^4 + m_2^4)a^2 + (m_2^4 + m_3^4)b^2 + (m_1^4 + m_3^4)c^2. \tag{62}$$

Moreover, the complex phase δ can be extracted from

$$\det(h_d) = pqr - a^2 r - b^2 q - c^2 p + 2abc \cos \delta, \tag{63}$$

whereas the remaining phases α and β are determined from

$$\frac{1}{2i} \text{tr}([h_d, \bar{h}_u] h_{\text{eff}}) = a(m_2^2 - m_1^2) r_1 r_2 \sin \alpha + b(m_1^2 - m_3^2) r_1 r_3 \sin(\beta - \delta)$$
$$+ c(m_3^2 - m_2^2) r_2 r_3 \sin(\alpha - \beta),$$
$$\frac{1}{2i} \text{tr}([h_d, \bar{h}_u] h_{\text{eff}}^2) = a(m_2^4 - m_1^4) r_1 r_2 \sin \alpha + b(m_1^4 - m_3^4) r_1 r_3 \sin(\beta - \delta)$$
$$+ c(m_3^4 - m_2^4) r_2 r_3 \sin(\alpha - \beta). \tag{64}$$

Note that although $\text{tr}([h_d, \bar{h}_u] h_{\text{eff}})$ is a CP-odd WBI of dimension 6, this does not contradict the discussion of section 3. In reality, using the notation of section 2.2 we have $\bar{h}_u \equiv (M_u M_u^\dagger)^{-1}(H_u^{(2)} - h_u^2)$, meaning that

$$\text{tr}([h_d, \bar{h}_u] h_{\text{eff}}) = (M_u M_u^\dagger)^{-1} \text{tr}([h_u, h_d] H_u^{(2)}), \tag{65}$$

so that the WBI in (64) has only such a low dimension due to a scaling factor $(M_u M_u^\dagger)^{-1} \sim 1/m_4^2$. It is not, therefore, a new WBI of lower dimension.

Thus, we have obtained all physical parameters of our model in terms of a minimal set WB invariants.

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