Physica Scripta

PAPER

Near-periodic behavior and parameter-insensitive dynamics of a giant atom interacting with a continuum

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Keywords: qubits, two-level systems, exponential decay, revival dynamics

OPEN ACCESS

RECEIVED
14 June 2024

REVISED
13 September 2024

ACCEPTED FOR PUBLICATION
24 September 2024

PUBLISHED
14 October 2024

Abstract

We consider the solutions of a two-level atom (qubit) coupled to a discretized continuum in the case when the system initially start out in a continuum state. By solving the model we explain the origin of two new pronounced features of a widely used model which has been used for numerous recent studies related to quantum information. In this work we document two new phenomena related to the dynamics which occur when the system is starting out in the continuum. First, dissipatively non-periodic dynamics is replaced by near-periodic oscillatory dynamics when the initial state is switched to be an initially populated continuum state, in combination with a qubit ground state. Second, when the qubit is coupled to two well separated points, the initial state exhibits an exact linear, stepwise decay which is completely insensitive to the coupling parameters.

1. Introduction

One of the most widely used mathematical models in time-dependent non-relativistic quantum mechanics is a two-level atom interacting with a continuum or a quasi-continuum consisting of a dense band of discrete states. In this model the continuum, bounded or unbounded, can in fact always be represented by a large or infinite set of discrete states with constant energy separation. When the coupling between the two states of the atom and a corresponding creation or annihilation of a continuum particle with arbitrary energy are given by a state-independent constant it was shown already in 1935 that the model is solvable analytically [1]. This has later been done more generally based on Fourier or Laplace transform techniques [2, 3] and using matrix algebra [4–6].

The realization of quantum computers and quantum information science has led to a revived interest in this model [7]. The two-level atom then represent a qubit which interacts with other qubits via photonic or phononic interaction channels [8–13], or via interactions with spins of surrounding nuclei [14–16]. In some realizations of a qubit it is coined a *giant atom*. The coined name refers to the fact that the giant atomic extension can be comparable or much larger than the wavelengths of the interacting photon field, in contrast to natural atomic radi and interacting wavelengths pertaining to the electromagnetic field. The giant atom can interact with phonon modes of a transmission line at well defined coupling points. In real setups the coupling points are technological devices which couple the giant atom in question to the field. For example, the superconducting device is coupled to the transmission line through interdigitated transducers. The small speed of sound and the relatively large distance between coupling points leads to signifiant time delays which have been experimentally observed [17].

The core mathematical model is a single state $|a\rangle$, which couples with constant or variable strength to a finite or infinite band of states $|b_n\rangle$. In the context of a giant atom (or an interacting qubit), state $|a\rangle$ is the excited state of the superconducting qubit and band states $|b_n\rangle$ represent the ground state and possible excitations of the photon/phonon field. In short, for an unbounded continuum with constant coupling, the following features are some of the known phenomena regarding the time development of the initially excited atomic state:

(i) The survival probability of the excited state $|a(t)|^2$ undergoes exponential decay up to a *revival time*, $\tau = 2\pi/\Delta$, where Δ is the energy separation between the band states. At the same time the population of the continuum obtaines a finite width and Lorentzian shape.

(ii) A new type of exponential decay also takes place for harmonic energy dependent couplings between the initial state and the continuum [18].

(iii) A sequence of revivals takes place at well defined integer n times τ, either due to the nature of the couplings or the density of states.

(iv) The revival peak strengths follow a decay pattern as well, sometimes polynomial [18]. This implies that, at large times, the population of the initial as well as a number of band states appears noisy.

The situation is only slightly more complicated when considering the coupling to two continua of band states, corresponding to two-point couplings [6]. In this case, we have two characteristic revival times τ_1 and τ_2, and the excited state revives on the multiple of each of these times, i.e. at $n\tau_1$ and $m\tau_2$ with $n,\ m \in \mathbb{N}$, but also on the combination of both, e.g. at $n\tau_1 + m\tau_2$. This dynamics can systematically be derived for higher-order couplings, as well.

Few, if any, studies have considered the detailed solution of this model when the system starts out in the continuum, which is the purpose of the present work. This corresponds to an initially excited phonon, or photon, mode which in turn can excite the two-level atoms it comes into contact with. At first one may think that no new dynamical features will occur since the basic processes would be identical to the situation above after an initial excitation of the atom. However, it turns out that the dynamical evolution in this case features some striking differences.

For constant coupling to a single band of states we find a near periodic oscillatory behavior of the initial state probability, in sharp contrast to the decaying revival dynamics of an initially excited atom. A second new phenomenon is the emergence of a pure stepwise linear decay of the initial state which is independent of the coupling constant. In contrast, for the corresponding initially excited atom dynamics the decay and long time dynamics is highly sensitive to the same coupling parameters. The model, and its general solution, will be described in the section 2, followed by section 3 where we explain the mechanisms behind these two phenomena. We conclude and give a brief outlook in section 4. Atomic units will be used throughout.

2. Model system

The Hilbert space is spanned by a set of orthonormal states $|a\rangle$, for the excited state, and $\{|b, \omega_n\rangle\}$, for the band states. The Hamiltonian consists of two parts, the non-interacting part H_0 and an interaction part, H_I. For the non-interaction part the matrix elements are $\langle a|H_0|a\rangle = \omega_{ab}$ and $\langle b,\ \omega_n|H_0|b,\ \omega_{n'}\rangle \ = \ \delta_{n,n'} n\Delta$, where Δ is the inverse density of states of the discrete band. The interaction between the excited state and the band states is given by $\langle a|H_I|b, \omega_n\rangle = \beta_n$. Couplings between all other states are zero. The state vector $|\Psi(t)\rangle$, which obeys the time-dependent Schrödinger equation, is then given by the superposition

$$|\Psi(t)\rangle \ = \ a(t)|a,\ 0\rangle \ + \ \sum_{n=\infty}^{\infty} b_n(t)|b,\ \omega_n\rangle. \tag{1}$$

The set of coupled Schrödinger equations for the coefficients $a(t)$ and $b_n(t)$ can be expressed as a first order coupled vector matrix equation,

$$i\frac{\mathrm{d}}{\mathrm{dt}}\begin{pmatrix} a \\ \mathbf{b} \end{pmatrix} = \begin{pmatrix} \omega_{ab} & \mathbf{C}^\dagger \\ \mathbf{C} & \mathbf{\Omega} \end{pmatrix}\begin{pmatrix} a \\ \mathbf{b} \end{pmatrix}, \tag{2}$$

where the vector $\mathbf{b}$ is a collection of all the expansion coefficients $b_n(t)$ and the elements of the vector $\mathbf{C}^\dagger$ are the coupling constants between the atomic states induced by the presence of the phonon field. Ω is a diagonal matrix, $\Omega_{n,n'} \ = \ \delta_{n,n'} n\Delta$. A single contact point between the two-level atom and the band results in a constant coupling (row) vector $\mathbf{C}^\dagger = \beta(1, 1, 1....)$. For two-point contacts, positioned at $x = \pm\, L/2$, the n-th component of the vector becomes $(\mathbf{C}^\dagger)_n \ = \ \beta\, \cos(n\Delta T/2)$, where the speed of wave-propagation in the band, v_b, determines the period $T = L/v_b$ [18].

For discrete bands representing a continuum the atomic level separation ω_{ab} plays no role for the dynamics of single point contact as long as an integer number of band states fill the energy interval $\omega_{ab} = M\Delta$ for some integer M. Alternatively, an extra time-independent phase, which seldom plays any important role, must be added [3]. Therefore, and without losing important physics, we can set $\omega_{ab} = 0$ and obtain the solution of equation (2) by diagonalizing the matrix. The energy eigenvalues, E_m, are the solution of the implicit expression

$$E_m = \sum_{n=-\infty}^{\infty} \frac{|\beta_n|^2}{E_m - n\Delta}. \tag{3}$$

As a consequence we can express each eigen-energy as $E_m = m\Delta + \phi_m$ where the phase ϕ_m decays with increasing $|m|$ and $\phi_{-m} = \phi_m$ [4]. Projecting onto the physical states of the Hamiltonian, the amplitudes are $a_m \equiv \langle a|\Psi_m\rangle = [1 + \sum_n |\beta_n/(E_m - \omega_n)|^2]^{-1/2}$ and $\langle b_n|\Psi_m\rangle = a_m\beta_n/(E_m - \omega_n)$. In the diagonal basis the wave function takes the form,

$$|\Psi(t)\rangle = \sum_{m=-\infty}^{\infty} \langle\Psi_m|\Psi(0)\rangle\,|\Psi_m\rangle\; e^{-iE_m t}, \tag{4}$$

where the initial condition is $|\Psi(0)\rangle$. In this paper we will consider two possible initial conditions, namely $|\Psi(0)\rangle = |a, 0\rangle$ or $|\Psi(0)\rangle = |b, \omega_0\rangle$.

For the single-point contact interaction, the characteristic time-scale of the problem is $\tau = 2\pi/\Delta$ [3, 6], which we will call the 'revival time'. It turns out that the dynamics naturally can be separated into time-intervals of increasing integer number of τ, i.e. $t = t' + N\tau$ with $N = 1, 2, \ldots$ and $0 \leqslant t' < \tau$.

In this work, we will track the concurrent evolution of the excited state of the giant atom, via the amplitude $a(t)$, and its closest band state, by means of the amplitude $b_0(t)$. For the initial configuration of an excited atom, $a(0) = 1$ while $b_n(0) = 0$ for all $n \geqslant 0$, we then obtain the following analytical expression for the initial state amplitudes when $|\Psi(0)\rangle = |a, 0\rangle$,

$$a^{(a)}(t) = \sum_m \tilde{a}_m(t') e^{-iN\phi_m\tau}, \tag{5}$$

$$b_0^{(a)}(t) = \sum_m \tilde{b}_{0,m}^{(a)}(t') e^{-iN\phi_m\tau}, \tag{6}$$

where the superscript '(a)' refers to the initial condition. Alternatively, when starting in the atomic ground state and the resonant band state which can drive the atomic excitation, $|\Psi(0)\rangle = |b, \omega_0\rangle$ we obtain,

$$a^{(b)}(t) = b_0^{(a)}(t), \tag{7}$$

$$b_0^{(b)}(t) = \sum_m \tilde{b}_{0,m}^{(b)}(t') e^{-iN\phi_m\tau}, \tag{8}$$

with a similar superscript convention as above. In the equations above, we introduced

$$\tilde{a}_m(t) = \frac{\beta^2}{\beta^2 + (\gamma/2)^2 + E_m^2} e^{-iE_m t}, \tag{9}$$

with $\tilde{b}_{0,m}^{(a)}(t) = \frac{\beta}{E_m}\tilde{a}_m(t)$, $\tilde{b}_{0,m}^{(b)}(t) = \frac{\beta^2}{E_m^2}\tilde{a}_m(t)$, and $\gamma \equiv \tau\beta^2$ [4]. Note, that the absolute values of the individual amplitudes $|\tilde{a}_m(t)|$ are simply constants.

In the first interval $N = 0$ and $t = t'$, $a(t) = \mathrm{e}^{-\gamma t}$ for $t \in (0, \tau)$, and leads to a Lorentzian distribution of $|b_{0,m}^{(a)}(t)|^2$ when we let $\beta \to 0$ at fixed γ. The equality in equation (7), relating the amplitudes of states of the same energy $\omega_0 = 0$ but at different initial conditions, is a direct consequence of detailed balance [19].

In the more widely used solution techniques based on analytic transformations, the final expression of the amplitudes above alternatively appear as a coherent summation of amplitudes from a sequence of delayed time intervals, eg. $a(t) = \sum_{n=0}^{\infty} a_n(t - N\tau)\Theta(t - N\tau)$. The delayed time contributions for $t > \tau$ arises from additive phasing effect of the evenly spaced states or coherent contributions from well separated contact points. Such effects has in many related works been discussed in terms of Markovian versus non-Markovian dynamics, referring to situations where the time-delay additions can be neglected (Markovian) or not (non-Markovian) [20]. When using matrix mechanics, the dynamics in the so-called "non-Markovian regime" is completely determined by the solution in the first time-interval. For each passage into a new interval $t \in [N\tau, (N + 1)\tau]$, the amplitudes are simply augmented by an extra phase.

In the following, we will pursue both techniques, i.e. the matrix-dynamics directly expressed through the wave-function and the solutions in terms of delayed rate equations, to shed further light on these features.

3. Results: oscillatory and parameter-insensitive dynamics

The time development for single point contact when starting in the excited atomic state is shown in the upper panels of figure 1. It includes the mentioned known features a polynomial decay of the revival peaks occurring after each passage of τ. In between the revival times the amplitudes $b_n(t)$ have the same pattern and phase.

When keeping the model parameters fixed and starting in the resonant band state, visualized in the lower panels of figure 1, the dynamic evolution becomes completely different and exhibits an almost periodic behavior. Neighboring band states now oscillate with opposite phases and the number of significantly contributing amplitudes are much smaller, cf also figure 3.

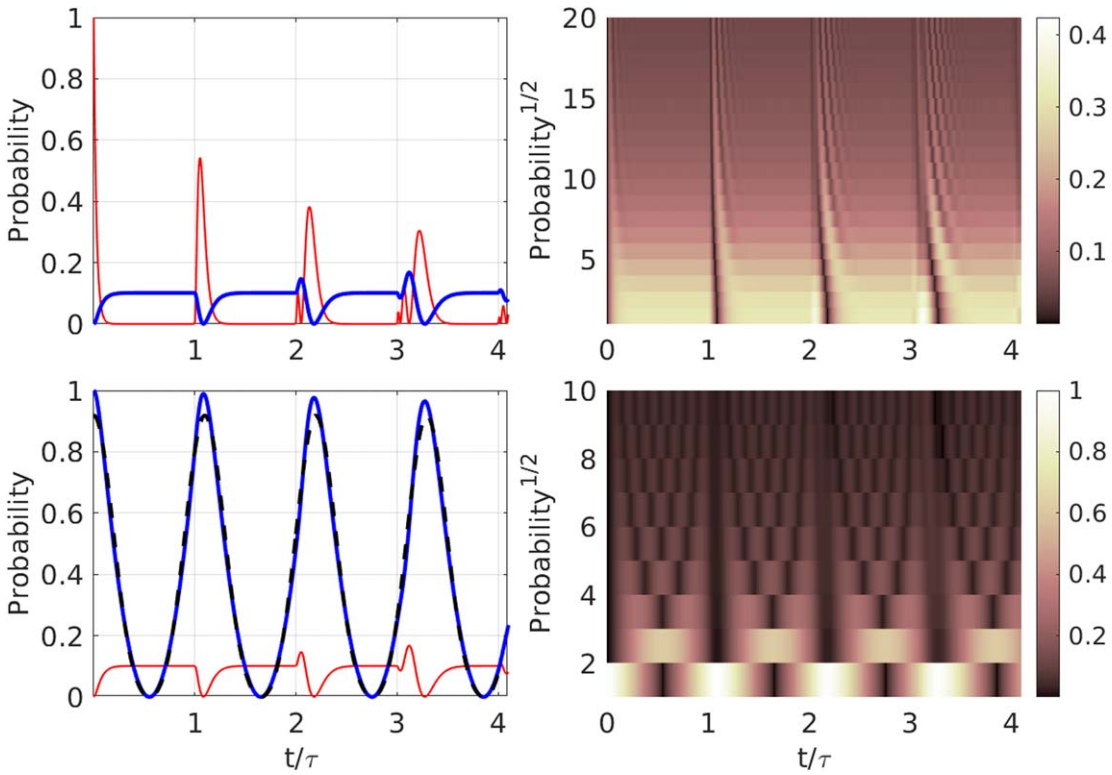

Figure 1. Upper panels: The left figure shows the initial state ($|a,0\rangle$) probability, $|a(t)|^2$ as function of reduced time (red thin curve) as well as the probability of populating the resonant state ($|b,\omega_0=0\rangle$), $|b_0(t)|^2$, of a timespan up to 4 revivals with constant coupling and density of states. The right figure shows the distribution of the states $|b_n(t)|$, where the index of the band state $n\in(0,20)$ is listed on the vertical axis while the color represents the value of the square of the probability (to enhance the visual pattern). **Lower panels**: Both figures show the same time evolution as above, except that the initial state is the resonant band state $|b_0\rangle$. The broken black curve display the approximate solution based on the pair of the three lowest eigenstates as discussed in the text, see equation (10). Parameters are chosen such that $\Delta=\beta=0.035$.

It is interesting to explore whether the periodicity indeed is a stable feature of the evolution equations and lasts far into the time region where the initially excited atom dynamics has become structureless. For that purpose we show the two initial state probabilities at a much later time interval in figure 2. The initially populated band state is seen still to remain oscillatory (blue curve) with an equally large revival amplitude as in figure 1. In comparison, the evolution of the initially populated excited state (red curve) at late times is polluted by interference from multiple peaks appearing at higher multiples of the revival times and becomes seemingly random.

The periodicity turns out to follow directly from the suppression of each amplitude $b^{(b)}_{n,m}$ with the factor $[E_m(E_m-\omega_n)]^{-1}\sim E_m^{-2}$, cf equations (7)–(8). Large eigen-energies will damp the amplitudes $b^{(b)}_{n,m}$ to a non-significant magnitude. It follows that much fewer terms takes part in the superposition. As an illustrative example the absolute value of the amplitudes $|\tilde{b}^{(b)}_{0,m}|$ and $|\tilde{a}_m|$, see equation (5), are compared in figure 3.

For the initially populated excited state, we clearly see that many $m\lesssim 20$ eigen-states of the energy are actively participating in the evolution. In contrast, for the initially populated band state, in the summation over m only a few $m\leqslant 3$ terms will play a significant role. The dynamics is then approximately given by a small number, m, of terms of the form,

$$b^{(b)}_0(t)\approx\sum_{m=1}^{m_{\max}}\frac{\beta^4}{\beta^2+(\gamma/2)^2+E_m^2}\frac{1}{E_m^2}\cos(E_m t') \tag{10}$$

In figures 1 and 2 the broken black curve shows the solution obtained with three such cosine terms, $m_{\max}=3$. The approximation is seen to be excellent within both the considered time regions. We conclude that the model demonstrates strikingly different long-term dynamics depending on chosen resonant initial state: An excited atomic state decays asymptotically into a broad distribution of all involved states. When the initial state is the ground state accompanied by a phonon state exactly matching the energy level of the excited state, the behavior remains oscillatory over arbitrarily long time periods.

Interestingly, the complex dynamics invoked by exciting a band state can also give access to regimes that are insensitive to the model parameters. In order to gain analytical insight, instead of formulating the problem in terms of matrix dynamics we shall here treat the problem as a coupled set of evolution equations for the state $a(t)$

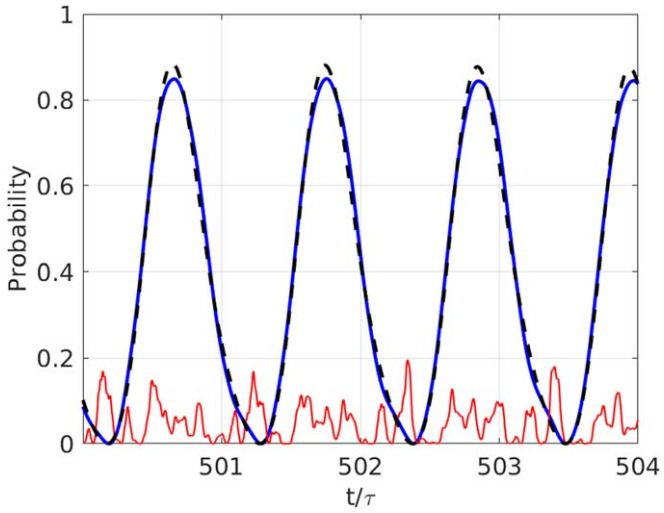

Figure 2. Initial state probabilities for the two initial state and parameters displayed in figure 1 at a similar time interval starting at $t = 500\tau$ (red curve relates to the red curve in the upper panel, and blue and black-dashed curves relate to the curves in the lower panel).

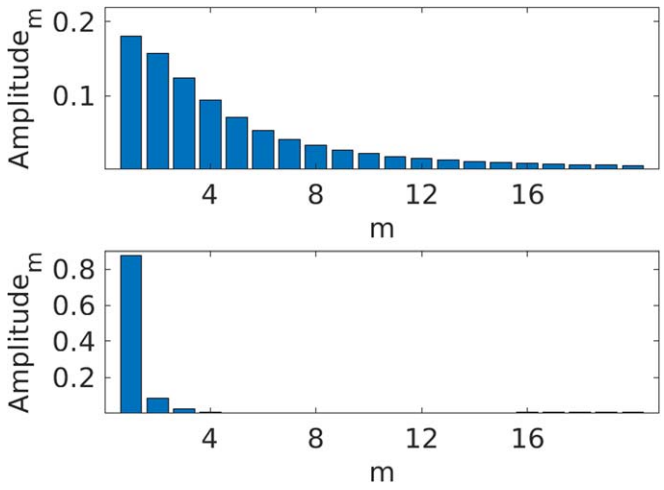

Figure 3. Numerical magnitude of the amplitudes of individual eigenstate coefficients of the initial state in two cases: when we initialize the system in the excited state, $|\tilde{a}_m|$ in equation (5) (upper panel), and when we initialize the system in the resonant band state $|\tilde{b}^{(b)}_{0,m}|$ in equation (8) (lower panel), for increasing $|m|$ in figure 1. Parameters as in figure 1.

coupled to a quasi-continuum $b_n(t)$ [2, 3]. For a single-point interaction (constant band coupling), we arrive at a delay differential equation for the excited state [3],

$$\dot{a}(t) = -i\beta \sum_{n=-\infty}^{\infty} \mathrm{e}^{-i\omega_n t} b_n(0) - \frac{\gamma}{2} a(t) - \gamma \sum_{n=1}^{\infty} a(t - n\tau)\Theta(t - n\tau). \tag{11}$$

We immediately note from the last term in equation (11) that memory effects, leading to non-Markovian time evolution, start to play a role at times $t \geqslant \tau$ (of course $a(t) = 0$ at $t < 0$). Solutions to these evolution equations can straightforwardly be found in Laplace space, and the evolution of the band states are found via $b_n(t) = \mathrm{e}^{-i\omega_n t} b_n(0) - i\beta_n \int_0^t \mathrm{dt}'\, \mathrm{e}^{-i\omega_n (t-t')} \mathrm{a(t')}$. In fact, in Laplace space, the solution is found to be

$$\tilde{a}(s) = \frac{a(0)}{s + \Pi(s)} + \sum_n \frac{(-i\beta_n) b_n(0)}{(s + i\omega_n)[s + \Pi(s)]} \tag{12}$$

where in this case $\Pi(s) = \frac{\gamma}{2} \coth\left(\frac{\tau}{2} s\right)$. The location of the poles is easily found, but the structure is complicated by the presence of higher-order poles [3].

In contrast, for a cosine-modulated coupling $\beta_n = \beta \cos(\omega_n T/2)$, the dynamics is more complicated and get the following delay equation,

$$\dot{a}(t) = -i\beta \sum_{n=-\infty}^{\infty} \cos\left(\frac{\omega_n T}{2}\right) e^{-i\omega_n t} b_n(0) - \frac{\gamma}{4} \sum_{n=\lceil \frac{T}{\tau} \rceil}^{\infty} a(t - T - n\tau) e^{-i\omega_0 \tau n} \Theta(t - T - n\tau) - \frac{\gamma}{4} \sum_{n=\lceil \frac{T}{\tau} \rceil}^{\infty} a(t + T - n\tau) e^{-i\omega_0 \tau n} \Theta(t + T - n\tau) - \frac{\gamma}{2}\left[\frac{a(t)}{2} + \sum_{n=1}^{\infty} a(t - n\tau) e^{-i\omega_0 \tau n} \Theta(t - n\tau)\right],$$

where $\lceil \ldots \rceil$ denotes the ceiling function. In this case, memory effects are associated both with the intrinsic revival period τ as for the previous case but also with the time-scale T related to the phonon propagation. Let us concretely consider a scenario where $T \ll \tau$, and look at early times, i.e. $t < \tau$. We end up with

$$\dot{a}(t) = -i\beta \sum_n \cos\left(\frac{\omega_n T}{2}\right) e^{-i\omega_n t} b_n(0) - \frac{\gamma}{4}[a(t) + a(t - T)\Theta(t - T)]. \tag{13}$$

In Laplace space, the solution of $\tilde{a}(s)$ is again given by equation (12), but now with $\Pi(s) = \frac{\gamma}{4}(e^{-sT} + 1)$.

Such differential equations are known to possess step-like behavior and can be solved considering independent intervals. An interesting phenomenon occurs for the cosine-modulated coupling when we consider the strong coupling limit $\gamma \gg 1$ while keeping $\tau \gg T$ constant. Starting the system dynamics from the excited state (i.e. $a(0) = 1$ while $b_n(0) = 0$ for all n), leads to the following evolution of the excited state

$$a(t) = \sum_{n=0}^{\infty} \left(-\frac{\gamma}{4}\right)^n \frac{(t - nT)^n}{n!} e^{-\gamma(t - nT)/4} \Theta(t - nT), \tag{14}$$

valid for $t \ll \tau$. For $\gamma T \gg 1$ this results in a 'spiky' pattern for $|a(t)|^2$, where the first maximum is at $t = 0$ with value 1. The next spike occurs at $T + 2/\gamma$ with value e^{-2} which is independent of the coupling [6], et cetera. For the lowest band state $b_0(t)$ in the same limit, this results in a cyclical 'on-and-off' behavior, i.e.

$$b_0(t) \simeq -i\beta \frac{4}{\gamma}[H_0(T, t) + H_2(T, t) + \ldots], \tag{15}$$

where $H_n(T, t) = \Theta((n + 1)T - t)\Theta(t - nT)$ is a 'step' function which is 1 at intervals $nT < t < (n + 1)T$ and 0 otherwise. The state is filled with a constant density only in the odd intervals of time, i.e. for $2n\,T < t < (2n + 1)T$. This behavior is clearly seen in figure 4 (upper, left panel). In the upper, right panel of figure 4 we have visualized the probability density of band states up to $n = 36$ (in symmetric combinations, i.e. $|b_n(t)|^2 + |b_{-n}(t)|^2$). The fact that high-$n$ band states are activated for any t/T is very different from the behavior for constant coupling in figure 1. Note also completely empty (dark) band states, which correspond to values of n that leads to vanishing coupling $\beta_n \approx 0$. The resulting dynamics is continuously dependent on the strength of the coupling: the 'spikes' in $a(t)$ get sharper and sharper, while the plateaus in $b_0(t)$ are more and more suppressed.

This is in stark contrast to a parameter-insensitive and stable behavior when starting the system dynamics from the band (i.e. $a(0) = 0, b_{n\neq 0}(0) = \delta_{n,0}$). In this case, the dominant terms are only the ones where we pick up the poles at the origin in Laplace space. Again this results in a cyclical evolution of the excited state, i.e. $a(t) \simeq -i\beta\frac{4}{\gamma}[\Theta(T - t)\Theta(t) + \Theta(3T - t)\Theta(t - 2T) + \ldots]$. This behavior is identical to that of $b_0(t)$ in the previous case due to detailed balance. Surprisingly, this results in a 'step-wise' descent of the occupancy of the band-state, i.e.

$$b_0(t) = \left(1 - \frac{4t}{\tau}\right) H_0(T, t) + \left(1 - \frac{4T}{\tau}\right) H_1(T, t) + \left(1 - \frac{4t}{\tau}\right) H_2(T, t) + \ldots, \tag{16}$$

which is independent on the coupling constant γ and driven merely by the delay time T of the two-point coupling. This evolution is plotted in figure 4 (lower panels). Note, that this behavior is of course modified when we approach $t \approx \tau$, where the additional terms will play a role. Perhaps surprisingly, for a constant coupling scenario, where τ plays the role of the recurrence time, this behavior does not occur.[2] In the lower, right panel of figure 4 we again have visualized the probability density of band states up to $n = 11$ (as above). This demonstrates, as in figure 1, the dominance of the first few band states $n \lesssim 4$ driving the dynamics of the whole system.

It is imperative to emphasize, that the pattern of evolving probabilities described above is independent of the coupling, γ, provided that it is large enough. In the first scenario the 'spikes' observed in $a(t)$ occur at $t \approx T$ with

[2] Instead, in terms of the expansion $a(t) = a^{(0)}(t) + \sum_{k=1}^{\infty} a^{(k)}(t)$, we get $a^{(0)}(t) = -i\beta\frac{2}{\gamma}$ and $a^{(k)}(t) = (-1)^{k+1} i\beta\frac{4}{\gamma}\Theta(t - k\tau)$ for $k \geqslant 1$. The mismatch of numerical pre-factors ruins the prefect cancellation between different τ-periods.

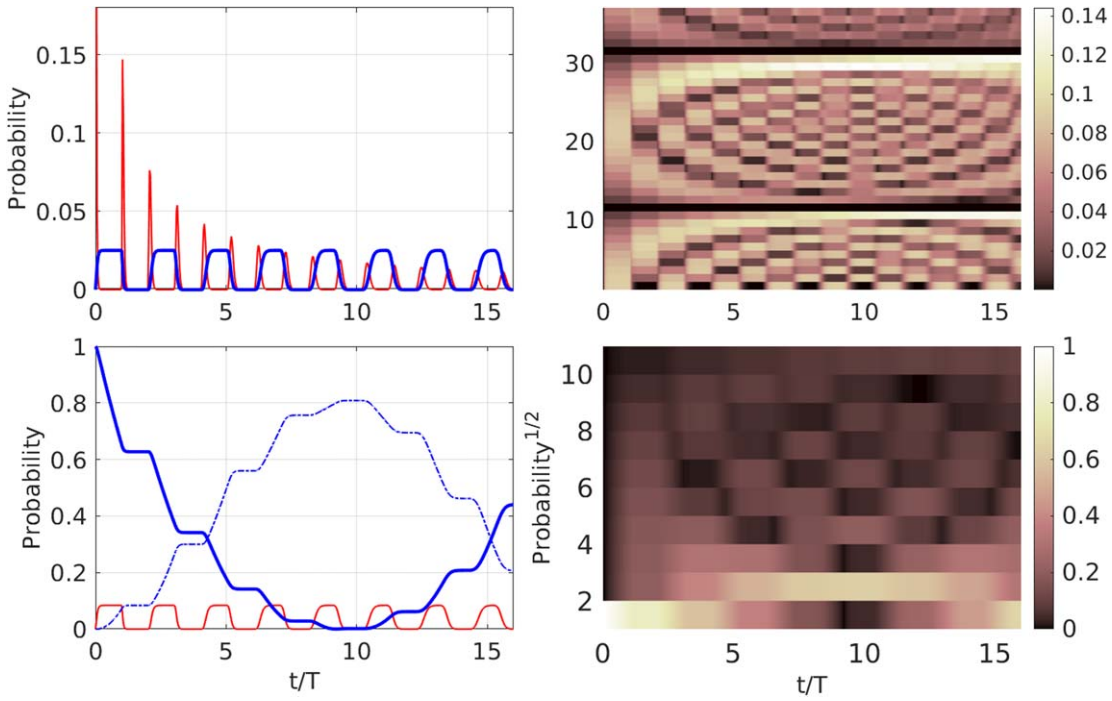

Figure 4. Analogous simulations as in figure 1 but for a giant atom coupled to the continuum via a harmonic coupling (two-point coupling). **Upper panels**: The left figure shows the initial state probability $|a(t)|^2$ as function of reduced time (red thin curve) as well as the probability of populating the resonant state $|b_0(t)|^2$ of a timespan up to 15 revivals. The right panel shows the distribution onto the states ($|b_n\rangle$) for $n \in (0, 36)$, as in figure 1. **Lower panels**: Both figures show the same time evolution as above, except that the initial state is the resonant band state ($|b_0\rangle$). The broken blue curve shows the probability of populating the states ($|b_1\rangle$, $|b_{-1}\rangle$). The right figure shows the distribution onto the states ($|b_n\rangle$) for $n \in (0, 11)$. Parameters are chosen to be $\Delta = 0.314$ and $\beta = 2.186$. The results in this panel apply to any β-value larger than the one used in the present simulation.

heights that are just pure numbers. In the second scenario, it is the ratio T/τ that governs the height of the 'steps' in the even intervals of $b_0(t)$ with a *linear* decay in the odd intervals.

4. Summary

The characteristic dynamics of a two-level atom interacting with a discretized bath is extremely sensitive to its initial state. When starting in the excited atomic state the decay is initially exponential followed by a pattern of decaying revivals towards a noisy behavior. The dynamics is always sensitive to the two parameters of the model, the energy separation Δ and the coupling strengths β_n. In this paper we have shown, based on solution of the Schrödinger equation using matrix algebra or solutions of delayed-time equations, that the initial band state are populated with a cosine like near oscillatory behavior over all time intervals. Furthermore, when introducing a harmonic (two-point) coupling, we have shown that the decay becomes stepwise linear and insensitive to the model parameters. This novel behavior could be realized in experiment when allowing the ground state atom (qubit) interact with an excited cavity of photons or phonons, i.e. tuned to the excited level [21].

Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

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