

Review

Fifty Years of the Dynamical Casimir Effect

V.V. Dodonov 

Institute of Physics and International Center for Physics, University of Brasilia, P.O. Box 04455, Brasília 70919-970, Brazil; vdodonov@fis.umb.br

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Abstract: This is a digest of the main achievements in the wide area, called the Dynamical Casimir Effect nowadays, for the past 50 years, with the emphasis on results obtained after 2010.

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1. Introduction

Fifty years ago, in June 1969, G.T. Moore finished his PhD thesis (prepared under the guidance of H.N. Pendleton III and submitted to the Brandeis University), part of which was published in the next year as [1]. Using a simplified one-dimensional model, Moore showed that motions of ideal boundaries of a one-dimensional cavity could result in a generation of quanta of the electromagnetic field from the initial *vacuum quantum state*. A few years later, DeWitt [2] demonstrated that moving boundaries could induce particle creation from a vacuum in a single-mirror set-up. A more detailed study was performed by Fulling and Davies [3,4]. Thus, step by step, year by year, more and more authors followed this direction of research. In 1989, two names were suggested for such kinds of phenomena: *dynamic (or non-adiabatic) Casimir effect* [5] and *Nonstationary Casimir Effect* [6]. Being supported in part by the authority of Schwinger [7], the first name gradually acquired the overwhelming popularity, so that now we have an established direction in the theoretical and experimental physics, known under the general name *Dynamical Casimir Effect* (DCE). This area became rather large by now: more than 300 papers containing the words “dynamical Casimir” have been published already, including more than 100 publications during the past decade. Moore’s paper [1] has been cited more than 400 times, and some authors use the name “Moore effect” instead of DCE (other names were “Mirror Induced Radiation” or “Motion Induced Radiation”).

To combine different studies under the same “roof”, it seems reasonable to assume the following definition of the Dynamical Casimir Effect: *Macroscopic phenomena caused by changes of vacuum quantum states of fields due to fast time variations of positions (or properties) of boundaries confining the fields (or other parameters)*. Such phenomena include, in particular, the modification of the Casimir force for moving boundaries. However, the most important manifestation is *the creation of the field quanta (photons) due to the motion of neutral boundaries*. The most important ingredients of the DCE are *quantum vacuum fluctuations* and *macroscopic manifestations*. The reference to vacuum fluctuations explains the appearance of Casimir’s name (by analogy with the famous *static* Casimir effect, which is also considered frequently as a manifestation of quantum vacuum fluctuations), although Casimir himself did not write anything on this subject. Therefore, the DCE can be considered as the specific subfield of a much bigger physical area, known nowadays under the name *Casimir Physics*. This whole area is outside the present study, so that we give only a few references to the relevant reviews and books [8–24].

In turn, the subject of the DCE can be divided in several sub-areas. In the strict (narrow) sense, one can think about the “single mirror DCE” or “cavity DCE”. In the most wide sense, the DCE

can be related to the *amplification of quantum (vacuum) fluctuations in macroscopic systems*, and many authors studied analogs of the DCE in parametrically excited media. The “intermediate” area of quantum circuits with time-dependent parameters is also frequently considered as belonging to the DCE. The aim of our review is to describe the main achievements in all those sub-areas for the past 50 years, as well as still unsolved and challenging problems. This is not the first review on the subject. The early history, including studies on the *classical fields* with moving boundaries, can be found in [25–28]. The most recent reviews were published almost a decade ago [29–31]. Therefore, here I tried to collect a more or less complete list of references to the publications of the decade after 2009. As for earlier papers, I have included only a small part of them, to maintain a reasonable length of this review.

2. One-Dimensional Models with Moving Boundaries

One-dimensional “toy” models were considered by many authors because they are much simpler than the realistic 3D ones; therefore, they admit exact and relatively simple analytical solutions, which can provide some insight into the real effects.

2.1. Classical Fields

The history of studies on classical fields in time-dependent domains began almost 100 years ago. The first exact solution of the wave equation (with $c = 1$)

$$\partial^2 A / \partial t^2 - \partial^2 A / \partial x^2 = 0, \quad (1)$$

in a time-dependent domain $0 < x < L(t)$, satisfying the boundary conditions

$$A(0, t) = A(L(t), t) = 0, \quad (2)$$

was obtained by Nicolai [32] for

$$L(t) = L_0(1 + \alpha t). \quad (3)$$

The solution was interpreted in terms of the transverse vibrations of a string with a variable length. A few years later, these results were published in [33], where the extension to the case of electromagnetic field was also made. A similar treatment was given by Havelock [34] in connection with the problem of radiation pressure. However, this subject hardly attracted the attention of researchers for several decades. The splash of new studies happened in the 1960s, probably due to the invention of lasers. We cite here only a few of relevant papers [35–38]. Much more references can be found in [26,39,40].

2.2. Quantum Fields

In the quantum case, one of the most interesting problems is the creation of quanta of different fields from the initial vacuum (ground) state due to the time variation of some parameters. The generation of massive particles by time-dependent gravitational fields was considered for the first time, probably, by Schrödinger [41]. Later on, this problem was treated by Imamura [42], Parker [43,44], and Zel’dovich [45]. However, the main subject of the present paper is the creation of *massless* quanta (photons). In this case, the simplest model is the so-called “scalar electrodynamics”, where the main object is the single component of the vector potential operator, perpendicular to the axis x (and parallel to the infinite plane surfaces confining the field). This operator satisfies, in the Heisenberg picture, the same Equation (1) as the classical field vector. If the field is confined between two ideal surfaces (ideal conducting walls) with coordinates $x = 0$ and $x = L(t) > 0$, then the condition of vanishing the electric field at each surface, *in the frame moving with the wall*, results in the boundary conditions (2).

It is assumed usually that both walls were at rest for $t \leq 0$, so that initially the field can be written in the form

$$A(x, t < 0) = \sum_{n=0}^{\infty} c_n \sin(n\pi x/L_0) \exp(-i\omega_n t), \quad \omega_n = n\pi/L_0, \quad (4)$$

where coefficients c_n are complex numbers in the classical case and operators in the quantum case. Then, one has to find the function (operator) $A(x, t)$ for $t > 0$. This problem can be solved in the frameworks of two approaches.

2.2.1. Moore's Approach

The specific feature of the wave equation in the single spatial dimension (1) is the existence of the well known form of the general solution: $A(x, t) = f(x - t) + g(x + t)$, where f and g can be arbitrary functions. How can these functions be chosen to satisfy the additional boundary conditions (2)? Moore [1] has found a complete set of solutions to the problem, satisfying the initial condition (4), in the form

$$A_n(x, t) = C_n \{ \exp[-i\pi n R(t - x)] - \exp[-i\pi n R(t + x)] \}, \quad (5)$$

where function $R(\xi)$ must satisfy the functional equation

$$R(t + L(t)) - R(t - L(t)) = 2. \quad (6)$$

As a matter of fact, a quite similar approach was used in [32–34], although for a *linear* function $L(t)$ only. Independently, Equation (6) was obtained by Vesnitskii [38].

It is easy to find an exact solution to Equation (6) for the uniform law of motion of the boundary (3). This was done by many authors, starting from [32–34]. We give it in the form obtained in [37]:

$$R_\alpha(\xi) = \frac{2 \ln |1 + \alpha \xi|}{\ln |(1 + v)/(1 - v)|}, \quad v = \alpha L_0. \quad (7)$$

Evidently, function (7) goes to $R_0(\xi) = \xi/L_0$ if $\alpha \rightarrow 0$. For an arbitrary nonrelativistic law of motion, one can find the solution in the form of the expansion over subsequent time derivatives of the wall displacement. Such an idea goes to the papers [1,38] whose results (with some corrections) can be written in the form [26]

$$R(\xi) = \xi \lambda(\xi) - \frac{1}{2} \xi^2 \dot{\lambda}(\xi) + \frac{1}{6} \xi \ddot{\lambda}(\xi) [\xi^2 - L^2(\xi)] + \dots, \quad \lambda(\xi) \equiv L^{-1}(\xi). \quad (8)$$

In the special case of $L(t) = L_0/(1 + \alpha t)$, when $\ddot{\lambda}(\xi) \equiv 0$, Equation (8) yields another exact solution, $R(\xi) = L_0^{-1} \left(\xi + \frac{1}{2} \alpha \xi^2 \right)$. Unfortunately, the expansions such as Label(8) cannot be used in the long-time limit $\xi \rightarrow \infty$, since the terms proportional to the derivatives of $\lambda(\xi)$ (which are supposed to be small corrections) become bigger than the unperturbed term $\xi \lambda(\xi)$.

Several exact solutions to the Moore equation were found with the aid of the “inverse” method [40,46,47], when one chooses some reasonable function $R(\xi)$ and determines the corresponding law of motion of the boundary $L(t)$ using the consequence of Equation (6),

$$\dot{L}(t) = \frac{R'[t - L(t)] - R'[t + L(t)]}{R'[t - L(t)] + R'[t + L(t)]}. \quad (9)$$

To solve differential Equation (9) with some simple functions, $R(\xi)$ is more easier than solving the functional Equation (6) for the given function $L(t)$. However, the dependence $L(t)$ does not appear to be admissible from the point of view of physics (the velocity may occur greater than the speed of light, or some discontinuities may arise) for any simple function $R(\xi)$. A large list of simple functions $R(\xi)$ (rational, exponential, logarithmic, hyperbolic, trigonometrical, and inverse trigonometrical) and, corresponding to them, functions $L(t)$ can be found in [40]. The cases considered in [47] correspond to

some monotonous displacements of the mirror from the initial to final positions. Typical functions $R(\xi)$ used in [47] were some combinations of ξ/L_0 and some trigonometric functions, such as $\sin(m\pi\xi/L_0)$. However, none of these functions can be used in the parametric resonance case. Exact solutions for some specific trajectories of the *single* mirror were found in [48–59]. Two mirrors moving with constant accelerations in opposite directions were considered in [60,61]. Massless spin-1/2 fields in a 1D box with moving boundaries were studied in [62], while the single mirror case was considered in [63]. The solutions of the 1D Klein–Gordon equation with one uniformly moving boundary (and another boundary at rest) were found in [64,65].

The *asymptotic* solution of the Moore equation for $L(t) = L_0 [1 + \epsilon \sin(\pi q t/L_0)]$, with $q = 1, 2, \dots$ and $|\epsilon| \ll 1$, was found in [66,67]. For $\epsilon t \gg 1$, it has the form (here $L_0 = 1$)

$$R(t) = t - \frac{2}{\pi q} \operatorname{Im} \{ \ln [1 + \zeta + \exp(i\pi q t)(1 - \zeta)] \}, \quad \zeta = \exp \left[(-1)^{q+1} \pi q \epsilon t \right], \quad (10)$$

which clearly demonstrates that the asymptotic mode structure in the resonance case is quite different from the mode structure inside the cavity with unmoving walls. The solution (10) was improved in [68]. The additional term

$$\Delta R(t) = \frac{2\zeta\epsilon(-1)^q z \sin(q\pi z)}{1 + \zeta^2 + (1 - \zeta^2) \cos(q\pi z)}, \quad -1 \leq z = t - 2p \leq 1, \quad p = 0, 1, 2, \dots \quad (11)$$

is negligible in the long-time limit, but it is crucial for the solution to satisfy the correct boundary condition at short times. The case of two moving boundaries, left $L(t)$ and right $R(t)$, was studied in [69]. Then, Equations (5) and (6) can be generalized as follows:

$$A_n(x, t) = C_n \{ \exp[-i\pi n F(t - x)] - \exp[-i\pi n G(t + x)] \}, \quad (12)$$

$$G(t + R(t)) - F(t - R(t)) = 2, \quad G(t + L(t)) - F(t - L(t)) = 0. \quad (13)$$

The solutions generalizing (10) and (11) were found in [69] for the boundaries oscillating with equal resonance frequencies, but different amplitudes and with some phase difference. Numerical solutions of Equations (13) were presented in [70]. The authors of [71,72] pointed out on a possible physical realization of one-dimensional models in the case of TEM modes in cylindrical waveguides. The methods of characteristics and circle maps were applied in [73–75]. The one-dimensional cavity with one and two oscillating mirrors was considered within the framework of the “optical” approach in [76]. Generalizations of Moore’s approach to the one-dimensional vibrating cavities were considered in [77]. A one-dimensional *uniformly contracting* cavity was studied in [78]. The computer program for the numerical solution of the Moore equation was presented in [79]. The *Floquet map* was applied in [80].

2.2.2. The Modification of the Casimir Force

How can the function $R(\xi)$ be used? The first application is connected with the modification of the static Casimir force due to the motion of boundaries. It was shown in [3,81] that the force pressing the moving wall (more precisely, the T_{11} component of the energy-momentum tensor of the field) is given by a simple formula (for the initial vacuum state of the field)

$$F = -g[t - L(t)] - g[t + L(t)], \quad (14)$$

where

$$g(\xi) = \frac{1}{24\pi} \left\{ \frac{R'''(\xi)}{R'(\xi)} - \frac{3}{2} \left[\frac{R''(\xi)}{R'(\xi)} \right]^2 \right\} + \frac{\pi}{48} [R'(\xi)]^2. \quad (15)$$

For the wall at rest, when $R_0(\xi) = \xi/L_0$, one gets the following expression for the stationary Casimir force in one dimension [3,81] (returning to the dimensional variables):

$$F = -\pi\hbar c/(24L_0^2). \quad (16)$$

If the distance between the boundaries $L(t)$ slowly varies with time (in the sense that $|dL/dt| \ll c$), then the approximate solution (8) yields [6] the following generalization of formula (16):

$$F = -\frac{\pi\hbar c}{24L^2(t)} \left[1 + \left(\frac{\dot{L}}{c} \right)^2 \left(\frac{7}{3} - \frac{1}{\pi^2} \right) - \frac{L\ddot{L}}{c^2} \left(\frac{2}{3} - \frac{2}{\pi^2} \right) \right]. \quad (17)$$

The Casimir force in a cavity moving with a constant acceleration was calculated in [82].

In the parametric resonance case, the force averaged over the period of oscillations was calculated in [67] (in the dimensionless units with $L_0 = 1$):

$$\langle F \rangle = -\frac{\pi}{24} \left[q^2 + \frac{1}{2} (1 - q^2) (\zeta + \zeta^{-1}) \right], \quad \zeta = \exp [(-1)^{q+1} \pi q \epsilon t]. \quad (18)$$

For $q \geq 2$, we do not have attraction, but an exponentially increasing pressure on the oscillating wall due to the creation of real photons in the cavity. An effective action approach to the problem of dynamical Casimir force in 1+1 dimensions was developed in [83].

2.2.3. Generation of Quanta inside the 1D Cavity with Moving Boundary

The knowledge of function $R(\xi)$ also permits one to calculate the mean number of created quanta inside a nonstationary cavity. The general scheme is as follows [1,81]. Taking into account only the electromagnetic modes whose vector potential is directed along z -axis (“scalar electrodynamics”), one can write down the field operator *in the Heisenberg representation* $\hat{A}(x, t)$ at $t \leq 0$ (when both the plates were at rest at the positions $x_{left} = 0$ and $x_{right} = L_0$) as (we assume $c = \hbar = 1$)

$$\hat{A}_{in} = 2 \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \sin \frac{n\pi x}{L_0} \hat{b}_n \exp(-i\omega_n t) + \text{h.c.} \quad (19)$$

where \hat{b}_n means the usual annihilation photon operator and $\omega_n = \pi n / L_0$. The choice of coefficients in Equation (19) corresponds to the standard form of the field Hamiltonian (in the Gaussian units)

$$\hat{H} \equiv \frac{1}{8\pi} \int_0^{L_0} dx \left[(\partial A / \partial t)^2 + (\partial A / \partial x)^2 \right] = \sum_{n=1}^{\infty} \omega_n \left(\hat{b}_n^\dagger \hat{b}_n + 1/2 \right). \quad (20)$$

For $t > 0$, the field operator can be written as

$$\hat{A}(x, t) = 2 \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \left[\hat{b}_n \psi^{(n)}(x, t) + \text{h.c.} \right], \quad (21)$$

where functions $\psi^{(n)}(x, t)$ are given by formula (5) with $C_n = -i/2$ (in order to satisfy the initial conditions). If the wall stops after some time T , then the field operator can be expanded again over the complete set of sine functions, like in Equation (19), but with the “physical” operators \hat{a}_m and \hat{a}_m^\dagger instead of \hat{b}_m and \hat{b}_m^\dagger . The two sets of operators are related by means of the Bogoliubov transformation

$$\hat{a}_m = \sum_{n=1}^{\infty} \left(\hat{b}_n \alpha_{nm} + \hat{b}_n^\dagger \beta_{nm}^* \right), \quad m = 1, 2, \dots, \quad (22)$$

which implies the unitarity conditions

$$\sum_{m=1}^{\infty} (\alpha_{nm}^* \alpha_{km} - \beta_{nm}^* \beta_{km}) = \delta_{nk}, \quad \sum_{n=1}^{\infty} (\alpha_{nm}^* \alpha_{nj} - \beta_{nm}^* \beta_{nj}) = \delta_{mj}, \quad \sum_{n=1}^{\infty} (\beta_{nm}^* \alpha_{nk} - \beta_{nk}^* \alpha_{nm}) = 0. \quad (23)$$

If the final position of moving boundary coincides with initial position L_0 , then the Bogoliubov coefficients can be found in the form [84] (here $c = 1$)

$$\left. \begin{array}{l} \alpha_{mn} \\ \beta_{mn} \end{array} \right\} = \frac{1}{2} \sqrt{m/n} \int_{t/L_0-1}^{t/L_0+1} dx \exp \{-i\pi [nR(L_0x) \mp mx]\}. \quad (24)$$

The mean number of photons in the m th mode equals the average value of the operator $\hat{a}_m^\dagger \hat{a}_m$ in the initial state $|\text{in}\rangle$, since namely this operator has a physical meaning at $t > T$:

$$\mathcal{N}_m \equiv \langle \text{in} | \hat{a}_m^\dagger \hat{a}_m | \text{in} \rangle = \sum_n |\beta_{nm}|^2 + \sum_{n,k} \left[(\alpha_{nm}^* \alpha_{km} + \beta_{nm}^* \beta_{km}) \langle \hat{b}_n^\dagger \hat{b}_k \rangle + 2\text{Re}(\beta_{nm} \alpha_{km} \langle \hat{b}_n \hat{b}_k \rangle) \right]. \quad (25)$$

The first sum on the right-hand side of (25) describes the effect of the photon creation from vacuum, while the other sums are different from zero only in the case of a non-vacuum initial state of the field.

In the resonance case, solution (10) leads after long calculations to the following simple formula for the rate of photon generation in the m th mode, when the wall vibrates at the twice frequency of the first resonator eigenmode, in the limit $\epsilon t \gg 1$ [67]:

$$d\mathcal{N}_m/dt \approx |\epsilon| [1 - (-1)^m] / (m\pi). \quad (26)$$

The energy density inside the cavity can be calculated as [3,81] $\langle T_{00}(x, t) \rangle = -g(t+x) - g(t-x)$, with the same function $g(\xi)$ (15). It was shown in [68,85–87] that, for $q \geq 2$, the energy density grows exponentially in the form of q traveling wave packets, which become narrower and higher as time increases. The total energy also grows exponentially. For two resonantly oscillating boundaries (with equal frequencies), the asymptotic photon production rate is given by the same formula (26), where ϵ should be replaced with $\tilde{\epsilon} = \epsilon_L + (-1)^{q+1} \epsilon_R \cos(\phi)$, where $\epsilon_{L,R}$ are relative oscillation amplitudes of each boundary and ϕ the phase difference [69].

The influence of initial states of the field and different boundary conditions on the energy density and radiation force was studied analytically and numerically in [88–91]. The case of two relativistic moving mirrors was studied in [92].

2.2.4. Expansions over the Instantaneous Basis

Another approach to the problem consists of expanding the function $\psi^{(n)}(x, t)$ in Equation (21) in a series with respect to the *instantaneous basis* [93–97]:

$$\psi^{(n)}(x, t > 0) = \sum_{k=1}^{\infty} Q_k^{(n)}(t) \sqrt{\frac{L_0}{L(t)}} \sin \left(\frac{\pi k [x - u(t)]}{L(t)} \right), \quad n = 1, 2, \dots \quad (27)$$

with the initial conditions $Q_k^{(n)}(0) = \delta_{kn}$, $\dot{Q}_k^{(n)}(0) = -i\omega_n \delta_{kn}$, $k, n = 1, 2, \dots$. This way, we satisfy automatically both the boundary conditions, $A(u(t), t) = A(u(t) + L(t), t) = 0$, and the initial condition (4). Putting expression (27) into the wave Equation (1), one can arrive after some algebra at an infinite set of coupled differential equations [97–99]

$$\ddot{Q}_k^{(n)} + \omega_k^2(t) Q_k^{(n)} = 2 \sum_{j=1}^{\infty} g_{kj}(t) \dot{Q}_j^{(n)} + \sum_{j=1}^{\infty} \dot{g}_{kj}(t) Q_j^{(n)} + \mathcal{O}(g_{kj}^2), \quad (28)$$

$$\omega_k(t) = k\pi/L(t), \quad g_{kj}(t) = -g_{jk}(t) = (-1)^{k-j} \frac{2kj(\dot{L} + \dot{u}\epsilon_{kj})}{(j^2 - k^2)L(t)}, \quad \epsilon_{kj} = 1 - (-1)^{k-j}, \quad j \neq k. \quad (29)$$

For $u = 0$ (the left wall at rest), the equations like (28)–(29) were derived in [95,100].

If the wall comes back to its initial position L_0 after some interval of time T , then the right-hand side of Equation (28) disappears, so, at $t > T$, one can write

$$Q_k^{(n)}(t) = \xi_k^{(n)} e^{-i\omega_k(t+\delta T)} + \eta_k^{(n)} e^{i\omega_k(t+\delta T)}, \quad k, n = 1, 2, \dots \quad (30)$$

where $\xi_k^{(n)}$ and $\eta_k^{(n)}$ are some constant complex coefficients. To find these coefficients, one has to solve an infinite set of coupled Equation (28) ($k = 1, 2, \dots$) with time-dependent parameters; moreover, each equation also contains an infinite number of terms. However, the problem can be essentially simplified, if the walls perform small oscillations at the frequency ω_w close to some unperturbed field eigenfrequency: $L(t) = L_0(1 + \epsilon_L \sin[p\omega_1(1 + \delta)t])$, $u(t) = \epsilon_u L_0 \sin[p\omega_1(1 + \delta)t + \varphi]$. Assuming $|\epsilon_L|, |\epsilon_u| \sim \epsilon \ll 1$, it is natural to look for the solutions to Equation (28) in the form

$$Q_k^{(n)}(t) = \xi_k^{(n)} e^{-i\omega_k(1+\delta)t} + \eta_k^{(n)} e^{i\omega_k(1+\delta)t}, \quad (31)$$

allowing now the coefficients $\xi_k^{(n)}$ and $\eta_k^{(n)}$ to be *slowly varying functions of time*. Then, using the standard method of slowly varying amplitudes [101], one can arrive [97] at the following set of equations, containing only three terms with simple *time independent* coefficients in the right-hand sides:

$$\frac{d}{d\tau} \xi_k^{(n)} = (-1)^p \left[(k+p)\xi_{k+p}^{(n)} - (k-p)\xi_{k-p}^{(n)} \right] + 2i\gamma k \xi_k^{(n)}, \quad (32)$$

$$\frac{d}{d\tau} \eta_k^{(n)} = (-1)^p \left[(k+p)\eta_{k+p}^{(n)} - (k-p)\eta_{k-p}^{(n)} \right] - 2i\gamma k \eta_k^{(n)}, \quad (33)$$

$$\tau = \epsilon\omega_1 t/2, \quad \gamma = \delta/\epsilon, \quad \xi_k^{(n)}(0) = \delta_{kn}, \quad \eta_k^{(n)}(0) = 0. \quad (34)$$

Equations (32) and (33) correspond to the case of $u = 0$ (the left wall at rest). Actually, the term \dot{u} in $g_{kj}(t)$ does not make any contribution to the simplified equations of motion for *even* values of integer parameter p , so that the rate of change of the cavity length \dot{L}/L_0 is only important in this case. On the contrary, if p is an *odd* number, then the field evolution depends on the velocity of the *center of the cavity* $v_c = \dot{u} + \dot{L}/2$, but does not depend on \dot{L} alone [102]. Therefore, one should simply replace \dot{L}/L_0 by $2v_c/L_0$, if p is an odd number.

The uncoupled Equations (32) and (33) hold only for $k \geq p$. This means that they describe the evolution of *all* the Bogoliubov coefficients, only if $p = 1$. Then *all* the functions $\eta_k^{(n)}(t)$ are *identically equal to zero* due to the initial conditions (34); consequently, no photon can be created *from vacuum* (in the lowest order approximation with respect to the small parameter ϵ). This special “semi-resonance” case was studied in detail in [97,103], where exact analytical solutions to Equation (32) were obtained in terms of the Jacobi polynomials. It was demonstrated that the total number of photons is an integral of motion in this specific case. (A similar phenomenon in the classical case was discussed in [46], whereas the quantum case was also considered in [96]). The exact formula for the total energy in all cavity modes for $p = 1$ (normalized by the fundamental cavity mode energy $\hbar\omega_1$) is as follows:

$$\mathcal{E}(\tau) = \mathcal{E}(0) + \frac{2 \sinh^2(a\tau)}{a^2} \left[\mathcal{E}(0) - \frac{\gamma}{2} \text{Im}(\mathcal{G}_1) \right] + \text{Re}(\mathcal{G}_1) \frac{\sinh(2a\tau)}{2a}, \quad (35)$$

$$a = \sqrt{1 - \gamma^2}, \quad \mathcal{G}_1 = 2 \sum_{n=1}^{\infty} \sqrt{n(n+1)} \langle \text{in} | \hat{b}_n^\dagger \hat{b}_{n+1} | \text{in} \rangle. \quad (36)$$

It can be shown that $|\dot{\mathcal{E}}(0)| \leq \mathcal{E}(0)$. Thus, the total energy grows exponentially, when $\tau \gg 1$ (provided $\gamma < 1$), although it can decrease at $\tau \ll 1$, if $\dot{\mathcal{E}}(0) = \text{Re}(\mathcal{G}_1) < 0$. Since the total number of photons is constant, such a behavior is explained by the effect of pumping the highest modes at the expense of the lowest ones (in the classical case this effect was noticed in 1960s: see [39]). This results in an effective cooling of the lowest electromagnetic modes. The absence of photon creation in the fundamental mode was related to the $SL(2, R)$ symmetry of the problem in Ref. [104].

If $p \geq 2$, we have, in addition to (32)–(33), $p - 1$ pairs of *coupled* equations for the coefficients with lower indexes $1 \leq k \leq p - 1$,

$$\frac{d}{d\tau} \xi_k^{(n)} = (-1)^p \left[(k+p) \xi_{k+p}^{(n)} - (p-k) \eta_{p-k}^{(n)} \right] + 2i\gamma k \xi_k^{(n)}, \quad (37)$$

$$\frac{d}{d\tau} \eta_k^{(n)} = (-1)^p \left[(k+p) \eta_{k+p}^{(n)} - (p-k) \xi_{p-k}^{(n)} \right] - 2i\gamma k \eta_k^{(n)}. \quad (38)$$

In this case, some functions $\eta_k^{(n)}(t)$ are not equal to zero at $t > 0$, thus we have the effect of photon creation from the vacuum. It is convenient to introduce a new set of coefficients $\rho_k^{(n)}$, whose lower indexes run over all integers from $-\infty$ to ∞ :

$$\rho_k^{(n)} = \begin{cases} \xi_k^{(n)} & k > 0 \\ 0 & k = 0 \\ -\eta_{-k}^{(n)} & k < 0 \end{cases} \quad (39)$$

Then, Equations (32), (33), (37) and (38) can be combined into a *single* set of equations ($k = \pm 1, \pm 2, \dots$) [97]

$$\frac{d}{d\tau} \rho_k^{(n)} = (-1)^p \left[(k+p) \rho_{k+p}^{(n)} - (k-p) \rho_{k-p}^{(n)} \right] + 2i\gamma k \rho_k^{(n)}, \quad \rho_k^{(n)}(0) = \delta_{kn}. \quad (40)$$

A remarkable feature of the set of Equation (40) is that its solutions satisfy *exactly* the unitarity conditions (23), which can be rewritten as ($m, j, n, k = 1, 2, \dots$)

$$\sum_{m=-\infty}^{\infty} m \rho_m^{(n)*} \rho_m^{(k)} = n \delta_{nk} \quad \sum_{n=1}^{\infty} \frac{m}{n} \left[\rho_m^{(n)*} \rho_j^{(n)} - \rho_{-m}^{(n)*} \rho_{-j}^{(n)} \right] = \delta_{mj} \quad \sum_{n=1}^{\infty} \frac{1}{n} \left[\rho_m^{(n)*} \rho_{-j}^{(n)} - \rho_j^{(n)*} \rho_{-m}^{(n)} \right] = 0.$$

Exact solutions to the set (40) were found in [97] in terms of the Gauss hypergeometric function. They can be expressed also in terms of the complete elliptic integrals, if $p = 2$ [105]. The most interesting consequences of these solutions are as follows.

There is no photon creation in the modes with numbers $p, 2p, \dots$. In the short-time limit ($\tau \ll 1$), $\mathcal{N}_{j+pq}^{(vac)} \sim \tau^{2q+1}$, but the photon generation rate in each mode tends to the constant value in the long-time limit:

$$\frac{d}{d\tau} \mathcal{N}_{j+pq}^{(vac)} \approx \frac{2ap^2 \sin^2(\pi j/p)}{\pi^2(j+pq)}, \quad ap\tau \gg 1. \quad (41)$$

The total number of photons created from vacuum in all modes increases in time approximately quadratically, both in the short-time and in the long-time limits (although with different coefficients):

$$\ddot{\mathcal{N}}^{(vac)} = \frac{1}{3} p(p^2 - 1), \quad |ap\tau| \ll 1, \quad \ddot{\mathcal{N}}^{(vac)} = 2a^2 p^3 / \pi^2, \quad ap\tau \gg 1, \quad a > 0. \quad (42)$$

At the same time, the total number of “nonvacuum” photons increases in time only *linearly* at $ap\tau \gg 1$.

The total energy in all modes for the initial vacuum state of field is given by the exact formula [97]

$$\mathcal{E}^{(vac)}(\tau) = \frac{p^2 - 1}{12a^2} \sinh^2(p\tau). \quad (43)$$

The total energy increases exponentially at $\tau \rightarrow \infty$, provided $\gamma < 1$. In the special case of $\gamma = 0$, such a behavior of the total energy was obtained also in the frameworks of other approaches in [100,106–108]. Note that the total “vacuum” and “nonvacuum” energies increase exponentially with time, if $\gamma < 1$, whereas the total number of photons increases only as τ^2 and τ , respectively, under the same conditions. This happens because the number of effectively excited modes increases in time exponentially [109]. The parametric instability in systems with moving boundaries was studied in [110–113].

The solutions found in [97] were used in [114,115] to study the effects of *squeezing* and the formation of narrow packets inside the 1D cavity with oscillating walls. The emission of narrow packets outside the 1D Fabry–Perot cavity was studied in [116–118]. The influence of the dispersion of the reflection coefficient of the fixed mirror (with the ideal moving one) and the frequency detuning on the formation of packets inside the 1D cavity was investigated for different initial conditions in [119–121]. The comparisons of numerical and analytical calculations in the one-dimensional case were made in [122–124]. It was shown that the coincidence of results is excellent for small enough values of parameter $\epsilon < 10^{-5}$ (remember that the realistic values are expected to be less than 10^{-7}). However, strong differences were observed for big values $\epsilon > 10^{-2}$. For other approaches to solving equations like (28), see, e.g., [99,125,126]. Recently, these equations were solved numerically in the 1D and 3D cases, for one and two moving boundaries, in the study [127].

The case of mixed boundary conditions—Dirichlet condition $\phi(t, L(t)) = 0$ for the moving wall and Neumann condition $\partial_x \phi(t, 0) = 0$ for the wall at rest—was solved, following the method of [105], in papers [128,129]. In this case, photons can be produced in all modes, differently from the Dirichlet–Dirichlet boundary conditions used in [105].

The analytical results shown above were obtained in the first order approximation with respect to the small parameter ϵ . Taking into account the higher order approximations, the resonance frequencies can be made smaller than the fundamental frequency ω_1 . For example, it was shown that, in the n -th order approximation, the resonances at the frequency $2\tilde{\omega}_1/n$ are possible [130] (the frequency $\tilde{\omega}_1$ is slightly shifted from ω_1 due to nonlinear effects). In particular, the photon creation from vacuum at the frequency close to ω_1 becomes possible in the second order approximation.

2.2.5. Quantum Regime of the Wall Motion

In all papers cited above, the law of motion of the wall (boundary) was prescribed. The inclusion of the moving boundary as a part of the dynamical system was made by Barton and Calogeracos [131,132]. However, they considered the boundary as a classical particle. The back reaction of the field on the motion of a classical wall was taken into account in [133,134]. The resonant energy exchange between a moving boundary and cavity modes was studied in [135]. Another approach was used in Ref. [136], where the mirror was replaced by an ensemble of electrons and ions, bounded by some effective parabolic potential.

The quantized motion of the boundary was considered in [137–141]. The authors of [139–141] considered a scalar field in a one-dimensional cavity with one fixed and one mobile wall; the latter was bound to an equilibrium position by a harmonic potential and its mechanical degrees of freedom were treated quantum mechanically. The presence of the moving wall yielded an effective interaction between the field modes, described by means of the interaction Hamiltonian

$$\hat{H}_{int} = \left(\hat{b} + \hat{b}^\dagger \right) \sum_{jk} C_{kj} \mathcal{N} \left[\left(\hat{a}_k + \hat{a}_k^\dagger \right) \left(\hat{a}_j + \hat{a}_j^\dagger \right) \right], \quad (44)$$

where \hat{b} and \hat{a}_j are bosonic operators, describing the mobile mirror and field modes, respectively, and \mathcal{N} is the normal ordering operator. Note that a simplified version of (44), $\hat{H}_{int} = \hbar G \left(\hat{b} + \hat{b}^\dagger \right) \hat{a}^\dagger \hat{a}$, was used to describe the ponderomotive effects of a strong laser field in cavities with oscillating walls, but under the condition that the mechanical oscillator frequency is many orders of magnitude smaller

than the cavity frequency, so that the DCE is negligible [142–145]. The recent progress in studies of such fully quantized optomechanical systems in connection with the DCE can be seen in [146–152].

2.3. Partially Transparent Mirrors and General Boundary Conditions in 1D

The DCE with partially reflecting mirrors in the one-dimensional models was considered in [53,102,117,131,132,153–159]. The comparison of the Dirichlet and Neumann boundary conditions in terms of the solutions to Moore’s Equation (6) was performed in paper [72]. Another approach to this problem was used in [160]. The so-called Robin boundary conditions of the form

$$(\phi - \gamma(t)\partial\phi/\partial x = 0) \mid \text{at the boundary} \quad (45)$$

were studied in connection with the DCE in [131,132,161–164]. The simplest model uses the complex reflection and transmission coefficients in the form [131,132]:

$$r = -\frac{i\gamma}{\omega + i\gamma}, \quad t = \frac{\omega}{\omega + i\gamma}. \quad (46)$$

This is equivalent to the model of the boundary as an equivalent delta-potential or a jellium-type plasma sheet of infinitely small thickness. Such kind of models was used e.g., in Refs. [165,166]. Quantum and classical effects produced by thin sheets of electrons, working as relativistic mirrors, were considered in [167–169].

The further generalization to the $\delta - \delta'$ potential was considered in [170]. In this model, the reflection and transmission coefficients have the form

$$r = \frac{2\omega\lambda - i\gamma}{\omega(\lambda^2 + 1) + i\gamma}, \quad t = \frac{\omega(\lambda^2 + 1)}{\omega(\lambda^2 + 1) + i\gamma}. \quad (47)$$

It was shown that a partially reflecting single moving mirror can produce a larger number of particles in comparison with a perfect one. The interference between the motion of a single mirror and the time-dependent Robin parameter was studied in [171]. The problem of creation of a time-dependent boundary was addressed in [172]. The catastrophic generation of quanta due to instantaneous appearance and disappearance of a wall in a cavity was shown in [173]. The explosive photon production due to instantaneous changes from the Neumann to the Dirichlet BC (or reversely) was considered in [174]. (A similar unphysical behavior was discovered in [175] in the case of instantaneous displacements of boundaries.) Microscopic models of mirrors as oscillators coupled bilinearly to quantum fields were considered in [176–180].

3. Three-Dimensional Models with Moving Boundaries

3.1. Single Mirror DCE

The effect of photon production from vacuum, caused by single mirrors moving in the three-dimensional space with *relativistic* velocities or with great accelerations, was studied in the papers [181–186]. The nonrelativistic motion of the plane mirror was considered in [187–191]. The radiation from dynamically deforming mirrors with different boundary conditions was calculated in [192–194]. Graphene-like mirrors were considered in [195]. Classical analogs of the DCE due to the oscillating motion of the plane surfaces containing dipole layers were considered in [196,197]. For the most recent publications, one can see [198–200].

Various quantum effects arising due to the motion of *dielectric boundaries* in three dimensions, including the modification of the Casimir force and creation of photons, have been studied in [201–208].

3.1.1. Quantum Friction for Moving Surfaces

The first calculations of the forces acting on the single mirrors moving with *nonrelativistic* velocities due to the vacuum or thermal fluctuations of the field were performed in the framework of the spectral approach (using the fluctuation–dissipation theorem) in [209] (three-dimensional case, the force proportional to the fifth-order derivative of the coordinate) and in [210] (one-dimensional model, the force proportional to the third-order derivative of the coordinate). These studies were continued in [25,153,211–217], where it was assumed that the velocity of the boundary is perpendicular to the surface.

The frictional force proportional to the constant relative velocity of two parallel plates, when this velocity is also parallel to the surfaces, was calculated by Teodorovich [218] in the van der Waals regime (i.e., neglecting the retardation effects). More general results were obtained by Levitov [219] and Polevoi [220]. In particular, Levitov had shown that the linear dependence on the velocity disappears at zero absolute temperature, but the friction force is still nonzero: it is proportional to v^3 for a very small separation between the plates, while it becomes velocity independent (like the dry friction) for a big enough separation. Later on, the theory of “Casimir friction” was the subject of studies [221–228], with controversial results. Moreover, the existence of such a friction force was questioned in [229] and especially in [230]. The latter paper triggered hot discussions [231–235]. More recent results can be found in [236–250]. In particular, it was shown in [246] that the friction force between two graphene sheets is nonzero only if the relative velocity is larger than the Fermi velocity of the charge carriers in graphene.

In 1971, Zel'dovich [251,252] predicted that a rotating object can amplify certain incident waves under the condition $\omega - L\Omega < 0$, where ω is the frequency of the field mode, L is its azimuthal quantum number, and Ω is the angular velocity of the rotating body (a cylinder). Furthermore, he conjectured that such an object should spontaneously emit radiation for these selected modes. This idea was further developed in [253]. The effect was called “superradiance” in [254,255] and “spontaneous superradiance” in [256]. Recently, a similar phenomenon—the friction force on the rotating bodies due to the interaction with vacuum (or thermal) fluctuations of the EM field—was studied in detail in papers [257–260]. Analogs in superfluids were considered in [261]. The wave instability of the electromagnetic field inside a rotating cylinder, resulting in exponential growth with time, was studied in [262].

3.2. Cavity DCE

3.2.1. Effective Hamiltonians

The instantaneous basis approach, described above, corresponds to the Heisenberg description of the quantum systems. It was believed for some time after Moore [1] that it is the only possible approach to the systems with moving boundaries. However, an equivalent Schrödinger description (although approximate) also exists. It is based on using some *effective Hamiltonians* [94–96,98–100,156,175,263–267]. The general structure of the effective infinite-dimensional quadratic Lagrangians and Hamiltonians arising in the canonical approach to the dynamical Casimir effect was analyzed and classified in [98,263].

Following [268], let us suppose that the set of Maxwell's equation in a medium with *time-independent* parameters and boundaries can be reduced to an equation of the form $\hat{\mathcal{K}}(\{L\})\mathbf{F}_\alpha(\mathbf{r};\{L\}) = \omega_\alpha^2(\{L\})\mathbf{F}_\alpha(\mathbf{r};\{L\})$, where $\{L\}$ means a set of parameters (for example, the distance between the walls or the dielectric permittivity inside the cavity), $\omega_\alpha(\{L\})$ is the eigenfrequency of the field mode labeled by the number (or a set of numbers) α and $\mathbf{F}_\alpha(\mathbf{r};\{L\})$ is some vector function describing the EM field (e.g., the vector potential). In the simplest cases, $\hat{\mathcal{K}}(\{L\})$ is reduced to the Laplace operator. Usually, the operator $\hat{\mathcal{K}}(\{L\})$ is self-adjoint, and the set of functions $\{\mathbf{F}_\alpha(\mathbf{r};\{L\})\}$ is orthonormal and complete in some sense.

Now, suppose that parameters L_1, L_2, \dots, L_n become time-dependent. If one can still satisfy automatically the boundary conditions, expanding the field $\mathbf{F}(\mathbf{r}, t)$ over “instantaneous” eigenfunctions $\mathbf{F}(\mathbf{r}, t) = \sum_{\alpha} q_{\alpha}(t) \mathbf{F}_{\alpha}(\mathbf{r}; \{L(t)\})$ (this is true, e.g., for the Dirichlet boundary conditions, which are equivalent in some cases to the TE polarization of the field modes), then the dynamics of the field is described completely by the generalized coordinates $q_{\alpha}(t)$, whose equations of motion can be derived from the *effective time-dependent Hamiltonian* [98]

$$H = \frac{1}{2} \sum_{\alpha} \left[p_{\alpha}^2 + \omega_{\alpha}^2 (\{L(t)\}) q_{\alpha}^2 \right] + \sum_{k=1}^n \frac{\dot{L}_k(t)}{L_k(t)} \sum_{\alpha \neq \beta} p_{\alpha} m_{\alpha \beta}^{(k)} q_{\beta}, \quad (48)$$

$$m_{\alpha \beta}^{(k)} = -m_{\beta \alpha}^{(k)} = L_k \int dV \frac{\partial \mathbf{F}_{\alpha}(\mathbf{r}; \{L\})}{\partial L_k} \mathbf{F}_{\beta}(\mathbf{r}; \{L\}). \quad (49)$$

Consequently, the field problem can be reduced to studying the dynamics of the infinite set of harmonic oscillators with time-dependent frequencies and bilinear specific (coordinate–momentum) time-dependent coupling. Methods of diagonalization of some specific Hamiltonians like (48) were considered in [269,270].

3.2.2. Parametric Oscillator Model

From the point of view of applications to the DCE, the most important cases are those where the parameters $L_k(t)$ vary in time *periodically*. In the case of small *harmonic* variations at the frequency close to the double unperturbed eigenfrequency of some mode $2\omega_0$, the equations of motion resulting from Hamiltonian (48) can be solved approximately with the aid of the method of slowly varying amplitudes [101]. If the difference $\omega_{\alpha} - \omega_{\beta}$ is not close to $2\omega_0$ for all those modes which have nonzero (or not very small) coupling coefficients $m_{\alpha \beta}$, then only the selected mode with label 0 can be excited *in the long-time limit*, and one can consider only this single resonance mode [27,105], whose excitation is described by the Hamiltonian

$$H = \left[p_0^2 + \omega_0^2 (\{L(t)\}) q_0^2 \right] / 2. \quad (50)$$

The theory of quantum nonstationary harmonic oscillator has been well developed since its foundation by Husimi in 1953 [271] (see, e.g., [272–274] for the reviews and references). It appears that all dynamical properties of the *quantum* oscillator are determined by the fundamental set of solutions of the *classical* equation of motion

$$\ddot{\varepsilon} + \omega^2(t) \varepsilon = 0. \quad (51)$$

For harmonic variations of the frequency in the form $\omega(t) = \omega_0 [1 + 2\kappa \cos(2\omega_0 t)]$, with $|\kappa| \ll 1$, Equation (51) can be solved approximately using, e.g., the method of averaging over fast oscillations or the method of slowly varying amplitudes [101]. The final result is a simple formula for the number of quanta created from the initial vacuum state [105,275]:

$$\mathcal{N} = \sinh^2(\omega_0 \kappa t). \quad (52)$$

Here, one can notice the striking difference between the parabolic dependence on time of the total number of created quanta in the 1D case, according to Equation (42), and the exponential dependence (52) in the 3D case. This is explained by the crucial difference in the eigenmode spectra: it is equidistant in the 1D case, but strongly non-equidistant in the generic 3D case. A smooth transition from one situation to another was demonstrated in [276]. Anharmonic and random periodic displacements of boundaries were studied in [277].

In the literature on the DCE, many authors frequently use an equivalent form of the single-mode effective Hamiltonian (50) in terms of annihilation and creation operators, derived by Law [96],

$$\hat{H} = \omega(t) \hat{a}^{\dagger} \hat{a} - i \chi_t \left(\hat{a}^2 - \hat{a}^{\dagger 2} \right), \quad \chi_t = (d\omega/dt)/(4\omega), \quad \hbar = 1. \quad (53)$$

Algebraic methods of solving the Schrödinger equation with Hamiltonian (53) were used in [278]. Recently, the harmonic oscillator model was re-discovered and applied to the circuit QED in [279].

3.2.3. The Role of Intermode Interactions

The single-mode approximation of the generic Hamiltonian (48) does not work, if the frequency difference between specific modes equals (almost) exactly twice the resonance frequency of some mode. For example, such a *resonance coupling* between two modes is possible in cubical cavities [280]. This case was studied in [280,281]. It was shown that the number of photons in both the coupled modes grows exponentially with time in the long time limit $\omega_0 kt \gg 1$, but the rate of photon generation (the argument of the exponential function) turns out to be *two times smaller* than the value of this rate in the absence of the resonance coupling. (Actually, this rate depends on the concrete values of the coupling coefficients $m_{\alpha\beta}$, but in any case it cannot exceed the ‘uncoupled’ values [281].) This example indicates that the resonance coupling between the modes should be avoided in order to achieve the maximal photon generation rate, at least in the case of TE modes. A detailed numerical study of this case for different sizes of the rectangular cavities was performed in [282]. The authors of paper [166] used numerical methods, taking into account the interaction between 50 lowest coupled modes in the rectangular cavity, bisected by a “plasma sheet” with a periodically varying number of free carriers. Some results of that paper show that the intermode coupling can increase the number of photons in the modes of EM field with the TM polarization.

The parametric excitation of the resonantly coupled modes of the scalar field in the cubic cavity was studied in [283]. The case of the TE polarization of the vector EM field was also considered there. The “swinging” cubic cavity, performing small amplitude periodical rotations along the z -axis, $\theta(t) = \epsilon \sin(\Omega t)$, was considered in [284] (in the model of scalar field). The frequency Ω was chosen in such a way that three modes, $(1,1,1)$, $(1,2,1)$ and $(2,1,1)$, were resonantly coupled.

The role of intermode interactions was also studied in paper [285]. There, the evolution of the classical electromagnetic field inside the cylindrical (rectangular) cavity with ideal boundaries was studied analytically and numerically in the case, when the conductivity σ and dielectric permittivity ϵ of a thin slab attached to the base of the cylinder can vary with time. It was shown that the single-mode model can be justified, if the perturbation of the field is small due to the smallness of σ and $\delta\epsilon$. However, no amplification can be achieved in this case for microwave fields, if σ and $\delta\epsilon$ are due to the creation of free carriers inside the slab.

3.2.4. Time-Dependent Casimir Force

Remember that the famous Casimir formula [286] for the attraction force per unit area between two ideal infinite plates in three dimensions reads

$$F/S = -\pi^2 \hbar c / (240 L_0^4). \quad (54)$$

It was generalized, using the Green function method, to the case of uniform motion of one boundary (in the direction perpendicular to its surface) in [287] for the scalar field and in [288] for the vector electromagnetic field. The results of two models turned out to be qualitatively different. Namely, the scalar model predicted the increase of the absolute value of the Casimir force in the nonrelativistic case:

$$F_{scal} = -\frac{\pi^2}{480 L^4(t)} \left[1 + \frac{8}{3} \left(\frac{v}{c} \right)^2 + \dots \right], \quad v/c \ll 1. \quad (55)$$

On the contrary, the account of the TM modes of the electromagnetic field results in the decrease of the absolute value of the Casimir force in the nonrelativistic case:

$$F_{EM} = -\frac{\pi^2}{240 L^4(t)} \left[1 + \left(\frac{v}{c} \right)^2 \left(\frac{2}{3} - \frac{10}{\pi^2} \right) + \dots \right], \quad v/c \ll 1. \quad (56)$$

Moreover, as a matter of fact, the force practically does not depend on the velocity (with accuracy about 10%), decreasing monotonously (by the absolute value) to the ultra-relativistic limit

$$F_{EM} = -\frac{3}{8\pi^2 L^4(t)} \left[1 + \frac{1}{16} \left(1 - v^2/c^2 \right)^2 + \dots \right], \quad 1 - (v/c)^2 \ll 1. \quad (57)$$

However, the force can be significantly amplified under the resonance conditions, either in the LC-contour [209] or in the Fabry–Perot cavity [210] (see also [84]). The time-dependent force between vibrating plates was considered in [289].

3.2.5. Vector Fields in 3D Cavities

Quantum properties of the electromagnetic field between two infinite plates, moving with constant relative velocity, with account of the field polarization, were considered in [290]. The photon generation in the TE and TM modes of electromagnetic field between two parallel vibrating plates was considered in [265]. It was shown that the photon generation rate in the TM modes is higher by one order of magnitude than that in the TE modes (see also [266]). The case of a three-dimensional rectangular cavity divided into two parts by an ideal mirror, which suddenly disappears, was considered in [291]. TE and TM modes in a resonant cavity bisected by a plasma sheet were considered in [166]. The DCE for the Dirac field inside the 3D cubic box with oscillating walls was studied in [292]. The spherical and cylindrical geometries were considered in [72,293–298]. General boundary conditions in 3D cavities were used in [299,300]. The scattering approach was applied to the DCE problem in [301]. Numerical algorithms were elaborated in [302].

3.2.6. Saturated Regimes

An unlimited growth of the number of generated “Casimir photons” happens in the simplified models only. The maximal number of quanta that could be generated under realistic conditions is limited due to many factors. One such factor is related to unavoidable nonlinearities in real systems [303–306]. Another mechanism, related to the temporal coherence, was studied in [307]. The saturation due to the finite reflectance spectral band of mirrors was considered in [308].

4. General Parametric DCE

4.1. Circuit DCE

In view of great difficulties in observing weak manifestations of the DCE, several authors came to the idea of simulating (modeling) this effect in more simple arrangements. Probably, the simplest possibility is to use some electrical circuits [309]. The idea to use quantum resonant oscillatory contours or Josephson junctions with time-dependent parameters (capacitance, inductance, magnetic flux, critical current, etc.) was put forward by Man’ko many years ago [310]. More elaborated proposals in the same direction were presented in [311–315]. The idea to use a superconducting coplanar waveguide in combination with a Josephson junction was developed in [316–319], and the experiments were reported in [320–323]. Their success is related to the possibility of achieving the effective velocity of the boundary up to 25% of the light velocity in vacuum (although the analogy with the motion of real boundaries is not perfect). A detailed review of this approach was given in [31]. The experiments [320–323] were performed in the frequency interval of a few GHz. Their arrangements can be considered as realizations of the *one-dimensional* models with time-dependent parameters. The experiment [320] was performed with an open strip-line waveguide, where the photon generation was achieved due to periodical fast changes of the boundary conditions on one side of the line. Although that boundary conditions can be interpreted as equivalent to a high effective velocity of an oscillating boundary, such an interpretation has a limited range of validity. The second experiment [322] used the parametric resonance excitation of quanta due to changes of the effective speed of light. This arrangement can be considered as a one-dimensional system with periodically

varying distributed parameters. The results of that experiments stimulated many theoretical papers, suggesting further improvements of the experimental schemes [324–337]. The circuit QED with “artificial atoms” (qubits) was the subject of studies [338–350]. The most recent review on parametric effects in circuit QED can be found in [351].

4.2. Analogs of DCE in Condensed Matter

Analogs of the DCE in Bose–Einstein condensates and ultracold gases with time-dependent parameters were considered in [352–364]. An obvious advantage of replacing EM waves with their sound analogs is a possibility of achieving high ratios of effective velocities to the sound speed, including the supersonic regimes [358,360]. The use of plasmon resonances in metallic nanoparticles surrounded by an amplifying medium, excited by femtosecond lasers, was suggested in [365]. The DCE with polaritons was studied in [366–370], and the DCE with magnons in [371,372]. The DCE for phonons was considered in [373–376].

4.3. DCE and Atomic Excitations

Many papers were devoted to the interaction between the “Casimir photons” and multilevel systems (“atoms” or “qubits”) inside cavities or quantum circuits [306,377–408]. In particular, two-level atoms (qubits) were considered in [306,378,380–382,384,386,391,392,394,400,401,403,404,408]. Two two-level atoms were studied in [390,396,405,407], and ensembles of many two-level atoms in [383,385,388,395,397,398,401,406]. Three-level systems were considered in [379,387,389,408]. The case of N -level atoms in equally spaced, resonant ladder configuration, was investigated in [388]. The circuit DCE with two-level “atoms” was studied in [409]. The case of a two-level atom near a single accelerated mirror was studied in [410]. A possibility of generation of quantum field states with the hyper-Poissonian statistics, as well as some other “exotic” quantum states, due to the DCE in the presence of “atoms”, was shown in [411,412]. The “anti-dynamic” Casimir effect (coherent annihilation of system excitations from common initial states due to parametric modulation) was predicted and studied in papers [304,340,413–418]. The effect of generation of pairs of atomic excitations instead of photons (“inverse DCE”) was considered in [304]. A possibility of exciting the Rydberg atoms passing near the boundary with oscillating parameters (“oscillating effective mirror”) was studied in [419]. The influence of the oscillating boundary on the spontaneous emission rate of an atom placed nearby this boundary was considered in [420,421]. The influence of the DCE on the fidelity of the quantum state transfer between two qubits in the ultrastrong coupling regime was studied in [405]. The generation of photons by accelerated neutral objects in cavities with static walls was considered, e.g., in [422].

5. Experimental Proposals for the Cavity DCE

5.1. Difficulties with Real Moving Boundaries

According to formula (52), a possibility of experimental verification of the DCE depends on the amplitude of the frequency variation $\Delta\omega = 2\kappa\omega_0$. The main difficulty is due to the very high frequency $2\omega_0$. The most exciting dream is to observe the “Casimir light” in the visible part of the electromagnetic spectrum. However, it seems to be very improbable using real mirrors oscillating at the frequency about 10^{15} Hz, due to the practical impossibility of exciting and maintaining the *high frequency oscillations* of the suspended plate with a large amplitude and for a long time.

It seems that the only possible way to realize the real motion of material boundaries at high frequencies is not to move the whole mirror, but to cause its surface to perform harmonic vibrations with the aid of some mechanism, e.g., using the piezo-effect. The amplitude of such vibrations ΔL is connected with the maximal relative deformation δ in a standing acoustic wave inside the wall as $\delta = \omega_w \Delta L / v_s$, where $v_s \sim 5 \cdot 10^3$ m/s is the sound velocity. Since usual materials cannot bear deformations exceeding the value $\delta_{max} \sim 10^{-2}$, the velocity of the boundary cannot exceed the

value $v_{max} \sim \delta_{max} v_s \sim 50$ m/s (independent on the frequency). The maximal possible frequency variation amplitude $\Delta\omega$ can be evaluated as $\Delta\omega \sim v_s \delta_{max} / (2L)$, where L is the cavity size. For the optical frequencies, taking $2L > 1 \mu\text{m}$, we obtain $\Delta\omega < 5 \times 10^7 \text{ s}^{-1}$, whereas, for the microwave frequencies (in the GHz band) with $L \sim 1 \text{ cm}$, we have $\Delta\omega < 10^3 \text{ s}^{-1}$. Since the time of excitation t must be bigger than $1/\Delta\omega$, the quality factor of the cavity Q must be not less than $Q_{min} \approx \omega_0 / \Delta\omega \approx (L/\lambda)4\pi c / (v_s \delta) \sim 10^8 (L/\lambda)$. Consequently, there are two main challenges: how to excite high frequency surface oscillations and how to maintain the high quality factor in the regime of strong surface vibrations. The excitation of high amplitude surface vibrations at the optical frequencies seems very problematic. Therefore, hardly the "Casimir light" in the visible region can be generated in systems with really moving boundaries. However, this seems to be possible in other schemes, where changes of some parameters can be interpreted as variations of an "effective length" of the cavity.

The GHz frequency band seems more promising. In such a case, the dimensions of cavities must be of the order of few centimeters. Superconducting cavities with the quality factors exceeding 10^{10} in the frequency band from 1 to 50 GHz have been available for a long time. Therefore, the most difficult problem is to excite the surface vibrations. At lower frequencies, it was solved long ago. Only recently, a significant progress was achieved in fabrication of the so called "film bulk acoustic resonators" (FBARs): piezoelectric devices working at the frequencies from 1 to 3 GHz [423]. They consist of an aluminum nitride (AlN) film of thickness corresponding to one half of the acoustic wavelength, sandwiched between two electrodes. It was suggested [424–426] to use such kind of devices to excite the surface vibrations of cavities in order to observe the DCE. However, no experimental results in this direction were reported until now. For the most recent proposals, see [427,428].

5.2. Simulations with Semiconductor Slabs

In view of difficulties of the excitation of oscillating motion of real boundaries, the ideas concerning the *imitation* of this motion attracted more and more attention with the course of time. The first concrete suggestion was made by Yablonovitch [5], who proposed to use a medium with a rapidly decreasing in time refractive index ("plasma window") to simulate the so-called Unruh effect. In addition, he pointed out that fast changes of dielectric properties can be achieved in semiconductors illuminated by subpicosecond optical pulses and supposed that "the moving plasma front can act as a moving mirror exceeding the speed of light." Similar ideas and different possible schemes based on fast changes of the carrier concentration in semiconductors illuminated by laser pulses were discussed in [429–431]. Yablonovitch [5,429] put emphasis on the excitation of *virtual electron-hole pairs* by optical radiation tuned to the transparent region just below the band gap in a semiconductor photodiode. He showed that big changes of the *real part* of the dielectric permittivity could be achieved in this way.

The key idea of the experiment named "MIR" in the university of Padua [432,433] was to imitate the motion of a boundary, using an effective "plasma mirror" formed by *real electron-hole pairs* in a thin film near the surface of a semiconductor slab, illuminated by a periodical sequence of short laser pulses. If the interval between pulses exceeds the recombination time of carriers in the semiconductor, a highly conducting layer will periodically appear and disappear on the surface of the slab. This can be interpreted as periodical displacements of the boundary. The basic physical idea was nicely explained in [433]: "...this effective motion is much more convenient than a mechanical motion, since in a metal mirror only the conduction electrons reflect the electromagnetic waves, whereas a great amount of power would be wasted in the acceleration of the much heavier nuclei." However, attempts to implement this idea in practice [434] met severe difficulties due to high losses in semiconductor materials. A rough explanation was given in [285,435]. It seems that the necessary condition for the photon generation, in addition to a high value of the slab conductivity, is the big *negative* change of the real part of the dielectric permeability. Probably, this regime was not reached in the experiments. An idea to use the resonance between the field mode and cyclotron transitions inside a semiconductor heterostructure in a strong and rapidly varying magnetic field was suggested in [436].

5.3. Simulations with Linear and Nonlinear Optical Materials

The main mechanism of the DCE is amplification of vacuum fluctuations due to fast variations of instantaneous eigenfrequencies of the field modes. These variations can happen either due to change of real dimensions of the cavity confining the field (DCE in narrow sense) or due to changes of the effective (optical) length, when dielectric properties of the medium inside the cavity depend on time (DCE in wide sense). This second possibility was investigated theoretically by many authors [96,275,437–460]. The main problem is how to realize fast variations of the dielectric permeability in real experiments. The idea to use laser beams passing through a material with nonlinear optical properties was considered in [5,429,438–440,448], and concrete experimental schemes were proposed in [461–464]. Although those schemes are rather different (Dezael and Lambrecht [461] and Hizhnyakov [463] considered nonlinear crystals with the second-order nonlinearity, whereas Faccio and Carusotto [462] proposed to exploit the third-order Kerr effect), their common feature is the prediction of generation of infrared [461,462] or even visible [463] photons, whose frequency is of the same order as the frequency of the laser beam. The experiment based on the suggestion [462] was realized recently [465]. A simulation of the DCE in photonic lattices or photonic crystals was suggested in papers [466,467].

The evaluation of a possibility to obtain parametric amplification of the *microwave* vacuum field, using a reentrant cavity enclosing a nonlinear crystal with a strong third-order nonlinearity, whose refractive index is modulated by near infrared high-intensity laser pulses, was performed in [468,469]. Such a configuration seems to be more adequate for the simulation of the DCE because the effective time-varying length of the cavity is created by infrared quanta, whereas an antenna put inside the cavity can select microwave (RF) quanta only, whose frequency is five orders of magnitude smaller. This could help experimentalists to get rid of various spurious effects due to other possible mechanisms of photon generation.

5.4. Interaction with Detectors

An important part of the experiments on the DCE is how to *detect* the “Casimir photons”. This problem was considered in [105,411,470–481]. Detectors modeled by harmonic oscillators were studied in [105,471,475–479]. Two-level and multilevel detectors were considered in [471,477,481]. Electron beams as detectors were suggested in [472]. Rydberg atoms as detectors were proposed in [474].

6. DCE and Other Quantum Phenomena

Various phenomena connected with the DCE were investigated in [482–486]. In particular, the DCE in curved spacetimes or in the presence of gravitational fields was studied in [487–491]. The role of the dynamical Casimir effect in the cosmological problems was studied in [158,492]. The DCE in an infinite uniformly accelerated medium was considered in [493]. The influence of gravitational fields on the DCE was studied in [494]. Many of these studies are connected with the general problem of the quantum radiation produced by time-varying metrics. It takes the origin in the papers published about 50 years ago: [2,4,43–45,495–497]. For the recent studies one can see Refs. [454,498]. The differences between the DCE and the *Unruh effect* [499–501] were discussed, e.g., in studies [31,80,500,502–505]. Intersections between the DCE and quantum thermodynamics were discussed in [138,415,417,506]. Interferometers and detectors of weak signals with moving mirrors were considered in [61,507,508]. Possible (although looking fantastic at the moment) applications of the DCE to the space flights were discussed in [509].

6.1. Damping and Decoherence

The necessity of taking into account the effects of damping in the DCE was clear from the very beginning. Various estimations of the influence of these effects on the photon production rate inside the cavity and in quantum circuits were made in [27,102,348,349,510–516].

The closely connected problem of *decoherence* due to the DCE was considered for the first time in [517,518]. For other publications in this direction, see, e.g., [380,519–522].

6.2. Entanglement

The problem of entanglement creation by means of the DCE was put forward in the study [523]. Various aspects of the problem of entanglement in systems with moving mirrors (or their analogs) were considered in Refs. [143,144,333,346,361,363,524–539]. The quantum discord in the DCE was the subject of studies [333,540]. Other aspects of the quantum information theory were investigated in connection with the DCE in [541–543]. Possible applications of the DCE in quantum simulations were discussed in [544].

6.3. Other Dynamical Effects

The *Dynamical Lamb Effect*, introduced in [545], consists of the excitation of atoms inside a cavity with moving walls, without creation of real photons in the cavity. For further studies, one can consult papers [348,349,523,533–535,546–548].

The *Dynamical Casimir–Polder Effects* are related to the influence of the EM field fluctuations on the interaction between atomic-size objects (or between such objects and walls) in non-stationary situations. The term was coined in papers [549,550]. For further developments, one can consult studies [419,551–558].

7. Conclusions

This short review demonstrates an impressive expansion of the front of research related to the DCE (or its analogs) in many different areas that happened during the past decade. Studies in the area of quantum circuits come first (Section 4.1), stimulated by experiments [320,322,323]. They are followed by many results and suggestions that connect the DCE with the condensed matter (Section 4.2) and atomic physics (Section 4.3). Here, the first experiment was reported in [353]. The first experimental results related to the analog of the DCE in an optical fiber were also obtained recently [465]. Thus, the fascinating dynamical Casimir physics continues to attract many scientists. However, an observation of the “real” DCE in cavities with moving boundaries is still a challenge (or dream).

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References

1. Moore, G.T. Quantum theory of the electromagnetic field in a variable-length one-dimensional cavity. *J. Math. Phys.* **1970**, *11*, 2679–2691.
2. DeWitt, B.S. Quantum field theory in curved spacetime. *Phys. Rep.* **1975**, *19*, 295–357.
3. Fulling, S.A.; Davies, P.C.W. Radiation from a moving mirror in two-dimensional space-time: Conformal anomaly. *Proc. R. Soc. Lond. A* **1976**, *348*, 393–414.
4. Davies, P.C.W.; Fulling, S.A. Radiation from moving mirrors and from black holes. *Proc. R. Soc. Lond. A* **1977**, *356*, 237–257.
5. Yablonovitch, E. Accelerating reference frame for electromagnetic waves in a rapidly growing plasma: Unruh–Davies–Fulling–De Witt radiation and the nonadiabatic Casimir effect. *Phys. Rev. Lett.* **1989**, *62*, 1742–1745.
6. Dodonov, V.V.; Klimov, A.B.; Man’ko, V.I. Nonstationary Casimir effect and oscillator energy level shift. *Phys. Lett. A* **1989**, *142*, 511–513.

7. Schwinger, J. Casimir energy for dielectrics. *Proc. Nat. Acad. Sci. USA* **1992**, *89*, 4091–4093.
8. Plunien, G.; Müller, B.; Greiner, W. The Casimir effect. *Phys. Rep.* **1986**, *134*, 87–193.
9. Milonni, P.W.; Shih, M.L. Casimir forces. *Contemp. Phys.* **1992**, *33*, 313–322.
10. Milonni, P.W. *The Quantum Vacuum: An Introduction to Quantum Electrodynamics*; Academic Press: New York, USA, 1993.
11. Lamoreaux, S.K. Resource letter GF-1: Casimir force. *Am. J. Phys.* **1999**, *67*, 850–861.
12. Bordag, M.; Mohideen, U.; Mostepanenko, V.M. New developments in the Casimir effect. *Phys. Rep.* **2001**, *353*, 1–205.
13. Milton, K.A. *The Casimir Effect: Physical Manifestations of Zero-Point Energy*; World Scientific: Singapore, 2001.
14. Feinberg, J.; Mann, A.; Revzen, M. Casimir effect: The classical limit. *Ann. Phys.* **2001**, *288*, 103–136.
15. Milton, K.A. The Casimir effect: Recent controversies and progress. *J. Phys. A Math. Gen.* **2004**, *37*, R209–R277.
16. Lamoreaux, S.K. The Casimir force: Background, experiments, and applications. *Rep. Prog. Phys.* **2005**, *68*, 201–236.
17. Brown-Hayes, M.; Brownell, J.H.; Dalvit, D.A.R.; Kim, W.J.; Lambrecht, A.; Lombardo, F.C.; Mazzitelli, F.D.; Middleman, S.M.; Nesvizhevsky, V.V.; Onofrio, R.; et al. Thermal and dissipative effects in Casimir physics. *J. Phys. A Math. Gen.* **2006**, *39*, 6195–6208.
18. Farina, C. The Casimir effect: Some aspects. *Braz. J. Phys.* **2006**, *36*, 1137–1149.
19. Buhmann, S.Y.; Welsch, D.-G. Dispersion forces in macroscopic quantum electrodynamics. *Prog. Quantum Electron.* **2007**, *31*, 51–130.
20. Bordag, M.; Klimchitskaya, G.L.; Mohideen, U.; Mostepanenko, V.M. *Advances in the Casimir Effect (International Series of Monographs on Physics vol 145)*; Oxford University Press: Oxford, UK, 2009.
21. Babb, J.F. Casimir effects in atomic, molecular, and optical physics. *Adv. At. Mol. Opt. Phys.* **2010**, *59*, 1–20.
22. Lambrecht, A.; Reynaud, S. Casimir effect: theory and experiments. *Int. J. Mod. Phys. A* **2012**, *27*, 1260013.
23. Palasantzas, G.; Dalvit, D.A.R.; Decca, R.; Svetovoy, V.B.; Lambrecht, A. Special issue on Casimir physics. *J. Phys. Condens. Matter* **2015**, *27*, no. 21.
24. Simpson, W.M.R.; Leonhardt, U. *Force of the Quantum Vacuum: An Introduction to Casimir Physics*; World Scientific: Singapore, 2015.
25. Jaekel, M.T.; Reynaud, S. Movement and fluctuations of the vacuum. *Rep. Prog. Phys.* **1997**, *60*, 863–887.
26. Dodonov, V.V. Nonstationary Casimir effect and analytical solutions for quantum fields in cavities with moving boundaries. In *Modern Nonlinear Optics (Advances in Chemical Physics Series Vol. 119, Part 1)*; Evans, M.W., Ed.; Wiley: New York, NY, USA, 2001; pp. 309–394.
27. Dodonov, V.V.; Dodonov, A.V. Quantum harmonic oscillator and nonstationary Casimir effect. *J. Rus. Laser Res.* **2005**, *26*, 445–83.
28. Dodonov, V.V. Dynamical Casimir effect: Some theoretical aspects. *J. Phys. Conf. Ser.* **2009**, *161*, 012027.
29. Dodonov, V.V. Current status of the Dynamical Casimir Effect. *Phys. Scr.* **2010**, *82*, 038105.
30. Dalvit, D.A.R.; Maia Neto, P.A.; Mazzitelli, F.D. Fluctuations, dissipation and the dynamical Casimir effect. In *Casimir Physics (Lecture Notes in Physics, Vol. 834)*; Dalvit, D., Milonni, P., Roberts, D., da Rosa, F., Eds.; Springer: Berlin, Germany, 2011; pp. 419–457.
31. Nation, P.D.; Johansson, J.R.; Blencowe, M.P.; Nori, F. Stimulating uncertainty: Amplifying the quantum vacuum with superconducting circuits. *Rev. Mod. Phys.* **2012**, *84*, 1–24.
32. Nicolai, E.L. On transverse vibrations of a portion of a string of uniformly variable length. *Ann. Petrograd Polytechn. Inst.* **1921**, *28*, 273–287 (in Russian).
33. Nicolai, E.L. On a dynamical illustration of the pressure of radiation. *Phil. Mag.* **1925**, *49*, 171–177.
34. Havelock, T.H. Some dynamical illustrations of the pressure of radiation and of adiabatic invariance. *Phil. Mag.* **1924**, *47*, 754–771.
35. Balazs, N.L. On the solution of the wave equation with moving boundaries. *J. Math. Anal. Appl.* **1961**, *3*, 472–484.
36. Greenspan, H.P. A string problem. *J. Math. Anal. Appl.* **1963**, *6*, 339–348.
37. Baranov, R.I.; Shirokov, Y.M. Electromagnetic Field in an Optical Resonator With a Movable Mirror. *Sov. Phys. JETP* **1968**, *26*, 1199–1202.
38. Vesnitskii, A.I. A one-dimensional resonator with movable boundaries. *Sov. Phys. - Radiophys. Quant. Electron.* **1971**, *14*, 1124–1129.

39. Krasilnikov, V.N. *Parametric Wave Phenomena in Classical Electrodynamics*; St. Petersburg University Publ.: St. Petersburg, Russia, 1996. (in Russian)
40. Vesnitskii, A.I. *Waves in Systems with Moving Boundaries and Loads*; Fizmatlit: Moscow, Russia, 2001. (in Russian)
41. Schrödinger, E. The proper vibrations of the expanding universe. *Physica* **1939**, *6*, 899–912.
42. Imamura, T. Quantized meson field in a classical gravitational field. *Phys. Rev.* **1960**, *118*, 1430–1434.
43. Parker, L. Quantized fields and particle creation in expanding universes, I. *Phys. Rev.* **1969**, *183*, 1057–1068.
44. Parker, L. Quantized fields and particle creation in expanding universes, II. *Phys. Rev. D* **1971**, *3*, 346–356.
45. Zel'dovich, Y.B. Particle production in cosmology. *JETP Lett.* **1970**, *12*, 307–311.
46. Vesnitskii, A.I. The inverse problem for a one-dimensional resonator the dimensions of which vary with time.
47. Castagnino, M.; Ferraro, R. The radiation from moving mirrors: the creation and absorption of particles. *Ann. Phys.* **1984**, *154*, 1–23.
48. Walker, W.R.; Davies, P.C.W. An exactly soluble moving mirror problem. *J. Phys. A Math. Gen.* **1982**, *15*, L477–L480.
49. Carlitz, R.D.; Willey, R.S. Reflections on moving mirrors. *Phys. Rev. D* **1987**, *36*, 2327–2335.
50. Hotta, M.; Shino, M.; Yoshimura, M. Moving mirror model of Hawking evaporation. *Prog. Theor. Phys.* **1994**, *91*, 839–869.
51. Nikishov, A.I.; Ritus, V.I. Emission of scalar photons by an accelerated mirror in 1+1-space and its relation to the radiation from an electrical charge in classical electrodynamics. *Zhurn. Eksp. Teor. Fiz.* **1995**, *108*, 1121–1140 [*JETP* **1995**, *81*, 615–624].
52. Ritus, V.I. Functional identity of the spectra of Bose- and Fermi radiation of an accelerated mirror in 1 + 1 space and the spectra of electric and scalar charges in 3 + 1 space, and its relation to radiative reaction. *Zhurn. Eksp. Teor. Fiz.* **1996**, *110*, 526–548 [*JETP* **1996**, *83*, 282–293].
53. Obadie, N.; Parentani, R. Uniformly accelerated mirrors. I. Mean fluxes. *Phys. Rev. D* **2003**, *67*, 024021.
54. Ford, L.H.; Roman, T.A. Energy flux correlations and moving mirrors. *Phys. Rev. D* **2004**, *70*, 125008.
55. Good, M.R.R.; Anderson, P.R.; Evans, C.R. Time dependence of particle creation from accelerating mirrors. *Phys. Rev. D* **2013**, *88*, 025023.
56. Good, M.R.R.; Yelshibekov, K.; Ong, Y.C. On horizonless temperature with an accelerating mirror. *JHEP* **2017**, *3*, 013.
57. Good, M.R.R.; Linder, E.V. Slicing the vacuum: New accelerating mirror solutions of the dynamical Casimir effect. *Phys. Rev. D* **2017**, *96*, 125010.
58. Good, M.R.R.; Linder, E.V. Eternal and evanescent black holes and accelerating mirror analogs. *Phys. Rev. D* **2018**, *97*, 065006.
59. Good, M.R.R.; Linder, E.V. Finite energy but infinite entropy production from moving mirrors. *Phys. Rev. D* **2019**, *99*, 025009.
60. Hosoya, A. Moving mirror effects in hadronic reactions. *Prog. Theor. Phys.* **1979**, *61*, 280–293.
61. Obadie, N.; Parentani, R. Uniformly accelerated mirrors. II. Quantum correlations. *Phys. Rev. D* **2003**, *67*, 024022.
62. Mazzitelli, F.D.; Paz, J.P.; Castagnino, M.A. Fermions between moving boundaries. *Phys. Lett. B* **1987**, *189*, 132–136.
63. Horibe, M. Thermal radiation of fermions by an accelerated wall. *Prog. Theor. Phys.* **1979**, *61*, 661–671.
64. Koehn, M. Solutions of the Klein–Gordon equation in an infinite square-well potential with a moving wall. *EPL* **2012**, *100*, 60008.
65. Bialynicki-Birula, I. Solutions of the d'Alembert and Klein–Gordon equations confined to a region with one fixed and one moving wall. *EPL* **2013**, *101*, 60003.
66. Dodonov, V.V.; Klimov, A.B., Long-time asymptotics of a quantized electromagnetic field in a resonator with oscillating boundary. *Phys. Lett. A* **1992**, *167*, 309–313.
67. Dodonov, V.V.; Klimov, A.B.; Nikonorov, D.E. Quantum phenomena in resonators with moving walls. *J. Math. Phys.* **1993**, *34*, 2742–2756.
68. Dalvit, D.A.R.; Mazzitelli, F.D. Renormalization-group approach to the dynamical Casimir effect. *Phys. Rev. A* **1998**, *57*, 2113–2119.

69. Dalvit, D.A.R.; Mazzitelli, F.D. Creation of photons in an oscillating cavity with two moving mirrors. *Phys. Rev. A* **1999**, *59*, 3049–3059.

70. Li, L.; Li, B.Z. Numerical solutions of the generalized Moore’s equations for a one-dimensional cavity with two moving mirrors. *Phys. Lett. A* **2002**, *300*, 27–32.

71. Crocce, M.; Dalvit, D.A.R.; Lombardo, F.C.; Mazzitelli, F.D. 2005 Hertz potentials approach to the dynamical Casimir effect in cylindrical cavities of arbitrary section. *J. Opt. B Quantum Semiclass. Opt.* **2005**, *7* S32–S39.

72. Dalvit, D.A.R.; Mazzitelli, F.D.; Millán, X.O. The dynamical Casimir effect for different geometries. *J. Phys. A Math. Gen.* **2006**, *39*, 6261–70.

73. de la Llave, R.; Petrov, N.P. Theory of circle maps and the problem of one-dimensional optical resonator with a periodically moving wall. *Phys. Rev. E* **1999**, *59*, 6637–6651.

74. Petrov, N.P.; de la Llave, R.; Vano, J.A. Torus maps and the problem of a one-dimensional optical resonator with a quasiperiodically moving wall. *Phys. D* **2003**, *180*, 140–184.

75. Petrov, N.P. The dynamical Casimir effect in a periodically changing domain: A dynamical systems approach. *J. Opt. B Quantum Semiclass. Opt.* **2005**, *7*, S89–S99.

76. Węgrzyn, P. An optical approach to the dynamical Casimir effect. *J. Phys. B At. Mol. Opt. Phys.* **2006**, *39*, 4895–4903.

77. Węgrzyn, P. Exact closed-form analytical solutions for vibrating cavities. *J. Phys. B At. Mol. Opt. Phys.* **2007**, *40*, 2621–2640.

78. Fedotov, A.M.; Lozovik, Y.E.; Narozhny, N.B.; Petrosyan, A.N. 2006 Dynamical Casimir effect in a one-dimensional uniformly contracting cavity. *Phys. Rev. A* **2006**, *74*, 013806.

79. Alves, D.T.; Granhen, E.R. A computer algebra package for calculation of the energy density produced via the dynamical Casimir effect in one-dimensional cavities. *Comp. Phys. Commun.* **2014**, *185*, 2101–2114.

80. Martin, I. Floquet dynamics of classical and quantum cavity fields. *Ann. Phys.* **2019**, *405*, 101–129.

81. Birrell, N.D.; Davies, P.C.W. *Quantum Fields in Curved Space*; Cambridge Univ. Press: Cambridge, UK, 1982.

82. Beyer, H.; Nitsch, J. A note on a Casimir effect in a uniformly accelerated reference frame. *Found. Phys.* **1990**, *20*, 459–469.

83. Nagatani, Y.; Shigetomi, K. Effective theoretical approach to back reaction of the dynamical Casimir effect in $1+1$ dimensions. *Phys. Rev. A* **2000**, *62*, 022117.

84. Dodonov, V.V.; Klimov, A.B.; Man’ko, V.I. Generation of squeezed states in a resonator with a moving wall. *Phys. Lett. A* **1990**, *149*, 225–228.

85. Wu, Y.; Chan, K.W.; Chu, M.-C.; Leung, P.T. Radiation modes of a cavity with a resonantly oscillating boundary. *Phys. Rev. A* **1999**, *59*, 1662–1666.

86. Węgrzyn, P.; Róż, T. Vacuum energy of a cavity with a moving boundary. *Acta Phys. Pol. B* **2001**, *32*, 129–146.

87. Węgrzyn, P. Parametric resonance in a vibrating cavity. *Phys. Lett. A* **2004**, *322*, 263–269.

88. Węgrzyn, P.; Róż, T. Photons produced inside a cavity with a moving wall. *Acta Phys. Pol. B* **2003**, *34*, 3887–3900.

89. Alves, D.T.; Granhen, E.R.; Lima, M.G.; Silva, H.O.; Rego, A.L.C. Time evolution of the energy density inside a one-dimensional non-static cavity with a vacuum, thermal and a coherent state. *J. Phys. Conf. Ser.* **2009**, *161*, 012032.

90. Alves, D.T.; Granhen, E.R.; Lima, M.G.; Rego, A.L.C. Quantum radiation force on a moving mirror for a thermal and a coherent field. *J. Phys. Conf. Ser.* **2009**, *161*, 012033.

91. Alves, D.T.; Granhen, E.R.; Silva, H.O.; Lima, M.G. Quantum radiation force on the moving mirror of a cavity, with Dirichlet and Neumann boundary conditions for a vacuum, finite temperature, and a coherent state. *Phys. Rev. D* **2010**, *81*, 025016.

92. Alves, D.T.; Granhen, E.R.; Pires, W.P. Quantum radiation reaction force on a one-dimensional cavity with two relativistic moving mirrors. *Phys. Rev. D* **2010**, *82*, 045028.

93. Grinberg, G.A. A method of approach to problems of the theory of heat conduction, diffusion and the wave theory and other similar problems in presence of moving boundaries and its applications to other problems.

94. Razavy, M.; Terning, J. Quantum radiation in a one-dimensional cavity with moving boundaries. *Phys. Rev. D* **1985**, *31*, 307–313.

95. Calucci, G. Casimir effect for moving bodies. *J. Phys. A Math. Gen.* **1992**, *25*, 3873–3882.

96. Law, C.K. Effective Hamiltonian for the radiation in a cavity with a moving mirror and a time-varying dielectric medium. *Phys. Rev. A* **1994**, *49*, 433–437.

97. Dodonov, V.V. Resonance photon generation in a vibrating cavity. *J. Phys. A Math. Gen.* **1998**, *31*, 9835–9854.

98. Schützhold, R.; Plunien, G.; Soff, G. Trembling cavities in the canonical approach. *Phys. Rev. A* **1998**, *57*, 2311–2318.

99. Ji, J.-Y.; Jung, H.-H.; Soh, K.-S. Interference phenomena in the photon production between two oscillating walls. *Phys. Rev. A* **1998**, *57*, 4952–4955.

100. Law, C.K. Interaction between a moving mirror and radiation pressure: A Hamiltonian formulation. *Phys. Rev. A* **1995**, *51*, 2537–2541.

101. Landau, L.D.; Lifshitz, E.M. *Mechanics*; Pergamon: Oxford, UK, 1969.

102. Lambrecht, A.; Jaekel, M.T.; Reynaud, S. Motion induced radiation from a vibrating cavity. *Phys. Rev. Lett.* **1996**, *77*, 615–618.

103. Dodonov, V.V. Resonance excitation and cooling of electromagnetic modes in a cavity with an oscillating wall. *Phys. Lett. A* **1996**, *213*, 219–225.

104. Wegrzyn, P. Quantum energy in a vibrating cavity. *Mod. Phys. Lett. A* **2004**, *19*, 769–774.

105. Dodonov, V.V.; Klimov, A.B. Generation and detection of photons in a cavity with a resonantly oscillating boundary. *Phys. Rev. A* **1996**, *53*, 2664–2682.

106. Law, C.K. Resonance response of the quantum vacuum to an oscillating boundary. *Phys. Rev. Lett.* **1994**, *73*, 1931–1934.

107. Cole, C.K.; Schieve, W.C. Radiation modes of a cavity with a moving boundary. *Phys. Rev. A* **1995**, *52*, 4405–4415.

108. Méplan, O.; Gignoux, C. Exponential growth of the energy of a wave in a 1D vibrating cavity: Application to the quantum vacuum. *Phys. Rev. Lett.* **1996**, *76*, 408–410.

109. Klimov, A.B.; Altuzar, V. Spectrum of photons generated in a one-dimensional cavity with oscillating boundary. *Phys. Lett. A* **1997**, *226*, 41–45.

110. Cooper, J. Long time behavior and energy growth for electromagnetic waves reflected by a moving boundary. *IEEE Trans. Antennas Prop.* **1993**, *41*, 1365–1370.

111. Dittrich, J.; Duclos, P.; Šeba, P. 1994, Instability in a classical periodically driven string. *Phys. Rev. E* **1994**, *49*, 3535–3538.

112. Dittrich, J.; Duclos, P.; Gonzalez, N. Stability and instability of the wave equation solutions in a pulsating domain. *Rev. Math. Phys.* **1998**, *10*, 925–962.

113. Wu, K.; Zhu, W.D. Parametric instability in a taut string with a periodically moving boundary. *ASME J. Appl. Mech.* **2014**, *81*, 061002.

114. Dodonov, V.V.; Andreata, M.A. Squeezing and photon distribution in a vibrating cavity. *J. Phys. A Math. Gen.* **1999**, *32*, 6711–6726.

115. Andreata, M.A.; Dodonov, V.V. Energy density and packet formation in a vibrating cavity. *J. Phys. A Math. Gen.* **2000**, *33*, 3209–3223.

116. Lambrecht, A.; Jaekel, M.-T.; Reynaud, S. Generating photon pulses with an oscillating cavity. *Europhys. Lett.* **1998**, *43*, 147–152.

117. Lambrecht, A.; Jaekel, M.-T.; Reynaud, S. Frequency up-converted radiation from a cavity moving in vacuum. *Eur. Phys. J. D* **1998**, *3*, 95–104.

118. Lambrecht, A. Electromagnetic pulses from an oscillating high-finesse cavity: Possible signatures for dynamic Casimir effect experiments. *J. Opt. B Quantum Semiclass. Opt.* **2005**, *7*, S3–S10.

119. Rosanov, N.N.; Fedorov, E.G.; Matskovsky, A.A. Parametric generation of radiation in a dynamic cavity with frequency dispersion. *Quantum Electron.* **2016**, *46*, 13–15.

120. Rosanov, N.N.; Fedorov, E.G. Influence of frequency detunings and form of the initial field distribution on parametric generation of radiation in a dynamic cavity. *Opt. Spectrosc.* **2016**, *120*, 803–807.

121. Fedorov, E.G.; Rosanov, N.N.; Malomed, B.A. The evolution of field distribution in a dynamic cavity. *Opt. Spectrosc.* **2017**, *123*, 454–462.

122. Ruser, M. Vibrating cavities: A numerical approach. *J. Opt. B Quantum Semiclass. Opt.* **2005**, *7*, S100–S115.

123. Ruser, M. Numerical approach to the dynamical Casimir effect. *J. Phys. A Math. Gen.* **2006**, *39*, 6711–6723.

124. Alves, D.T.; Granhen, E.R.; Silva, H.O.; Lima, M.G. Exact behavior of the energy density inside a one-dimensional oscillating cavity with a thermal state. *Phys. Lett. A* **2010**, *374*, 3899–3907.

125. Ji, J.-Y.; Jung, H.-H.; Park, J.-W.; Soh, K.-S. Production of photons by the parametric resonance in the dynamical Casimir effect. *Phys. Rev. A* **1997**, *56*, 4440–4444.

126. Ji, J.-Y.; Soh, K.-S.; Cai, R.-G.; Kim, S.P. Electromagnetic fields in a three-dimensional cavity and in a waveguide with oscillating walls. *J. Phys. A Math. Gen.* **1998**, *31*, L457–L462.

127. Villar, P.I.; Soba, A.; Lombardo, F.C. Numerical approach to simulating interference phenomena in a cavity with two oscillating mirrors. *Phys. Rev. A* **2017**, *95*, 032115.

128. Alves, D.T.; Farina, C.; Granhen, E.R. Dynamical Casimir effect in a resonant cavity with mixed boundary conditions. *Phys. Rev. A* **2006**, *73*, 063818.

129. Alves, D.T.; Granhen, E.R. Energy density and particle creation inside an oscillating cavity with mixed boundary conditions. *Phys. Rev. A* **2008**, *77*, 015808.

130. Ordaz-Mendoza, B.E.; Yelin, S.F. Resonant frequency ratios for the dynamical Casimir effect. *Phys. Rev. A* **2019**, *100*, 033815.

131. Barton, G.; Calogeracos, A. On the quantum electrodynamics of a dispersive mirror. I. Mass shifts, radiation, and radiative reaction. *Ann. Phys.* **1995**, *238*, 227–267.

132. Calogeracos, A.; Barton, G. On the quantum electrodynamics of a dispersive mirror. II. The boundary condition and the applied force via Diracs theory of constraints. *Ann. Phys.* **1995**, *238*, 268–285.

133. Oku, K.; Tsuchida, Y. Back-reaction in the moving mirror effects. *Prog. Theor. Phys.* **1979**, *62*, 1756–1767.

134. Colanero, K.; Chu, M.C. Energy focusing inside a dynamical cavity. *Phys. Rev. E* **2000**, *62*, 8663–8667.

135. Cole, C.K.; Schieve, W.C. Resonant energy exchange between a moving boundary and radiation modes of a cavity. *Phys. Rev. A* **2001**, *64*, 023813.

136. Saito, H.; Hyuga, H. Dynamical Casimir effect without boundary conditions. *Phys. Rev. A* **2002**, *65*, 053804.

137. Sinyukov, Y.M. Radiation processes in quantum systems with boundary. *J. Phys. A Math. Gen.* **1982**, *15*, 2533–2545.

138. Helfer, A.D. Moving mirrors and thermodynamic paradoxes. *Phys. Rev. D* **2001**, *63*, 025016.

139. Butera, S.; Passante, R. Field fluctuations in a one-dimensional cavity with a mobile wall. *Phys. Rev. Lett.* **2013**, *111*, 060403.

140. Armata, F.; Passante, R. Vacuum energy densities of a field in a cavity with a mobile boundary. *Phys. Rev. D* **2015**, *91*, 025012.

141. Armata, F.; Kim, M.S.; Butera, S.; Rizzuto, L.; Passante, R. Nonequilibrium dressing in a cavity with a movable reflecting mirror. *Phys. Rev. D* **2017**, *96*, 045007.

142. Mancini, S.; Man'ko, V.I.; Tombesi, P. Ponderomotive control of quantum macroscopic coherence. *Phys. Rev. A* **1997**, *55*, 3042–3050.

143. Mancini, S.; Giovannetti, V.; Vitali, D.; Tombesi, P. Entangling macroscopic oscillators exploiting radiation pressure. *Phys. Rev. Lett.* **2002**, *88*, 120401.

144. Vitali, D.; Gigan, S.; Ferreira, A.; Böhm, H.R.; Tombesi, P.; Guerreiro, A.; Vedral, V.; Zeilinger, A.; Aspelmeyer, M. Optomechanical entanglement between a movable mirror and a cavity field. *Phys. Rev. Lett.* **2007**, *98*, 030405.

145. Aspelmeyer, M.; Kippenberg, T.J.; Marquardt, F. Cavity optomechanics. *Rev. Mod. Phys.* **2014**, *86*, 1391–1452.

146. Mahajan, S.; Aggarwal, N.; Kumar, T.; Bhattacharjee, A.B.; Mohan, M. Dynamics of an optomechanical resonator containing a quantum well induced by periodic modulation of cavity field and external laser beam. *Can. J. Phys.* **2015**, *93*, 716–724.

147. Macrì, V.; Ridolfo, A.; Di Stefano, O.; Kockum, A.F.; Nori, F.; Savasta, S. Nonperturbative dynamical Casimir effect in optomechanical systems: Vacuum Casimir–Rabi splittings. *Phys. Rev. X* **2018**, *8*, 011031.

148. Jansen, E.; Machado, J.D.P.; Blanter, Y.M. Realization of a degenerate parametric oscillator in electromechanical systems. *Phys. Rev. B* **2019**, *99*, 045401.

149. Di Stefano, O.; Settineri, A.; Macrì, V.; Ridolfo, A.; Stassi, R.; Kockum, A.F.; Savasta, S.; Nori, F. Interaction of mechanical oscillators mediated by the exchange of virtual photon pairs. *Phys. Rev. Lett.* **2019**, *122*, 030402.

150. Butera, S.; Carusotto, I. Mechanical backreaction effect of the Casimir emission. *Phys. Rev. A* **2019**, *99*, 053815.

151. Settineri, A.; Macrì, V.; Garziano, L.; Di Stefano, O.; Nori, F.; Savasta, S. Conversion of mechanical noise into correlated photon pairs: Dynamical Casimir effect from an incoherent mechanical drive. *Phys. Rev. A* **2019**, *100*, 022501.

152. Del Grosso, N.F.; Lombardo, F.C.; Villar, P.I. Photon generation via the dynamical Casimir effect in an optomechanical cavity as a closed quantum system. *Phys. Rev. A* **2019**, *100*, 062516.

153. Jaekel, M.T.; Reynaud, S. Fluctuations and dissipation for a mirror in vacuum. *Quant. Opt.* **1992**, *4*, 39–53.

154. Nicolaevici, N. Quantum radiation from a partially reflecting moving mirror. *Class. Quant. Grav.* **2001**, *18*, 619–628.

155. Obadia, N.; Parentani, R. Notes on moving mirrors. *Phys. Rev. D* **2001**, *64*, 044019.

156. Haro, J.; Elizalde, E. Physically sound Hamiltonian formulation of the dynamical Casimir effect. *Phys. Rev. D* **2007**, *76*, 065001.

157. Haro, J.; Elizalde, E. Black hole collapse simulated by vacuum fluctuations with a moving semitransparent mirror. *Phys. Rev. D* **2008**, *77*, 045011.

158. Elizalde, E. 2008 Dynamical Casimir effect with semi-transparent mirrors, and cosmology. *J. Phys. A Math. Theor.* **2008**, *41*, 164061.

159. Nicolaevici, N. Semitransparency effects in the moving mirror model for Hawking radiation. *Phys. Rev. D* **2009**, *80*, 125003.

160. Sarabadani, J.; Miri, M.F. Motion-induced radiation in a cavity with conducting and permeable plates. *Phys. Rev. A* **2007**, *75*, 055802.

161. Mintz, B.; Farina, C.; Maia Neto, P.A.; Rodrigues, R.B. Casimir forces for moving boundaries with Robin conditions. *J. Phys. A Math. Gen.* **2006**, *39*, 6559–6565.

162. Mintz, B.; Farina, C.; Maia Neto, P.A.; Rodrigues, R.B. Particle creation by a moving boundary with a Robin boundary condition. *J. Phys. A Math. Gen.* **2006**, *39*, 11325–11333.

163. Silva, H.O.; Farina, C. Simple model for the dynamical Casimir effect for a static mirror with time-dependent properties. *Phys. Rev. D* **2011**, *84*, 045003.

164. Fosco, C.D.; Lombardo, F.C.; Mazzitelli, F.D. Vacuum fluctuations and generalized boundary conditions. *Phys. Rev. D* **2013**, *87*, 105008.

165. Crocce, M.; Dalvit, D.A.R.; Lombardo, F.C.; Mazzitelli, F.D. Model for resonant photon creation in a cavity with time-dependent conductivity. *Phys. Rev. A* **2004**, *70*, 033811.

166. Naylor, W.; Matsuki, S.; Nishimura, T.; Kido, Y. Dynamical Casimir effect for TE and TM modes in a resonant cavity bisected by a plasma sheet. *Phys. Rev. A* **2009**, *80*, 043835.

167. Kulagin, V.V.; Cherepenin, V.A. Generation of squeezed states on reflection of light from a system of free electrons. *JETP Lett.* **1996**, *63*, 170–175.

168. Kulagin, V.V.; Cherepenin, V.A.; Hur, M.S.; Suk, H. Flying mirror model for interaction of a super-intense nonadiabatic laser pulse with a thin plasma layer: Dynamics of electrons in a linearly polarized external field. *Phys. Plasmas* **2007**, *14*, 113101.

169. Bulanov, S.V.; Esirkepov, T.Zh.; Kando, M.; Pirozhkov, A.S.; Rosanov, N.N. Relativistic mirrors in plasmas. Novel results and perspectives. *Phys. - Uspekhi* **2013**, *56*, 429–464.

170. Silva, J.D.L.; Braga, A.N.; Alves, D.T. Dynamical Casimir effect with $\delta - \delta'$ mirrors. *Phys. Rev. D* **2016**, *94*, 105009.

171. Lima Silva, J.D.; Braga, A.N.; Rego, A.L.C.; Alves, D.T. Interference phenomena in the dynamical Casimir effect for a single mirror with Robin conditions. *Phys. Rev. D* **2015**, *92*, 025040.

172. Brown, E.G.; Louko, J. Smooth and sharp creation of a Dirichlet wall in 1+1 quantum field theory: how singular is the sharp creation limit? *JHEP* **2015**, *8*, 061.

173. Harada, T.; Kinoshita, S.; Miyamoto, U. Vacuum excitation by sudden appearance and disappearance of a Dirichlet wall in a cavity. *Phys. Rev. D* **2016**, *94*, 025006.

174. Miyamoto, U. Explosive particle creation by instantaneous change of boundary conditions. *Phys. Rev. D* **2019**, *99*, 025012.

175. Sassaroli, E.; Srivastava, Y.N.; Widom, A. Photon production by the Dynamical casimir effect. *Phys. Rev. A* **1994**, *50*, 1027–1034.

176. Galley, C.R.; Behunin, R.O.; Hu, B.L. Oscillator-field model of moving mirrors in quantum optomechanics. *Phys. Rev. A* **2013**, *87*, 043832.

177. Wang, Q.; Unruh, W.G. Motion of a mirror under infinitely fluctuating quantum vacuum stress. *Phys. Rev. D* **2014**, *89*, 085009.

178. Volovik, G.E.; Zubkov, M.A. Mirror as polaron with internal degrees of freedom. *Phys. Rev. D* **2014**, *90*, 087702.

179. Wang, Q.; Unruh, W.G. Mirror moving in quantum vacuum of a massive scalar field. *Phys. Rev. D* **2015**, *92*, 063520.

180. Lin, S.-Y. Unruh-DeWitt detectors as mirrors: Dynamical reflectivity and Casimir effect. *Phys. Rev. D* **2018**, *98*, 105010.

181. Candelas, P.; Deutsch, D. On the vacuum stress induced by uniform acceleration or supporting the ether. *Proc. Roy. Soc. Lond. A* **1977**, *354*, 79–99.

182. Frolov, V.P.; Serebriany, E.M. Quantum effects in systems with accelerated mirrors. *J. Phys. A Math. Gen.* **1979**, *12*, 2415–2428.

183. Frolov, V.P.; Serebriany, E.M. Quantum effects in systems with accelerated mirrors: II. electromagnetic field. *J. Phys. A Math. Gen.* **1980**, *13*, 3205–3211.

184. Anderson, W.G.; Israel, W. Quantum flux from a moving spherical mirror. *Phys. Rev. D* **1999**, *60*, 084003.

185. Frolov, V.; Singh, D. Quantum radiation of uniformly accelerated spherical mirrors. *Class. Quantum Grav.* **2001**, *18*, 3025–3038.

186. Fosco, C.D.; Lombardo, F.C.; Mazzitelli, F.D. 2007 Quantum dissipative effects in moving mirrors: A functional approach. *Phys. Rev. D* **2007**, *76*, 085007.

187. Ford, L.H.; Vilenkin, A. Quantum radiation by moving mirrors. *Phys. Rev. D* **1982**, *25*, 2569–2575.

188. Maia Neto, P.A. Vacuum radiation pressure on moving mirrors. *J. Phys. A Math. Gen.* **1994**, *27*, 2167–2180.

189. Maia Neto, P.A.; Machado, L.A.S. Radiation reaction force for a mirror in vacuum. *Braz. J. Phys.* **1995**, *25*, 324–334.

190. Maia Neto, P.A.; Machado, L.A.S. Quantum radiation generated by a moving mirror in free space. *Phys. Rev. A* **1996**, *54*, 3420–3427.

191. Mendonça, J.P.F.; Maia Neto, P.A.; Takakura, F.I. Quantum photon emission from a moving mirror in the nonperturbative regime. *Opt. Commun.* **1999**, *160*, 335–343.

192. Miri, F.; Golestanian, R. Motion-induced radiation from a dynamically deforming mirror. *Phys. Rev. A* **1999**, *59*, 2291–2294.

193. Montazeri, M.; Miri, M.F. Motion-induced radiation from a dynamically deforming mirror: Neumann boundary condition. *Phys. Rev. A* **2005**, *71*, 063814.

194. Sarabadani, J.; Miri, M.F. Mechanical response of the quantum vacuum to dynamic deformations of a cavity. *Phys. Rev. A* **2006**, *74*, 023801.

195. Fosco, C.D.; Lombardo, F.C.; Mazzitelli, F.D.; Remaggi, M.L. Quantum dissipative effects in graphenelike mirrors. *Phys. Rev. D* **2013**, *88*, 105004.

196. Fosco, C.D.; Mazzitelli, F.D. Radiation from a moving planar dipole layer: Patch potentials versus dynamical Casimir effect. *Phys. Rev. A* **2014**, *89*, 062513.

197. Fosco, C.D.; Lombardo, F.C. Oscillating dipole layer facing a conducting plane: A classical analogue of the dynamical Casimir effect. *Eur. Phys. J. C* **2015**, *75*, 598.

198. Stargen, D.J.; Kothawala, D.; Sriramkumar, L. Moving mirrors and the fluctuation-dissipation theorem. *Phys. Rev. D* **2016**, *94*, 025040.

199. Arkhipov, M.V.; Babushkin, I.; Pul'kin, N.S.; Arkhipov, R.M.; Rosanov, N.N. On the emission of radiation by an isolated vibrating metallic mirror. *Opt. Spectrosc.* **2017**, *122*, 670–674.

200. Fosco, C.D.; Remaggi, M.L.; Rodríguez, M.C. Vacuum fluctuation effects due to an Abelian gauge field in 2+1 dimensions, in the presence of moving mirrors. *Phys. Lett. B* **2019**, *797*, 134838.

201. Barton, G.; Eberlein, C. On quantum radiation from a moving body with finite refractive index. *Ann. Phys.* **1993**, *227*, 222–274.

202. Salamone, G.M. Virtual-photon-cloud creation and emission of radiation from a dielectric slab in arbitrary motion. *Phys. Rev. A* **1994**, *49*, 2280–2289.

203. Salamone, G.M.; Barton, G. Quantum radiative reaction on a dispersive mirror in one dimension. *Phys. Rev. A* **1995**, *51*, 3506–3512.

204. Barton, G. The quantum radiation from mirrors moving sideways. *Ann. Phys.* **1996**, *245*, 361–388.

205. Barton, G.; North, C.A. Peculiarities of quantum radiation in three dimensions from moving mirrors with high refractive index. *Ann. Phys.* **1996**, *252*, 72–114.

206. Gütig, R.; Eberlein, C. Quantum radiation from moving dielectrics in two, three, and more spatial dimensions. *J. Phys. A Math. Gen.* **1998**, *31*, 6819–6838.

207. Frolov, V.; Singh, D. Quantum effects in the presence of expanding semi-transparent spherical mirrors. *Class. Quant. Grav.* **1999**, *16*, 3693–3716.

208. Frolov, V.; Singh, D. Quantum radiation of a uniformly accelerated refractive body. *Class. Quantum Grav.* **2000**, *17*, 3905–3916.
209. Braginsky, V.B.; Khalili, F.Y. Friction and fluctuations produced by the quantum ground state. *Phys. Lett. A* **1991**, *161*, 197–201.
210. Jaekel, M.T.; Reynaud, S. Motional Casimir force. *J. Phys. I (Paris)* **1992**, *2*, 149–165.
211. Jaekel, M.T.; Reynaud, S. Causality, stability and passivity for a mirror in vacuum. *Phys. Lett. A* **1992**, *167*, 227–232.
212. Jaekel, M.T.; Reynaud, S. Friction and inertia for a mirror in a thermal field. *Phys. Lett. A* **1993**, *172*, 319–324.
213. Jaekel, M.T.; Reynaud, S. Inertia of Casimir energy. *J. Phys. I (Paris)* **1993**, *3*, 1093–1104.
214. Eberlein, C. Radiation-reaction force on a moving mirror. *J. Phys. I (Paris)* **1993**, *3*, 2151–2159.
215. Maia Neto, P.A.; Reynaud, S. Dissipative force on a sphere moving in vacuum. *Phys. Rev. A* **1993**, *47*, 1639–1646.
216. Alves, D.T.; Farina, C.; Maia Neto, P.A. Dynamical Casimir effect with Dirichlet and Neumann boundary conditions. *J. Phys. A Math. Gen.* **2003**, *36*, 11333–11342.
217. Alves, D.T.; Granhen, E.R.; Lima, M.G. Quantum radiation force on a moving mirror with Dirichlet and Neumann boundary conditions for a vacuum, finite temperature, and a coherent state. *Phys. Rev. D* **2008**, *77*, 125001.
218. Teodorovich, E.V. On the contribution of macroscopic van Der Waals interactions to frictional force. *Proc. R. Soc. Lond. A* **1978**, *362*, 71–77.
219. Levitov, L.S. Van der Waals friction. *Europhys. Lett.* **1989**, *8*, 499–504.
220. Polevoi, V.G. Tangential molecular forces caused between moving bodies by a fluctuating electromagnetic field. *Sov. Phys. - JETP* **1990**, *71*, 1119–1124.
221. Mkrtchian, V.E. Interaction between moving macroscopic bodies: viscosity of the electromagnetic vacuum. *Phys. Lett. A* **1995**, *207*, 299–302.
222. Golestanian, R.; Kardar, M. Mechanical response of vacuum. *Phys. Rev. Lett.* **1997**, *78*, 3421–3425.
223. Pendry, J.B. Shearing the vacuum—quantum friction. *J. Phys. Condens. Matter* **1997**, *9*, 10301–10320.
224. Golestanian, R.; Kardar, M. Path integral approach to the dynamic Casimir effect with fluctuating boundaries. *Phys. Rev. A* **1998**, *58*, 1713–1722.
225. Volokitin, A.I.; Persson, B.N.J. Theory of friction: The contribution from a fluctuating electromagnetic field. *J. Phys. Condens. Matter* **1999**, *11*, 345–359.
226. M. Kardar and R. Golestanian, The “friction” of vacuum and other fluctuation-induced forces. *Rev. Mod. Phys.* **1999**, *71*, 1233–1245.
227. Kenneth, O.; Nussinov, S. Small object limit of the Casimir effect and the sign of the Casimir force. *Phys. Rev. D* **2002**, *65*, 085014.
228. Volokitin, A.I.; Persson, B.N.J. Near-field radiative heat transfer and noncontact friction. *Rev. Mod. Phys.* **2007**, *79*, 1291–1329.
229. Schaich, W.L.; Harris, J. Dynamic corrections to Van der Waals potentials. *J. Phys. F: Met. Phys.* **1981**, *11*, 65–78.
230. Philbin, T.G.; Leonhardt, U. No quantum friction between uniformly moving plates. *New J. Phys.* **2009**, *11*, 033035.
231. Pendry, J.B. Quantum friction—fact or fiction? *New J. Phys.* **2010**, *12*, 033028.
232. Leonhardt, U. Comment on ‘Quantum Friction—Fact or Fiction?’ *New J. Phys.* **2010**, *12*, 068001.
233. Pendry, J.B. Reply to comment on ‘Quantum friction—fact or fiction?’ *New J. Phys.* **2010**, *12*, 068002.
234. Volokitin, A.I.; Persson, B.N.J. Comment on ‘No quantum friction between uniformly moving plates’. *New J. Phys.* **2011**, *13*, 068001.
235. Philbin, T.G.; Leonhardt, U. Reply to comment on ‘No quantum friction between uniformly moving plates’. *New J. Phys.* **2011**, *13*, 068002.
236. Dedkov, G.V.; Kyasov, A. A. Conservative-dissipative forces and heating mediated by fluctuation electromagnetic field: Two plates in relative nonrelativistic motion. *Surf. Sci.* **2010**, *604*, 562–567.
237. Fosco, C.D.; Lombardo, F.C.; Mazzitelli, F.D. Quantum dissipative effects in moving imperfect mirrors: Sidewise and normal motions. *Phys. Rev. D* **2011**, *84*, 025011.
238. Despoja, V.; Echenique, P.M.; Šunjić, M. Nonlocal microscopic theory of quantum friction between parallel metallic slabs. *Phys. Rev. B* **2011**, *83*, 205424.

239. Horsley, S.A.R. Canonical quantization of the electromagnetic field interacting with a moving dielectric medium. *Phys. Rev. A* **2012**, *86*, 023830.

240. Maslovski, S.I.; Silveirinha, M.G. Quantum friction on monoatomic layers and its classical analog. *Phys. Rev. B* **2013**, *88*, 035427.

241. Mkrtchian, V.E.; Henkel, C. On non-equilibrium photon distributions in the Casimir effect. *Ann. Der Phys.* **2014**, *526*, 87–101.

242. Høye, J.S.; Brevik, I. Casimir friction at zero and finite temperatures. *Eur. Phys. J. D* **2014**, *68*, 61.

243. Silveirinha, M.G. Theory of quantum friction. *New J. Phys.* **2014**, *16*, 063011.

244. Volokitin, A.I.; Persson, B.N.J. Quantum Vavilov-Cherenkov radiation from shearing two transparent dielectric plates. *Phys. Rev. B* **2016**, *93*, 035407.

245. Milton, K.A.; Høye, J.S.; Brevik, I. The reality of Casimir friction. *Symmetry* **2016**, *8*, 29.

246. Farías, M.B.; Fosco, C.D.; Lombardo, F.C.; Mazzitelli, F.D. Quantum friction between graphene sheets. *Phys. Rev. D* **2017**, *95*, 065012.

247. Dedkov, G.V.; Kyasov, A.A. Friction Force and Radiative Heat Exchange in a System of Two Parallel Plates in Relative Motion: Corollaries of the Levine–Polevoi–Rytov Theory. *Phys. Solid State* **2018**, *60*, 2349–2357.

248. Despoja, V.; Echenique, P.M.; Šunjić, M. Quantum friction between oscillating crystal slabs: Graphene monolayers on dielectric substrates. *Phys. Rev. B* **2018**, *98*, 125405.

249. Butera, S.; Carusotto, I. Quantum fluctuations of the friction force induced by the dynamical Casimir emission. *EPL* **2019**, *128*, 24002.

250. Farías, M.B.; Lombardo, F.C.; Soba, A.; Villar, P.I.; Decca, R.S. Towards detecting traces of non-contact quantum friction in the corrections of the accumulated geometric phase. *arXiv* **2019**, arXiv:1912.05513.

251. Zel'dovich, Y.B. Generation of waves by a rotating body. *JETP Lett.* **1971**, *14*, 180–181.

252. Zel'dovich, Y.B. Amplification of cylindrical electromagnetic waves reflected from a rotating body. *Sov. Phys. JETP* **1972**, *35*, 1085–1087.

253. Starobinskii, A.A. Amplification of waves during reflection from a rotating “black hole”. *Sov. Phys. JETP* **1973**, *37*, 28–32.

254. Press, W.H.; Teukolsky, S.A. Floating orbits, superradiant scattering and black-hole bomb. *Nature* **1972**, *238*, 211–212.

255. Unruh, W.G. Second quantization in the Kerr metric. *Phys. Rev. D* **1974**, *10*, 3194–3205.

256. Bekenstein, J.D.; Schiffer, M. The many faces of superradiance. *Phys. Rev. D* **1998**, *58*, 064014.

257. Manjavacas, A.; García de Abajo, F.J. Thermal and vacuum friction acting on rotating particles. *Phys. Rev. A* **2010**, *82*, 063827.

258. Maghrebi, M.F.; Jaffe, R.L.; Kardar, M. Spontaneous emission by rotating objects: a scattering approach. *Phys. Rev. Lett.* **2012**, *108*, 230403.

259. Zhao, R.; Manjavacas, A.; García de Abajo, F.J.; Pendry, J.B. Rotational quantum friction. *Phys. Rev. Lett.* **2012**, *109*, 123604.

260. Maghrebi, M.F.; Jaffe, R.L.; Kardar, M. Nonequilibrium quantum fluctuations of a dispersive medium: Spontaneous emission, photon statistics, entropy generation, and stochastic motion. *Phys. Rev. A* **2014**, *90*, 012515.

261. Calogeracos, A.; Volovik, G.E. Rotational quantum friction in superfluids: Radiation from object rotating in superfluid vacuum. *JETP Lett.* **1999**, *69*, 281–287.

262. Lannebère, S.; Silveirinha, M.G. Wave instabilities and unidirectional light flow in a cavity with rotating walls. *Phys. Rev. A* **2016**, *94*, 033810.

263. Johnston, H.; Sarkar, S. A re-examination of the quantum theory of optical cavities with moving mirrors. *J. Phys. A Math. Gen.* **1996**, *29*, 1741–1746.

264. Chizhov, A.V.; Schrade, G.; Zubairy, M.S. Quantum statistics of vacuum in a cavity with a moving mirror. *Phys. Lett. A* **1997**, *230*, 269–275.

265. Mundarain, D.F.; Maia Neto, P.A. Quantum radiation in a plain cavity with moving mirrors. *Phys. Rev. A* **1998**, *57*, 1379–1390.

266. Uhlmann, M.; Plunien, G.; Schützhold, R.; Soff, G. Resonant cavity photon creation via the dynamical Casimir effect. *Phys. Rev. Lett.* **2004**, *93*, 193601.

267. Levchenko, A.V.; Trifanov, A.I. Regimes of photon generation in Dynamical Casimir Effect under various resonance conditions. *J. Phys. Conf. Ser.* **2015**, *643*, 012093.

268. Dodonov, A.V.; Dodonov, V.V. Resonance generation of photons from vacuum in cavities due to strong periodical changes of conductivity in a thin semiconductor boundary layer. *J. Opt. B Quantum Semiclass. Opt.* **2005**, *7*, S47–S58.

269. Wu, Y.; Chu, M.-C.; Leung, P.T. Dynamics of the quantized radiation field in a cavity vibrating at the fundamental frequency. *Phys. Rev. A* **1999**, *59*, 3032–3037.

270. Yang, X.-X.; Wu, Y. Dynamics of the quantized radiation field in an oscillating cavity in the harmonic resonance case. *J. Phys. A Math. Gen.* **1999**, *32*, 7375–7392.

271. Husimi, K. Miscellanea in elementary quantum mechanics. II. *Prog. Theor. Phys.* **1953**, *9*, 381–402.

272. Dodonov, V.V.; Man'ko, V.I. *Invariants and the Evolution of Nonstationary Quantum Systems (Proceedings of the Lebedev Physics Institute vol 183)*; Nova Science: Commack, New York, USA, 1989.

273. Dodonov, V.V.; Man'ko, O.V.; Man'ko, V.I. Quantum nonstationary oscillator: Models and applications. *J. Russ. Laser Res.* **1995**, *16*, 1–56.

274. Dodonov, V.V. Parametric excitation and generation of nonclassical states in linear media. In *Theory of Nonclassical States of Light*; Dodonov, V.V., Man'ko, V.I., Eds.; Taylor & Francis: London, UK, 2003; pp. 153–218.

275. Dodonov, V.V.; Klimov, A.B.; Nikonov, D.E. Quantum phenomena in nonstationary media. *Phys. Rev. A* **1993**, *47*, 4422–4429.

276. Dodonov, A.V.; Dodonov, V.V. Dynamical Casimir effect in a cavity with a weakly non-equidistant spectrum. *Phys. Lett. A* **2012**, *376*, 1903–1906.

277. Dodonov, A.V.; Dodonov, E.V.; Dodonov, V.V. Photon generation from vacuum in nondegenerate cavities with regular and random periodic displacements of boundaries. *Phys. Lett. A* **2003**, *317*, 378–388.

278. Fujii, K.; Suzuki, T. Rotating wave approximation of the Law's effective hamiltonian on the dynamical Casimir effect. *Int. J. Geom. Meth. Mod. Phys.* **2014**, *11*, 1450003.

279. Brown, K.; Lowerstein, A.; Mathur, H. Effect of forcing on vacuum radiation. *Phys. Rev. A* **2019**, *99*, 022504.

280. Crocce, M.; Dalvit, D.A.R.; Mazzitelli, F.D. Resonant photon creation in a three-dimensional oscillating cavity. *Phys. Rev. A* **2001**, *64*, 013808.

281. Dodonov, A.V.; Dodonov, V.V. Nonstationary Casimir effect in cavities with two resonantly coupled modes *Phys. Lett. A* **2001**, *289*, 291–300.

282. Ruser, M. Numerical investigation of photon creation in a three-dimensional resonantly vibrating cavity: Transverse electric modes. *Phys. Rev. A* **2006**, *73*, 043811.

283. Yuce, C.; Ozcakmakli, Z. The dynamical Casimir effect for two oscillating mirrors in 3D. *J. Phys. A Math. Theor.* **2008**, *41*, 265401.

284. Yuce, C.; Ozcakmakli, Z. Dynamical Casimir effect for a swinging cavity. *J. Phys. A Math. Theor.* **2009**, *42*, 035403.

285. Dodonov, V.V.; Dodonov, A.V. Excitation of the classical electromagnetic field in a cavity containing a thin slab with a time-dependent conductivity. *J. Russ. Laser Res.* **2016**, *37*, 107–122.

286. Casimir, H.B.G. On the attraction between two perfectly conducting plates. *Proc. K. Ned. Akad. Wet.* **1948**, *51*, 793–795.

287. Bordag, M.; Petrov, G.; Robaschik, D. Calculation of the Casimir effect for scalar fields with the simplest nonstationary boundary conditions. *Sov. J. Nucl. Phys.* **1984**, *39*, 828–831.

288. Bordag, M.; Dittes, F.-M.; Robaschik, D. The Casimir effect with uniformly moving mirrors. *Sov. J. Nucl. Phys.* **1986**, *43*, 1034–1038.

289. Hanke, A. Non-Equilibrium Casimir Force between Vibrating Plates. *PLoS ONE* **2013**, *8*, e53228.

290. Villarreal, C.; Hacyan, S.; Jáuregui, R. Generation of particles and squeezed states between moving conductors. *Phys. Rev. A* **1995**, *52*, 594–601.

291. Cirone, M.A.; Rzążewski, K. Electromagnetic radiation in a cavity with a time-dependent mirror. *Phys. Rev. A* **1999**, *60*, 886–892.

292. Setare, M.R.; Seyedzahedi, A. Fermion particle production in dynamical Casimir effect in a three-dimensional box. *Int. J. Mod. Phys. A* **2012**, *27*, 1250176.

293. Mkrtchian, V.E.; von Baltz, R. Dynamical electromagnetic modes for an expanding sphere. *J. Math. Phys.* **2000**, *41*, 1956–1960.

294. Mazzitelli, F.D.; Millán, X.O. Photon creation in a spherical oscillating cavity. *Phys. Rev. A* **2006**, *73*, 063829.

295. Pascoal, F.; Céleri, L.C.; Mizrahi, S.S.; Moussa, M.H.Y. Dynamical Casimir effect for a massless scalar field between two concentric spherical shells. *Phys. Rev. A* **2008**, *78*, 032521.

296. Pascoal, F.; Céleri, L.C.; Mizrahi, S.S.; Moussa, M.H.Y.; Farina, C. Dynamical Casimir effect for a massless scalar field between two concentric spherical shells with mixed boundary conditions. *Phys. Rev. A* **2009**, *80*, 012503.

297. Naylor, W. Towards particle creation in a microwave cylindrical cavity. *Phys. Rev. A* **2012**, *86*, 023842.

298. Setare, M.R.; Seyedzahedi, A. Fermion particle production as a dynamical Casimir effect inside a three-dimensional sphere. *J. Phys. Conf. Ser.* **2013**, *410*, 012150.

299. Rego, A.L.C.; Mintz, B.W.; Farina, C.; Alves, D.T. Inhibition of the dynamical Casimir effect with Robin boundary conditions. *Phys. Rev. D* **2013**, *87*, 045024.

300. Fosco, C.D.; Giraldo, A.; Mazzitelli, F.D. Dynamical Casimir effect for semitransparent mirrors. *Phys. Rev. D* **2017**, *96*, 045004.

301. Maghrebi, M.F.; Golestanian, R.; Kardar, M. Scattering approach to the dynamical Casimir effect. *Phys. Rev. D* **2013**, *87*, 025016.

302. Villar, P.I.; Soba, A. Adaptive numerical algorithms to simulate the dynamical Casimir effect in a closed cavity with different boundary conditions. *Phys. Rev. E* **2017**, *96*, 013307.

303. Srivastava, Y.N.; Widom, A.; Sivasubramanian, S.; Pradeep Ganesh, M. Dynamical Casimir effect instabilities. *Phys. Rev. A* **2006**, *74*, 032101.

304. de Sousa, I.M.; Dodonov, A.V. Microscopic toy model for the cavity dynamical Casimir effect. *J. Phys. A Math. Theor.* **2015**, *48*, 245302.

305. Román-Ancheyta, R.; González-Gutiérrez, C.; Récamier, J. Influence of the Kerr nonlinearity in a single nonstationary cavity mode. *J. Opt. Soc. Am. B* **2017**, *34*, 1170–1176.

306. Paredes, A.; Récamier, J. Study of the combined effects of a Kerr nonlinearity and a two-level atom upon a single nonstationary cavity mode. *J. Opt. Soc. Am. B* **2019**, *36*, 1538–1543.

307. Mendorça, J.T.; Brodin, G.; Marklund, M. The influence of temporal coherence on the dynamical Casimir effect. *Phys. Lett. A* **2011**, *375*, 2665–2669.

308. Rosanov, N.N.; Matskovskii, A.A.; Malevich, V.L.; Sinitsyn, G.V. Parametric field excitation in a cavity with oscillating mirrors. *2015, Opt. Spectrosc.* **2015**, *119*, 89–91.

309. Srivastava, Y.; Widom, A. Quantum electrodynamic processes in electrical engineering circuits. *Phys. Rep.* **1987**, *148*, 1–65.

310. Man'ko, V.I. The Casimir effect and quantum vacuum generator. *J. Sov. Laser Res.* **1991**, *12*, 383–5.

311. Man'ko, O.V. Correlated squeezed states of a Josephson junction. *J. Kor. Phys. Soc.* **1994**, *27*, 1–4.

312. Segev, E.; Abdo, B.; Shtempluck, O.; Buks, E.; Yurke, B. Prospects of employing superconducting stripline resonators for studying the dynamical Casimir effect experimentally. *Phys. Lett. A* **2007**, *370*, 202–206.

313. Takashima, K.; Hatakenaka, N.; Kurihara, S.; Zeilinger, A. Squeezing of a quantum flux in a double rf-SQUID system. *J. Phys. A Math. Theor.* **2008**, *41*, 164036.

314. Fujii, T.; Matsuo, S.; Hatakenaka, N.; Kurihara, S.; Zeilinger, A. Quantum circuit analog of the dynamical Casimir effect. *Phys. Rev. B* **2011**, *84*, 174521.

315. Berdiyorov, G.R.; Milošević, M.V.; Savel'ev, S.; Kusmartsev, F.; Peeters, F.M. Parametric amplification of vortex-antivortex pair generation in a Josephson junction. *Phys. Rev. B* **2014**, *90*, 134505.

316. Dodonov, A.V. Photon creation from vacuum and interactions engineering in nonstationary circuit QED. *J. Phys. Conf. Ser.* **2009**, *161*, 012029.

317. Johansson, J.R.; Johansson, G.; Wilson, C.M.; Nori, F. Dynamical Casimir effect in a superconducting coplanar waveguide. *Phys. Rev. Lett.* **2009**, *103*, 147003.

318. Johansson, J.R.; Johansson, G.; Wilson, C.M.; Nori, F. Dynamical Casimir effect in superconducting microwave circuits. *Phys. Rev. A* **2010**, *82*, 052509.

319. Wilson, C.M.; Duty, T.; Sandberg, M.; Persson, F.; Shumeiko, V.; Delsing, P. Photon generation in an electromagnetic cavity with a time-dependent boundary. *Phys. Rev. Lett.* **2010**, *105*, 233907.

320. Wilson, C.M.; Johansson, G.; Pourkabirian, A.; Simoen, M.; Johansson, J.R.; Duty, T.; Nori, F.; Delsing, P. Observation of the dynamical Casimir effect in a superconducting circuit. *Nature* **2011**, *479*, 376–379.

321. Johansson, J.R.; Johansson, G.; Wilson, C.M.; Delsing, P.; Nori, F. Nonclassical microwave radiation from the dynamical Casimir effect. *Phys. Rev. A* **2013**, *87*, 043804.

322. Lähteenmäki, P.; Paraoanu, G.S.; Hassel, J.; Hakonen, P.J. Dynamical Casimir effect in a Josephson metamaterial. *Proc. Nat. Acad. Sci. USA* **2013**, *110*, 4234–8.

323. Svensson, I.-M.; Pierre, M.; Simoen, M.; Wustmann, W.; Krantz, P.; Bengtsson, A.; Johansson, G.; Bylander, J.; Shumeiko, V.; Delsing, P. Microwave photon generation in a doubly tunable superconducting resonator. *J. Phys. Conf. Ser.* **2018**, *969*, 012146.

324. Wustmann, W.; Shumeiko, V. Parametric resonance in tunable superconducting cavities. *Phys. Rev. B* **2013**, *87*, 184501.

325. Rego, A.L.C.; Alves, J.P.daS.; Alves, D.T.; Farina, C. Relativistic bands in the spectrum of created particles via the dynamical Casimir effect. *Phys. Rev. A* **2013**, *88*, 032515.

326. Rego, A.L.C.; Silva, H.O.; Alves, D.T.; Farina, C. New signatures of the dynamical Casimir effect in a superconducting circuit. *Phys. Rev. D* **2014**, *90*, 025003.

327. Lindkvist, J.; Sabin, C.; Fuentes, I.; Dragan, A.; Svensson, I.-M.; Delsing, P.; Johansson, G. Twin paradox with macroscopic clocks in superconducting circuits. *Phys. Rev. A* **2014**, *90*, 052113.

328. Zhang, Y.N.; Luo, X.W.; Zhou, Z.W. Dynamical Casimir effect in dissipative superconducting circuit system. *Sci. China - Phys. Mech. Astron.* **2014**, *57*, 2251–2258.

329. Andersen, C.K.; Mølmer, K. Multifrequency modes in superconducting resonators: Bridging frequency gaps in off-resonant couplings. *Phys. Rev. A* **2015**, *91*, 023828.

330. Doukas, J.; Louko, J. Superconducting circuit boundary conditions beyond the dynamical Casimir effect. *Phys. Rev. D* **2015**, *91*, 044010.

331. Corona-Ugalde, P.; Martín-Martínez, E.; Wilson, C.M.; Mann, R.B. Dynamical Casimir effect in circuit QED for nonuniform trajectories. *Phys. Rev. A* **2016**, *93*, 012519.

332. Lombardo, F.C.; Mazzitelli, F.D.; Soba, A.; Villar, P.I. Dynamical Casimir effect in superconducting circuits: A numerical approach. *Phys. Rev. A* **2016**, *93*, 032501.

333. Samos-Saenz de Buruaga, D.N.; Sabín, C. Quantum coherence in the dynamical Casimir effect. *Phys. Rev. A* **2017**, *95*, 022307.

334. Sabín, C.; Peropadre, B.; Lamata, L.; Solano, E. Simulating superluminal physics with superconducting circuit technology. *Phys. Rev. A* **2017**, *96*, 032121.

335. Lombardo, F.C.; Mazzitelli, F.D.; Soba, A.; Villar, P.I. Dynamical Casimir effect in a double tunable superconducting circuit. *Phys. Rev. A* **2018**, *98*, 022512.

336. Bosco, S.; Lindkvist, J.; Johansson, G. Simulating moving cavities in superconducting circuits. *Phys. Rev. A* **2019**, *100*, 023817.

337. Ma, S.; Miao, H.; Xiang, Y.; Zhang, S. Enhanced dynamic Casimir effect in temporally and spatially modulated Josephson transmission line. *Laser Photonics Rev.* **2019**, *1900164*.

338. Dodonov, A.V. Asymptotic mean excitation numbers due to anti-rotating term (AMENDART) in Markovian circuit QED. *J. Phys. Conf. Ser.* **2011**, *274*, 012137.

339. Dodonov, A.V. Analytical description of nonstationary circuit QED in the dressed-states basis. *J. Phys. A Math. Theor.* **2014**, *47*, 285303.

340. Veloso, D.S.; Dodonov, A.V. Prospects for observing dynamical and anti-dynamical Casimir effects in circuit QED due to fast modulation of qubit parameters. *J. Phys. B At. Mol. Opt. Phys.* **2015**, *48*, 165503.

341. Felicetti, S.; Sabin, C.; Fuentes, I.; Lamata, L.; Romero, G.; Solano, E. Relativistic motion with superconducting qubits. *Phys. Rev. B* **2015**, *92*, 064501.

342. Hoi, I.-C.; Kockum, A.F.; Tornberg, L.; Pourkabirian, A.; Johansson, G.; Delsing, P.; Wilson, C.M. Probing the quantum vacuum with an artificial atom in front of a mirror. *Nat. Physics* **2015**, *11*, 1045–1049.

343. Rossatto, D.Z.; Felicetti, S.; Eneriz, H.; Rico, E.; Sanz, M.; Solano, E. Entangling polaritons via dynamical Casimir effect in circuit quantum electrodynamics. *Phys. Rev. B* **2016**, *93*, 094514.

344. Dodonov, A.V.; Militello, B.; Napoli, A.; Messina, A. Effective Landau-Zener transitions in the circuit dynamical Casimir effect with time-varying modulation frequency, *Phys. Rev. A* **2016**, *93*, 052505.

345. Silva, E.L.S.; Dodonov, A.V. Analytical comparison of the first- and second-order resonances for implementation of the dynamical Casimir effect in nonstationary circuit QED. *J. Phys. A Math. Theor.* **2016**, *49*, 495304.

346. García-Álvarez, L.; Felicetti, S.; Rico, E.; Solano, E.; Sabin, C. Entanglement of superconducting qubits via acceleration radiation. *Sci. Rep.* **2017**, *7*, 657.

347. Gu, X.; Kockum, A.F.; Miranowicz, A.; Liu, Y.-X.; Nori, F. Microwave photonics with superconducting quantum circuits. *Phys. Rep.* **2017**, *718–719*, 1–102.

348. Zhukov, A.A.; Shapiro, D.S.; Remizov, S.V.; Pogosov, W.V.; Lozovik, Y.E. Superconducting qubit in a nonstationary transmission line cavity: Parametric excitation, periodic pumping, and energy dissipation. *Phys. Lett. A* **2017**, *381*, 592–596.

349. Zhukov, A.A.; Remizov, S.V.; Pogosov, W.V.; Shapiro, D.S.; Lozovik, Y.E. Superconducting qubit systems as a platform for studying effects of nonstationary electrodynamics in a cavity. *JETP Lett.* **18**, *108*, 63–70.

350. Dessano, H.; Dodonov, A.V. One- and three-photon dynamical Casimir effects using a nonstationary cyclic qutrit. *Phys. Rev. A* **2018**, *98*, 022520.

351. Wustmann, W.; Shumeiko, V. Parametric effects in circuit quantum electrodynamics. *Low Temp. Phys.* **2019**, *45*, 848–869.

352. Carusotto, I.; Balbinot, R.; Fabbri, A.; Recati, A. Density correlations and analog dynamical Casimir emission of Bogoliubov phonons in modulated atomic Bose–Einstein condensates. *Eur. Phys. J. D* **2010**, *56*, 391–404.

353. Jaskula, J.-C.; Partridge, G.B.; Bonneau, M.; Lopes, R.; Ruaudel, J.; Boiron, D.; Westbrook, C.I. Acoustic analog to the dynamical Casimir effect in a Bose–Einstein condensate. *Phys. Rev. Lett.* **2012**, *109*, 220401.

354. Balbinot, R.; Fabbri, A. Amplifying the Hawking signal in BECs. *Advances High Energy Phys.* **2014**, *713574*.

355. Mendonça, J.T.; Dodonov, V.V. Time crystals in ultracold matter. *J. Rus. Laser Res.* **2014**, *35*, 93–100.

356. Dodonov, V.V.; Mendonça, J.T. Dynamical Casimir effect in ultra-cold matter with a time-dependent effective charge. *Phys. Scr.* **2014**, *T160*, 014008.

357. Mahajan, S.; Aggarwal, N.; Bhattacherjee, A.B.; ManMohan. Dynamical Casimir effect in superradiant light scattering by Bose–Einstein condensate in an optomechanical cavity. *Chin. Phys. B* **2014**, *23*, 020315.

358. Marino, J.; Recati, A.; Carusotto, I. Casimir forces and quantum friction from Ginzburg radiation in atomic Bose–Einstein condensates. *Phys. Rev. Lett.* **2017**, *118*, 045301.

359. Rosanov, N.N.; Vysotina, N.V. Dynamics of hysteresis for a Bose–Einstein condensate soliton in a dynamic trap. *Opt. Spectrosc.* **2017**, *123*, 918–927.

360. Eckel, S.; Kumar, A.; Jacobson, T.; Spielman, I.B.; Campbell, G.K. A rapidly expanding Bose–Einstein condensate: An expanding universe in the lab. *Phys. Rev. X* **2018**, *8*, 021021.

361. Tian, Z.; Cha, S.-Y.; Fischer, U.R. Roton entanglement in quenched dipolar Bose–Einstein condensates. *Phys. Rev. A* **2018**, *97*, 063611.

362. Motazedifard, A.; Dalafi, A.; Naderi, M.H.; Roknizadeh, R. Controllable generation of photons and phonons in a coupled Bose–Einstein condensate-optomechanical cavity via the parametric dynamical Casimir effect. *Ann. Phys.* **2018**, *396*, 202–219.

363. Lange, K.; Peise, J.; Luecke, B.; Gruber, T.; Sala, A.; Polls, A.; Ertmer, W.; Juliá-Díaz, B.; Santos, L.; Klempert, C. Creation of entangled atomic states by an analogue of the Dynamical Casimir effect. *New J. Phys.* **2018**, *20*, 103017.

364. Michael, M.H.; Schmiedmayer, J.; Demler, E. From the moving piston to the dynamical Casimir effect: Explorations with shaken condensates. *Phys. Rev. A* **2019**, *99*, 053615.

365. Lawandy, N.M.; 2006 Scattering of vacuum states by dynamic plasmon singularities: Generating photons from vacuum. *Opt. Lett.* **2006**, *31*, 3650–2.

366. Ciuti, C.; Bastard, G.; Carusotto, I. Quantum vacuum properties of the intersubband cavity polariton field. *Phys. Rev. B* **2005**, *72*, 115303.

367. Koghee, S.; Wouters, M. Dynamical Casimir emission from polariton condensates. *Phys. Rev. Lett.* **2014**, *112*, 036406.

368. Koghee, S.; Wouters, M. Dynamical quantum depletion in polariton condensates. *Phys. Rev. B* **2015**, *92*, 195309.

369. Hizhnyakov, V.; Loot, A.; Azizabadi, S.C. Dynamical Casimir effect for surface plasmon polaritons. *Phys. Lett. A* **2015**, *379*, 501–505.

370. Naylor, W. Vacuum-excited surface plasmon polaritons. *Phys. Rev. A* **2015**, *91*, 053804.

371. Saito, H.; Hyuga, H. 2008 Dynamical Casimir effect for magnons in a spinor Bose–Einstein condensate. *Phys. Rev. A* **2008**, *78*, 033605.

372. Zhao, X.-D.; Zhao, X.; Jing, H.; Zhou, L.; Zhang, W. Squeezed magnons in an optical lattice: Application to simulation of the dynamical Casimir effect at finite temperature. *Phys. Rev. A* **2013**, *87*, 053627.

373. Ford, L.H.; Svaiter, N.F. The phononic Casimir effect: An analog model. *J. Phys. Conf. Ser.* **2009**, *161*, 012034.

374. Motazedifard, A.; Naderi, M.H.; Roknizadeh, R. Dynamical Casimir effect of phonon excitation in the dispersive regime of cavity optomechanics. *J. Opt. Soc. Am. B* **2017**, *34*, 642–652.

375. Wang, X.; Qin, W.; Miranowicz, A.; Savasta, S.; Nori, F. Unconventional cavity optomechanics: Nonlinear control of phonons in the acoustic quantum vacuum. *Phys. Rev. A* **2019**, *100*, 063827.

376. Wittemer, M.; Hakelberg, F.; Kiefer, P.; Schröder, J.-P.; Fey, C.; Schätz, T. Phonon pair creation by inflating quantum fluctuations in an ion trap. *Phys. Rev. Lett.* **2019**, *123*, 180502.

377. Jáuregui, R.; Villarreal, C. Transition probabilities of atomic systems between moving walls. *Phys. Rev. A* **1996**, *54*, 3480–3488.

378. Janowicz, M. Evolution of wave fields and atom-field interactions in a cavity with one oscillating mirror. *Phys. Rev. A* **1998**, *57*, 4784–4790.

379. Carusotto, I.; Antezza, M.; Bariani, F.; De Liberato, S.; Ciuti, C. Optical properties of atomic Mott insulators: From slow light to dynamical Casimir effects. *Phys. Rev. A* **2008**, *77*, 063621.

380. Werlang, T.; Dodonov, A.V.; Duzzioni, E.I.; Villas-Bôas, C.J. 2008, Rabi model beyond the rotating-wave approximation: Generation of photons from vacuum through decoherence. *Phys. Rev. A* **2008**, *78*, 053805.

381. De Liberato, S.; Gerace, D.; Carusotto, I.; Ciuti, C. Extracavity quantum vacuum radiation from a single qubit. *Phys. Rev. A* **2009**, *80*, 053810.

382. Dodonov, A.V. How ‘cold’ can a Markovian dissipative cavity QED system be? *Phys. Scr.* **2010**, *82*, 038102.

383. Zhang, X.; Zheng, T.Y.; Tian, T.; Pan, S.M. The dynamical Casimir effect versus collective excitations in atom ensemble. *Chin. Phys. Lett.* **2011**, *28*, 064202.

384. Dodonov, A.V.; Lo Nardo, R.; Migliore, R.; Messina, A.; Dodonov, V.V. Analytical and numerical analysis of the atom-field dynamics in non-stationary cavity QED. *J. Phys. B At. Mol. Opt. Phys.* **2011**, *44*, 225502.

385. Vacanti, G.; Pugnetti, S.; Didier, N.; Paternostro, M.; Palma, G.M.; Fazio, R.; Vedral, V. Photon production from the vacuum close to the superradiant transition: Linking the dynamical Casimir effect to the Kibble-Zurek mechanism. *Phys. Rev. Lett.* **2012**, *108*, 093603.

386. Dodonov, A.V.; Dodonov, V.V. Approximate analytical results on the cavity dynamical Casimir effect in the presence of a two-level atom. *Phys. Rev. A* **2012**, *85*, 015805.

387. Carusotto, I.; De Liberato, S.; Gerace, D.; Ciuti, C. Back-reaction effects of quantum vacuum in cavity quantum electrodynamics. *Phys. Rev. A* **2012**, *85*, 023805.

388. Dodonov, A.V.; Dodonov, V.V. Dynamical Casimir effect in a cavity with an N-level detector or N-1 two-level atoms. *Phys. Rev. A* **2012**, *86*, 015801.

389. Dodonov, A.V.; Dodonov, V.V. Dynamical Casimir effect in a cavity in the presence of a three-level atom. *Phys. Rev. A* **2012**, *85*, 063804.

390. Dodonov, A.V.; Dodonov, V.V. Dynamical Casimir effect in two-atom cavity QED. *Phys. Rev. A* **2012**, *85*, 055805.

391. Fujii, K.; Suzuki, T. An approximate solution of the dynamical Casimir effect in a cavity with a two-level atom. *Int. J. Geom. Meth. Mod. Phys.* **2013**, *10*, 1350035.

392. Garziano, L.; Ridolfo, A.; Stassi, R.; Di Stefano, O.; Savasta, S. Switching on and off of ultrastrong light-matter interaction: Photon statistics of quantum vacuum radiation. *Phys. Rev. A* **2013**, *88*, 063829.

393. Impens, F.; Ttira, C.C.; Maia Neto, P.A. Non-additive dynamical Casimir atomic phases. *J. Phys. B At. Mol. Opt. Phys.* **2013**, *46*, 245503.

394. Sheremetev, V.O.; Trifanova, E.S.; Trifanov, A.I. Conditional evolution of vacuum state in dynamical Casimir effect. *J. Phys. Conf. Ser.* **2014**, *541*, 012105.

395. Zhang, X.; Yang, H.; Zheng, T.Y.; Pan, S.M. Linking the dynamical Casimir effect to the collective excitation effect at finite temperature. *Int. J. Theor. Phys.* **2014**, *53*, 510–518.

396. Jin, Z.; Han, T.T.; Zheng, T.Y.; Zhang, X.; Pan, S.M.; Asilbieke, B. Influence of interaction of two Atoms on the dynamical Casimir effect. *Int. J. Theor. Phys.* **2015**, *54*, 1627–1632.

397. Aggarwal, N.; Bhattacherjee, A.B.; Banerjee, A.; Man Mohan. Influence of periodically modulated cavity field on the generation of atomic-squeezed states. *J. Phys. B At. Mol. Opt. Phys.* **2015**, *48*, 115501.

398. Asilbieke, B.; Xue, Z.; Jie, F.; Liu, H.; Pan, S.M.; Zheng, T.Y.; Yang, H. Dynamical Casimir effect and collective excitation effect at finite temperature without the rotating-wave approximation. *Int. J. Theor. Phys.* **2015**, *54*, 2762–2770.

399. Trautmann, N.; Hauke, P. Quantum simulation of the dynamical Casimir effect with trapped ions. *New J. Phys.* **2016**, *18*, 043029.

400. Hoeb, F.; Angaroni, F.; Zoller, J.; Calarco, T.; Strini, G.; Montangero, S.; Benenti, G. Amplification of the parametric dynamical Casimir effect via optimal control. *Phys. Rev. A* **2017**, *96*, 033851.

401. Remizov, S.V.; Zhukov, A.A.; Shapiro, D.S.; Pogosov, W.V.; Lozovik, Y.E. Parametrically driven hybrid qubit-photon systems: Dissipation-induced quantum entanglement and photon production from vacuum. *Phys. Rev. A* **2017**, *96*, 043870.

402. Silveri, M.P.; Tuorila, J.A.; Thuneberg, E.V.; Paraoanu, G.S. Quantum systems under frequency modulation. *Rep. Prog. Phys.* **2017**, *80*, 056002.

403. Lo, L.; Law, C.K. Quantum radiation from a shaken two-level atom in vacuum. *Phys. Rev. A* **2018**, *98*, 063807.

404. González-Gutiérrez, C.; de los Santos-Sánchez, O.; Román-Ancheyta, R.; Berrondo, M.; Récamier, J. Lie algebraic approach to a nonstationary atom-cavity system. *J. Opt. Soc. Am. B* **2018**, *35*, 1979–1984.

405. Benenti, G.; Stramacchia, M.; Strini, G. Dynamical Casimir effect and state transfer in the ultrastrong coupling regime. *MDPI Proceedings* **2019**, *12*, 12.

406. Pan, S.M.; Asilibieke, B.; Zheng, L.; Tian, T.; Zhang, X.; Zheng, T.Y. The dynamical Casimir effect in squeezed vacuum state. *Int. J. Theor. Phys.* **2019**, *58*, 22–30.

407. Liu, H.; Wang, Q.; Zhang, X.; Long, Y.M.; Pan, S.M.; Zheng, T.Y.; Sun, C.; Xue, K. The dynamical behaviors of the two-atom and the dynamical Casimir effect in a non-stationary cavity. *Int. J. Theor. Phys.* **2019**, *58*, 786–798.

408. Dodonov, A.V. Dynamical Casimir effect via four- and five-photon transitions using a strongly detuned atom. *Phys. Rev. A* **2019**, *100*, 032510.

409. Abdel-Khalek, S.; Berrada, K. Entanglement, nonclassical properties, and geometric phase in circuit quantum electrodynamics with relativistic motion. *Solid State Commun.* **2019**, *290*, 31–36.

410. Svidzinsky, A.A.; Ben-Benjamin, J.S.; Fulling, S.A.; Page, D.N. Excitation of an atom by a uniformly accelerated mirror through virtual transitions. *Phys. Rev. Lett.* **2018**, *121*, 071301.

411. Dodonov, A.V.; Dodonov, V.V. Strong modifications of the field statistics in the cavity dynamical Casimir effect due to the interaction with two-level atoms and detectors. *Phys. Lett. A* **2011**, *375*, 4261–4267.

412. Benenti, G.; Siccardi, S.; Strini, G. Exotic states in the dynamical Casimir effect. *Eur. Phys. J. D* **2014**, *68*, 139.

413. Motazedifard, A.; Naderi, M.H.; Roknizadeh, R. Analogue model for controllable Casimir radiation in a nonlinear cavity with amplitude-modulated pumping: Generation and quantum statistical properties. *J. Opt. Soc. Am. B* **2015**, *32*, 1555–1563.

414. Monteiro, L.C.; Dodonov, A.V. Anti-dynamical Casimir effect with an ensemble of qubits. *Phys. Lett. A* **2016**, *380*, 1542–1546.

415. Dodonov, A.V.; Valente, D.; Werlang, T. Antidynamical Casimir effect as a resource for work extraction. *Phys. Rev. A* **2017**, *96*, 012501.

416. Dodonov, A.V.; Díaz-Guevara, J.J.; Napoli, A.; Militello, B. Speeding up the antidynamical Casimir effect with nonstationary qutrits. *Phys. Rev. A* **2017**, *96*, 032509.

417. Dodonov, A.V.; Valente, D.; Werlang, T. Quantum power boost in a nonstationary cavity-QED quantum heat engine. *J. Phys. A Math. Theor.* **2018**, *51*, 365302.

418. Angaroni, F.; Benenti, G.; Strini, G. Applications of Picard and Magnus expansions to the Rabi model. *Eur. Phys. J. D* **2018**, *72*, 188.

419. Antezza, M.; Braggio, C.; Carugno, G.; Noto, A.; Passante, R.; Rizzato, L.; Ruoso, G.; Spagnolo, S. Optomechanical Rydberg-atom excitation via dynamic Casimir-Polder coupling. *Phys. Rev. Lett.* **2014**, *113*, 023601.

420. Glaetze, A.W.; Hammerer, K.; Daley, A.J.; Platt, R.; Zoller, P. A single trapped atom in front of an oscillating mirror. *Opt. Commun.* **2010**, *283*, 758–765.

421. Ferreri, A.; Domina, M.; Rizzato, L.; Passante, R. Spontaneous emission of an atom near an oscillating mirror. *Symmetry* **2019**, *11*, 1384.

422. Scully, M.O.; Kocharovskiy, V.V.; Belyanin, A.; Fry, E.; Capasso, F. Enhancing acceleration radiation from ground-state atoms via cavity quantum electrodynamics. *Phys. Rev. Lett.* **2003**, *91*, 243004.

423. Tadigadapa, S.; Mateti, K. Piezoelectric MEMS sensors: State-of-the-art and perspectives. *Meas. Sci. Technol.* **2009**, *20*, 092001.

424. Kim, W.-J.; Brownell, J.H.; Onofrio, R. Detectability of dissipative motion in quantum vacuum via superradiance. *Phys. Rev. Lett.* **2006**, *96*, 200402.

425. Brownell, J.H.; Kim, W.J.; Onofrio, R. Modelling superradiant amplification of Casimir photons in very low dissipation cavities. *J. Phys. A Math. Theor.* **2008**, *41*, 164026.

426. Sanz, M.; Wieczorek, W.; Gröblacher, S.; Solano, E. Electro-mechanical Casimir effect. *Quantum* **2018**, *2*, 91.

427. Wang, H.; Blencowe, M.P.; Wilson, C.M.; Rimberg, A.J. Mechanically generating entangled photons from the vacuum: A microwave circuit-acoustic resonator analog of the oscillatory Unruh effect. *Phys. Rev. A* **2019**, *99*, 053833.

428. Qin, W.; Macrì, V.; Miranowicz, A.; Savasta, S.; Nori, F. Emission of photon pairs by mechanical stimulation of the squeezed vacuum. *Phys. Rev. A* **2019**, *100*, 062501.

429. Yablonovitch, E.; Heritage, J.P.; Aspnes, D.E.; Yafet, Y. Virtual photoconductivity. *Phys. Rev. Lett.* **1989**, *63*, 976–9.

430. Okushima, T.; Shimizu, A. Photon emission from a false vacuum of semiconductors. *Japan. J. Appl. Phys.* **1995**, *34*, 4508–10.

431. Lozovik, Y.E.; Tsvetus, V.G.; Vinogradov, E.A. Parametric excitation of vacuum by use of femtosecond laser pulses. *Phys. Scripta* **1995**, *52*, 184–90.

432. Braggio, C.; Bressi, G.; Carugno, G.; Lombardi, A.; Palmieri, A.; Ruoso, G.; Zanello, D. Semiconductor microwave mirror for a measurement of the dynamical Casimir effect. *Rev. Sci. Instrum.* **2004**, *75*, 4967–70.

433. Braggio, C.; Bressi, G.; Carugno, G.; Del Noce, C.; Galeazzi, G.; Lombardi, A.; Palmieri, A.; Ruoso, G.; Zanello, D. A novel experimental approach for the detection of the dynamic Casimir effect. *Europhys. Lett.* **2005**, *70*, 754–60.

434. Agnesi, A.; Braggio, C.; Bressi, G.; Carugno, G.; Galeazzi, G.; Pirzio, F.; Reali, G.; Ruoso, G.; Zanello, D. MIR status report: An experiment for the measurement of the dynamical Casimir effect. *J. Phys. A Math. Theor.* **2008**, *41*, 164024.

435. Dodonov, V.V. Dynamical Casimir effect meets material science. *IOP Conf. Ser. Mater. Sci. Eng.* **2019**, *474*, 012009.

436. Hagenmuller, D. All-optical dynamical Casimir effect in a three-dimensional terahertz photonic band gap. *Phys. Rev. B* **2016**, *93*, 235309.

437. Bialynicka-Birula, Z.; Bialynicki-Birula, I. Space-time description of squeezing. *J. Opt. Soc. Am. B* **1987**, *4*, 1621–6.

438. Lobashov, A.A.; Mostepanenko, V.M. Quantum Effects in Nonlinear Insulating Materials in the Presence of a Nonstationary Electromagnetic Field. *Theor. Math. Phys.* **1991**, *86*, 303–309.

439. Lobashov, A.A.; Mostepanenko, V.M. Quantum Effects Associated With Parametric Generation of Light and the Theory of Squeezed States. *Theor. Math. Phys.* **1991**, *88*, 913–925.

440. Hizhnyakov, V.V. Quantum emission of a medium with a time-dependent refractive index. *Quant. Opt.* **1992**, *4*, 277–80.

441. Johnston, H.; Sarkar, S. Moving mirrors and time-varying dielectrics. *Phys. Rev. A* **1995**, *51*, 4109–4115.

442. Artoni, M.; Bulatov, A.; Birman, J. Zero-point noise in a nonstationary dielectric cavity. *Phys. Rev. A* **1996**, *53*, 1031–1035.

443. Saito, H.; Hyuga, H. The dynamical Casimir effect for an oscillating dielectric model. *J. Phys. Soc. Jpn.* **1996**, *65*, 3513–3523.

444. Cirone, M.; Rzażewski, K.; Mostowski, J. Photon generation by time-dependent dielectric: A soluble model. *Phys. Rev. A* **1997**, *55*, 62–66.

445. Mendonça, J.T.; Guerreiro, A.; Martins, A.M. Quantum theory of time refraction. *Phys. Rev. A* **2000**, *62*, 033805.

446. Braunstein, S.L. A quantum optical shutter. *J. Opt. B Quantum Semiclass. Opt.* **2005**, *7*, S28–S31.

447. Mendonça, J.T.; Guerreiro, A. Time refraction and the quantum properties of vacuum. *Phys. Rev. A* **2005**, *72*, 063805.

448. Hizhnyakov, V.; Kaasik, H. Emission by dielectric with oscillating refractive index. *J. Phys. Conf. Ser.* **2005**, *21*, 155–60.

449. De Liberato, S.; Ciuti, C.; Carusotto, I. Quantum vacuum radiation spectra from a semiconductor microcavity with a time-modulated vacuum Rabi frequency. *Phys. Rev. Lett.* **2007**, *98*, 103602.

450. Bialynicki-Birula, I.; Bialynicka-Birula, Z. 2008 Dynamical Casimir effect in oscillating media. *Phys. Rev. A* **2007**, *78*, 042109.

451. Choi, J.R.; Kim, D.; Chaabi, N.; Maamache, M.; Menouar, S. Zero-point fluctuations of quantized electromagnetic fields in time-varying linear media. *J. Korean Phys. Soc.* **2010**, *56*, 775–81.

452. Bei, X.-M.; Liu, Z.-Z. Quantum radiation in time-dependent dielectric media. *J. Phys. B At. Mol. Opt. Phys.* **2011**, *44*, 205501.

453. Ueta, T. The dynamic Casimir effect within a vibrating metal photonic crystal. *Appl. Phys. A* **2014**, *116*, 863–871.

454. Westerberg, N.; Cacciatori, S.; Belgiorno, F.; Dalla Piazza, F.; Faccio, D. Experimental quantum cosmology in time-dependent optical media. *New J. Phys.* **2014**, *16*, 075003.

455. Larré, P.-E.; Carusotto, I. Propagation of a quantum fluid of light in a cavityless nonlinear optical medium: General theory and response to quantum quenches. *Phys. Rev. A* **2015**, *92*, 043802.

456. Lotfipour, H.; Allameh, Z.; Roknizadeh, R.; Heydari, H. Two schemes for characterization and detection of the squeezed light: Dynamical Casimir effect and nonlinear materials. *J. Phys. B At. Mol. Opt. Phys.* **2016**, *49*, 065503.

457. Hasan, F.; O'Dell, D.H.J. Parametric amplification of light in a cavity with a moving dielectric membrane: Landau-Zener problem for the Maxwell field. *Phys. Rev. A* **2016**, *94*, 043823.

458. Ameri, V.; Eghbali-Arani, M.; Soltani, M. Perturbative approach to dynamical Casimir effect in an interface of dielectric mediums. *Eur. Phys. J. D* **2016**, *70*, 254.

459. Westerberg, N.; Prain, A.; Faccio, D.; Öhberg, P. Vacuum radiation and frequency-mixing in linear light-matter systems. *J. Phys. Commun.* **2019**, *3*, 065012.

460. Hizhnyakov, V. Emission of photon pairs in optical fiber - effect of zero-point fluctuations. *arXiv* **2019**, arXiv 1907.04321.

461. Dezael, F.X.; Lambrecht, A. Analogue Casimir radiation using an optical parametric oscillator. *EPL* **2010**, *89*, 14001.

462. Faccio, D.; Carusotto, I. Dynamical Casimir Effect in optically modulated cavities. *EPL* **2011**, *96*, 24006.

463. Hizhnyakov, V.; Kaasik, H.; Tehver, I. Spontaneous nonparametric down-conversion of light. *Appl. Phys. A* **2014**, *115*, 563–568.

464. Hizhnyakov, V.; Loot, A.; Azizabadi, S.C. Enhanced dynamical Casimir effect for surface and guided waves. *Appl. Phys. A* **2016**, *122*, 333.

465. Vezzoli, S.; Mussot, A.; Westerberg, N.; Kudlinski, A.; Saleh, H.D.; Prain, A.; Biancalana, F.; Lantz, E.; Faccio, D. Optical analogue of the dynamical Casimir effect in a dispersion-oscillating fibre. *Commun. Phys.* **2019**, *2*, 84.

466. Román-Ancheyta, R.; Ramos-Prieto, I.; Perez-Leija, A.; Busch, Kurt.; León-Montiel, R.deJ. Dynamical Casimir effect in stochastic systems: Photon harvesting through noise. *Phys. Rev. A* **2017**, *96*, 032501.

467. Tanaka, S.; Kanki, K. The dynamical Casimir effect in a dissipative optomechanical cavity interacting with photonic crystal. *Physics* **2020**, *2*, 34–48.

468. Dodonov, V.V. Dynamical Casimir effect in microwave cavities containing nonlinear crystals. *J. Phys. Condens. Matter* **2015**, *27*, 214009.

469. Braggio, C.; Carugno, G.; Borghesani, A.F.; Dodonov, V.V.; Pirzio, F.; Ruoso, G. Generation of microwave fields in cavities with laser-excited nonlinear media: Competition between the second- and third-order optical nonlinearities. *J. Opt.* **2018**, *20*, 095502.

470. Grove, P.G. On the detection of particle and energy fluxes in two dimensions. *Class. Quantum Grav.* **1986**, *3*, 793–800.

471. Dodonov, V.V. Photon creation and excitation of a detector in a cavity with a resonantly vibrating wall. *Phys. Lett. A* **1995**, *207*, 126–132.

472. Sarkisyan, E.M.; Petrosyan, K.G.; Oganesyan, K.B.; Hakobyan, A.A.; Saakyan, V.A.; Gevorkian, S.G.; Izmailyan, N.S.; Hu, C.K. Detection of Casimir photons with electrons. *Laser Phys.* **2008**, *18*, 621–624.

473. Braggio, C.; Bressi, G.; Carugno, G.; Della Valle, F.; Galeazzi, G.; Ruoso, G. Characterization of a low noise microwave receiver for the detection of vacuum photons. *Nucl. Instr. Meth. Phys. Res. A* **2009**, *603*, 451–455.

474. Kawakubo, T.; Yamamoto, K. Photon creation in a resonant cavity with a nonstationary plasma mirror and its detection with Rydberg atoms. *Phys. Rev. A* **2011**, *83*, 013819.

475. de Castro, A.S.M.; Cacheffo, A.; Dodonov, V.V. Influence of the field-detector coupling strength on the dynamical Casimir effect. *Phys. Rev. A* **2013**, *87*, 033809.

476. Dodonov, A.V.; Dodonov, V.V. Photon statistics in the dynamical Casimir effect modified by a harmonic oscillator detector. *Phys. Scr.* **2013**, *T153*, 014017.

477. Dodonov, A.V. Continuous intracavity monitoring of the dynamical Casimir effect. *Phys. Scr.* **2013**, *87*, 038103.

478. de Castro, A.S.M.; Dodonov, V.V. Parametric excitation of a cavity field mode coupled to a harmonic oscillator detector. *J. Phys. A Math. Theor.* **2013**, *46*, 395304.

479. de Castro, A.S.M.; Dodonov, V.V. Continuous monitoring of the dynamical Casimir effect with a damped detector. *Phys. Rev. A* **2014**, *89*, 063816.

480. Miroshnichenko, G.P.; Trifanova, E.S.; Trifanov, A.I. An indirect measurement protocol of intracavity mode quadratures dispersion in dynamical Casimir effect. *Eur. Phys. J. D* **2015**, *69*, 137.

481. Angaroni, F.; Benenti, G.; Strini, G. Reconstruction of electromagnetic field states by a probe qubit. *Eur. Phys. J. D* **2016**, *70*, 225.

482. Mendonça, J.T. Time refraction and the perturbed quantum vacuum. *J. Rus. Laser Res.* **2011**, *32*, 445–453.

483. Friis, N.; Lee, A.R.; Louko, J. Scalar, spinor, and photon fields under relativistic cavity motion. *Phys. Rev. D* **2013**, *88*, 064028.

484. Benenti, G.; Strini, G. Dynamical Casimir effect and minimal temperature in quantum thermodynamics. *Phys. Rev. A* **2015**, *91*, 020502.

485. Merlin, R. Orthogonality catastrophes in quantum electrodynamics. *Phys. Rev. A* **2017**, *95*, 023802.

486. de Melo e Souza, R.; Impens, F.; Maia Neto, P.A. Microscopic dynamical Casimir effect. *Phys. Rev. A* **2018**, *97*, 032514.

487. Ottewill, A.C.; Takagi, S. Radiation by moving mirrors in curved space-time. *Prog. Theor. Phys.* **1988**, *79*, 429–441.

488. Ruser, M.; Durrer, R. Dynamical Casimir effect for gravitons in bouncing braneworlds. *Phys. Rev. D* **2000**, *76*, 104014.

489. Céleri, L.C.; Pascoal, F.; Moussa, M.H.Y. Action of the gravitational field on the dynamical Casimir effect. *Class. Quantum Grav.* **2009**, *26*, 105014.

490. Zhitnitsky, A.R. Dynamical Casimir effect in a small compact manifold for the Maxwell vacuum. *Phys. Rev. D* **2015**, *91*, 105027.

491. Lock, M.P.E.; Fuentes, I. Dynamical Casimir effect in curved spacetime. *New J. Phys.* **2017**, *19*, 073005.

492. Brevik, I.; Milton, K.A.; Odintsov, S.D.; Osetrin, K.E. Dynamical Casimir effect and quantum cosmology. *Phys. Rev. D* **2000**, *62*, 064005.

493. Rudnicki, L.; Bialynicki-Birula, I. Dynamical Casimir effect in uniformly accelerated media. *Opt. Commun.* **2010**, *283*, 644–649.

494. Sorge, F.; Wilson, J.H. Casimir effect in free fall towards a Schwarzschild black hole. *Phys. Rev. D* **2019**, *100*, 105007.

495. Hawking, S.W. Black hole explosions? *Nature* **1974**, *248*, 30–31.

496. Hawking, S.W. Particle creation by black holes. *Commun. Math. Phys.* **1975**, *43*, 199–220.

497. Frolov, V.P. Black holes and quantum processes in them. *Sov. Phys. - Uspekhi* **1976**, *19*, 244–262.

498. Bialynicki-Birula, I.; Bialynicka-Birula, Z. Electromagnetic radiation by gravitating bodies. *Phys. Rev. A* **2008**, *77*, 052103.

499. Unruh W.G. Notes on black-hole evaporation. *Phys. Rev. D* **1976**, *14*, 870–892.

500. Crispino, L.C.B.; Higuchi, A.; Matsas, G.E.A. The Unruh effect and its applications. *Rev. Mod. Phys.* **2008**, *80*, 787–838.

501. Ben-Benjamin, J.S.; Scully, M.O.; Fulling, S.A.; Lee, D.M.; Page, D.N.; Svidzinsky, A.A.; Zubairy, M.S.; Duff, M.J.; Glauber, R.; Schleich, W.P.; et al. Unruh acceleration radiation revisited. *Int. J. Mod. Phys. A* **2019**, *34*, 1941005.

502. Sorge, F. Dynamical Casimir Effect in a kicked box. *Int. J. Mod. Phys. A* **2006**, *21*, 6173–6182.

503. Mendonça, J.T.; Brodin, G.; Marklund, M. Vacuum effects in a vibrating cavity: Time refraction, dynamical Casimir effect, and effective Unruh acceleration. *Phys. Lett. A* **2008**, *372*, 5621–5624.

504. Guerreiro, A. On the quantum space-time structure of light. *J. Plasma Phys.* **2010**, *76*, 833–843.

505. Good, M.R.R. On spin-statistics and Bogoliubov transformations in flat space-time with acceleration conditions. *Int. J. Mod. Phys. A* **2013**, *28*, 1350008.

506. Fei, Z.; Zhang, J.; Pan, R.; Qiu, T.; Quan, H.T. Quantum work distributions associated with the dynamical Casimir effect. *Phys. Rev. A* **2019**, *99*, 052508.

507. Luis, A.; Sánchez-Soto, L.L. Multimode quantum analysis of an interferometer with moving mirrors. *Phys. Rev. A* **1992**, *45*, 8228–8234.

508. Brif, C.; Mann, A. Quantum statistical properties of the radiation field in a cavity with a movable mirror. *J. Opt. B Quantum Semiclass. Opt.* **2000**, *2*, 53–61.

509. Maclay, G.J.; Forward, R.L. A Gedanken Spacecraft that Operates Using the Quantum Vacuum (Dynamic Casimir Effect). *Found. Phys.* **2004**, *34*, 477–500.

510. Dodonov, V.V. Dynamical Casimir effect in a nondegenerate cavity with losses and detuning. *Phys. Rev. A* **1998**, *58*, 4147–4152.

511. Dodonov, V.V. Time-dependent quantum damped oscillator with ‘minimal noise’: Application to the nonstationary Casimir effect in nonideal cavities. *J. Opt. B Quantum Semiclass. Opt.* **2005**, *7*, S445–S451.

512. Dodonov, V.V.; Dodonov, A.V. The nonstationary Casimir effect in a cavity with periodical time-dependent conductivity of a semiconductor mirror. *J. Phys. A Math. Gen.* **2006**, *39*, 6271–6281.

513. Dodonov, V.V. Photon distribution in the dynamical Casimir effect with an account of dissipation. *Phys. Rev. A* **2009**, *80*, 023814.

514. Lombardo, F.C.; Mazzitelli, F.D. The quantum open systems approach to the dynamical Casimir effect. *Phys. Scr.* **2010**, *82*, 038113.

515. Dodonov, V.V. Quantum damped nonstationary oscillator and Dynamical Casimir Effect. *Rev. Mex. Fis. S* **2011**, *57*, 120–127.

516. Settineri, A.; Macrì, V.; Ridolfo, A.; Di Stefano, O.; Kockum, A.F.; Nori, F.; Savasta, S. Dissipation and thermal noise in hybrid quantum systems in the ultrastrong-coupling regime. *Phys. Rev. A* **2018**, *98*, 053834.

517. Dalvit, D.A.R.; Maia Neto, P.A. Decoherence via the dynamical Casimir effect. *Phys. Rev. Lett.* **2000**, *84*, 798–801.

518. Maia Neto, P.A.; Dalvit, D.A.R. Radiation pressure as a source of decoherence. *Phys. Rev. A* **2000**, *62*, 042103.

519. Schützhold, R.; Tiersch, M. Decoherence versus dynamical Casimir effect. *J. Opt. B Quantum Semiclass. Opt.* **2005**, *7*, S120–S125.

520. Dodonov, V.V.; Andreata, M.A.; Mizrahi, S.S. Decoherence and transfer of quantum states of field modes in a one-dimensional cavity with an oscillating boundary. *J. Opt. B Quantum Semiclass. Opt.* **2005**, *7*, S468–479.

521. Céleri, L.C.; Pascoal, F.; de Ponte, M.A.; Moussa, M.H.Y. Number of particle creation and decoherence in the nonideal dynamical Casimir effect at finite temperature. *Ann. Phys.* **2009**, *324*, 2057–2073.

522. Román-Ancheyta, R.; de los Santos-Sánchez, O.; González-Gutiérrez, C. Damped Casimir radiation and photon correlation measurements. *J. Opt. Soc. Am. B* **2018**, *35*, 523–527.

523. Narozhny, N.B.; Fedotov, A.M.; Lozovik, Y.E. Dynamical Casimir and Lamb effects and entangled photon states. *Laser Phys.* **2003**, *13*, 298–304.

524. Dodonov, A.V.; Dodonov, V.V.; Mizrahi, S.S. Separability dynamics of two-mode Gaussian states in parametric conversion and amplification. *J. Phys. A Math. Gen.* **2005**, *38*, 683–696.

525. Andreata, M.A.; Dodonov, V.V. Dynamics of entanglement between field modes in a one-dimensional cavity with a vibrating boundary. *J. Opt. B Quantum Semiclass. Opt.* **2005**, *7*, S11–S20.

526. Bruschi, D.E.; Louko, J.; Faccio, D. Entanglement generation in relativistic cavity motion. *J. Phys. Conf. Ser.* **2013**, *442*, 012024.

527. Bruschi, D.E.; Louko, J.; Faccio, D.; Fuentes, I. Mode-mixing quantum gates and entanglement without particle creation in periodically accelerated cavities. *New J. Phys.* **2013**, *15*, 073052.

528. Busch, X.; Parentani, R. Dynamical Casimir effect in dissipative media: When is the final state nonseparable? *Phys. Rev. D* **2013**, *88*, 045023.

529. Busch, X.; Parentani, R.; Robertson, S. Quantum entanglement due to a modulated dynamical Casimir effect. *Phys. Rev. A* **2014**, *89*, 063606.

530. Felicetti, S.; Sanz, M.; Lamata, L.; Romero, G.; Johansson, G.; Delsing, P.; Solano, E. Dynamical Casimir effect entangles artificial atoms. *Phys. Rev. Lett.* **2014**, *113*, 093602.

531. Finazzi, S.; Carusotto, I. Entangled phonons in atomic Bose-Einstein condensates. *Phys. Rev. A* **2014**, *90*, 033607.

532. Sinha, K.; Lin, S.-Y.; Hu, B.L. Mirror-field entanglement in a microscopic model for quantum optomechanics. *Phys. Rev. A* **2015**, *92*, 023852.

533. Berman, O.L.; Kezerashvili, R.Y.; Lozovik, Y.E. Quantum entanglement for two qubits in a nonstationary cavity. *Phys. Rev. A* **2016**, *94*, 052308.

534. Amico, M.; Berman, O.L.; Kezerashvili, R.Y. Tunable quantum entanglement of three qubits in a nonstationary cavity. *Phys. Rev. A* **2017**, *96*, 032328.

535. Amico, M.; Berman, O.L.; Kezerashvili, R.Y. Dissipative quantum entanglement dynamics of two and three qubits due to the dynamical Lamb effect. *Phys. Rev. A* **2018**, *98*, 042325.

536. Agustí, A.; Solano, E.; Sabín, C. Entanglement through qubit motion and the dynamical Casimir effect. *Phys. Rev. A* **2019**, *99*, 052328.

537. Zhang, X.; Liu, H.; Wang, Z.; Zheng, T. Asymmetric quantum correlations in the dynamical Casimir effect. *Sci. Rep.* **2019**, *9*, 9552.

538. Romualdo, I.; Hackl, L.; Yokomizo, N. Entanglement production in the dynamical Casimir effect at parametric resonance. *Phys. Rev. D* **2019**, *100*, 065022.

539. Cong, W.; Tjoa, E.; Mann, R.B. Entanglement harvesting with moving mirrors. *JHEP* **2019**, *06*, 021.

540. Sabin, C.; Fuentes, I.; Johansson, G. Quantum discord in the dynamical Casimir effect. *Phys. Rev. A* **2015**, *92*, 012314.

541. Benenti, G.; D’Arrigo, A.; Siccardi, S.; Strini, G. Dynamical Casimir effect in quantum-information processing. *Phys. Rev. A* **2014**, *90*, 052313.

542. Stassi, R.; De Liberato, S.; Garziano, L.; Spagnolo, B.; Savasta, S. Quantum control and long-range quantum correlations in dynamical Casimir arrays. *Phys. Rev. A* **2015**, *92*, 013830.

543. Sabin, C.; Adesso, G. Generation of quantum steering and interferometric power in the dynamical Casimir effect. *Phys. Rev. A* **2015**, *92*, 042107.

544. Peropadre, B.; Huh, J.; Sabin, C. Dynamical Casimir effect for Gaussian boson sampling. *Sci. Rep.* **2018**, *8*, 3751.

545. Narozhny, N.B.; Fedotov, A.M.; Lozovik, Y.E. Dynamical Lamb effect versus dynamical Casimir effect. *Phys. Rev. A* **2001**, *64*, 053807.

546. Shapiro, D.S.; Zhukov, A.A.; Pogosov, W.V.; Lozovik, Y.E. Dynamical Lamb effect in a tunable superconducting qubit-cavity system. *Phys. Rev. A* **2015**, *91*, 063814.

547. Zhukov, A.A.; Shapiro, D.S.; Pogosov, W.V.; Lozovik, Y.E. Dynamical Lamb effect versus dissipation in superconducting quantum circuits. *Phys. Rev. A* **2016**, *93*, 063845.

548. Amico, M.; Berman, O.L.; Kezerashvili, R.Y. Dynamical Lamb effect in a superconducting circuit. *Phys. Rev. A* **2019**, *100*, 013841.

549. Passante, R.; Persico, F. Time-dependent Casimir-Polder forces and partially dressed states. *Phys. Lett. A* **2003**, *312*, 319–323.

550. Rizzato, L.; Passante, R.; Persico, F. Dynamical Casimir-Polder energy between an excited- and a ground-state atom. *Phys. Rev. A* **2004**, *70*, 012107.

551. Westlund, P.O.; Wennerstrom, H. Photon emission from translational energy in atomic collisions: A dynamic Casimir-Polder effect. *Phys. Rev. A* **2005**, *71*, 062106.

552. Vasile, R.; Passante, R. Dynamical Casimir-Polder force between an atom and a conducting wall. *Phys. Rev. A* **2008**, *78*, 032108.

553. Tian, T.; Zheng, T.Y.; Wang, Z.H.; Zhang, X. Dynamical Casimir-Polder force in a one-dimensional cavity with quasimodes. *Phys. Rev. A* **2010**, *82*, 013810.

554. Messina, R.; Vasile, R.; Passante, R. Dynamical Casimir-Polder force on a partially dressed atom near a conducting wall. *Phys. Rev. A* **2010**, *82*, 062501.

555. Dedkov, G.V.; Kyasov, A.A. Dynamical Casimir-Polder atom-surface interaction. *Surface Sci.* **2012**, *606*, 46–52.

556. Haakh, H.R.; Henkel, C.; Spagnolo, S.; Rizzato, L.; Passante, R. Dynamical Casimir-Polder interaction between an atom and surface plasmons. *Phys. Rev. A* **2014**, *89*, 022509.

557. Armata, F.; Vasile, R.; Barcellona, P.; Buhmann, S.Y.; Rizzato, L.; Passante, R. Dynamical Casimir-Polder force between an excited atom and a conducting wall. *Phys. Rev. A* **2016**, *94*, 042511.

558. Passante, R. Dispersion interactions between neutral atoms and the quantum electrodynamical vacuum. *Symmetry* **2018**, *10*, 735.

