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Scalar polarization window in gravitational-wave signals

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Scalar polarization modes of gravitational waves, which are often introduced in the context of the viable extension of gravity, have been actively searched for. However, couplings of the scalar modes to matter are strongly constrained by fifth-force experiments. Thus, the amplitude of scalar polarization in the observed gravitational-wave signal must be significantly suppressed compared to that of the tensor modes. Here, we discuss the implications of the experiments in the solar system on the detectability of scalar modes in gravitational waves from compact binary coalescences, taking into account the whole processes from the generation to the observation of gravitational waves. We first claim that the energy carried by the scalar modes at the generation is, at most, that of the tensor modes from the observed phase evolution of the inspiral gravitational waves. Next, we formulate general gravitational-wave propagation and point out that the energy flux hardly changes through propagation as long as the background changes slowly compared to the wavelength of the propagating waves. Finally, we show that the possible magnitude of scalar polarization modes detected by ground-based gravitational-wave telescopes is already severely constrained by the existing gravity tests in the solar system.

Subject Index E01, E02, E03

1. Introduction

Observations of gravitational waves (GWs) from compact binary coalescences [1–3] by advanced LIGO [4] and advanced Virgo [5] made it possible to test general relativity (GR) in an unprecedentedly extreme regime [6–9]. In alternative theories of gravity, additional polarization modes of GWs are often predicted [10,11]. There exist only two tensor modes (+ and × modes) in GR, whereas there exist four additional polarization modes: two vector modes (x and y modes) and two scalar modes (b and l modes), in the theories beyond GR. Detection of anomalous modes offers a clear signature of the violation of GR. The polarization modes beyond GR from compact binary coalescences have been actively searched for in the actual GW data [6–9,12–14].

Since the maximum number of polarization modes that can be simultaneously tested is determined by the number of GW detectors, the current search is limited to the case in which only one additional scalar mode is present [15]. Thus, we focus on the scalar polarization modes. In this paper, we investigate the implication of the experiments in the solar system on the scalar-polarization search in GWs from compact binary coalescences, asking the question of whether there exists a window among the extensions of GR in which a detectable magnitude of scalar modes can be generated.

First, we look at the generation process. The energy loss through the radiation of extra modes modifies the orbital evolution of inspiralling compact binaries and hence the phase evolution of the GW tensor modes [16]. In Ref. [17], constraints on the deviation from GR in the energy loss rate are placed by evaluating the phase correction in a model-independent manner. Recently, the possibility of a mixture of polarization modes, i.e. the tensor modes along with subdominant scalar modes, has been analyzed under the assumption that the waveform of the scalar mode is identical to the inspiral waveform for the tensor polarization modes [13]. In the analysis, the effect of the phase correction due to the scalar-mode radiation is more important than the existence of the scalar polarization mode itself for GW170814 [18] and GW170817 [19]. These observational constraints show that the energy flux of the scalar modes from compact binary systems is, at most, that of the tensor modes.

Second, we consider the propagation of GWs between the source and the observer. We assume that the propagating eigenmodes are described by linear combinations of the six polarization modes. Here, our key assumption is that the propagation speed of all propagating modes that we are concerned with is identical to the speed of light. This is because we focus on the possibility that additional scalar modes are detected simultaneously with the tensor modes using the GR waveform templates. Hence, we exclude most of the viable models with an extra scalar field in which a screening mechanism [20–27] is crucial to circumvent the experimental constraints. Such models introduce non-vanishing mass or a modified kinetic term for the screening to work, and thus the propagation speed of the extra mode differs from the speed of light.

Instead of considering concrete examples of modified gravity models, our discussion introduces a framework which is a little more generic and includes polarization mixing, allowing the tensor modes to be transformed to scalar polarization modes during the propagation to avoid the energy-loss constraint from the GW phase evolution. We show that the energy flux hardly changes as long as the background changes adiabatically, although the amplitude of the polarization may change during GW propagation.

Finally, we investigate the constraint on the sensitivities of GW detectors to scalar modes from the experiments in the solar system. The basic idea is that if GW detectors are sensitive to the scalar modes, we should also be able to probe the presence of scalar modes by exploring the additional gravitation force, i.e. a fifth force [28], which is mediated by scalar modes. The strength of the fifth force has been constrained by various experiments [29]. These experiments place a strong upper bound on the coupling between the scalar modes and the matter, implying that the coupling of GW detectors to the scalar modes should be significantly weaker than that to the standard tensor modes. Hence, the contribution of the scalar mode in the GW signal must be suppressed because of the weakness of this coupling. Combining all these points, we give an upper bound on the possible amplitude of the scalar polarization modes observed by GW detectors.

This paper is organized as follows. In Sect. 2, we consider the GW generation process to evaluate the constraint on the energy loss due to scalar modes from the phase evolution of the inspiral GWs. In Sect. 3, we consider the GW propagation process and formulate general propagating modes, taking into account the possibility of polarization mixing. In Sect. 4, we investigate the GW observation process. We discuss two different simplified cases as to the variation of polarization modes and the background field configurations at the position of observers. In the first case, we consider the possibility of observing independent scalar modes generated at the source or during propagation. Here, we assume that the local background is isotropic for simplicity. In the second case, we consider the possibility of observing the scalar polarization mode induced by the local anisotropic backgrounds from the tensor polarization modes. Here, we assume that there are only two propagating modes. Finally, Sect. 5 is devoted to discussions and a conclusion. We compare the derived possible amplitude of the scalar mode with the detection limit by the ground-based GW detectors to clarify the detectability of the scalar polarization modes. Throughout this paper, we use the units $c = 1$ and $m_{\rm Pl}^2/2 := (16\pi G)^{-1} = 1$. Parentheses are used for symmetrization for indices, i.e. $A_{(ij)} = (A_{ij} + A_{ji})/2$.

2. Generation process

In this section, we consider the GW generation process and evaluate the constraint on energy loss due to additional scalar modes from the observed phase evolution of the inspiral GWs. The modified energy loss from the compact binary system changes the evolution of the binary motion. It results in phase correction of the inspiral GWs. The modification of the energy flux $\dot{E}$ is often expressed as [16,17]

$$\dot{E} = \dot{E}_{\rm GR}(1 + B_q v^{2q})\,, \tag{1}$$

where $\dot{E}_{\rm GR}$ is the energy flux in GR, v is the binary orbital velocity, and B_q is the phenomenological deviation parameter at the q-th post-Newtonian (PN) order for the energy loss from the compact binary system. Through the stationary phase approximation, we can calculate the phase correction by extra energy loss due to radiation of the ℓ-th harmonic of the inspiral GW [16],

$$\delta\Psi_q^{(\ell)} = \frac{15}{64} B_q \frac{\ell}{(4-q)(5-2q)} \eta^{-2q/5} (2\pi \mathcal{M} F)^{(2q-5)/3}\,, \tag{2}$$

where $\mathcal{M}$ is the chirp mass and F is the orbital frequency that is related to the GW frequency as $f = \ell F$.

The phase corrections of inspiral GWs have been constrained in terms of the deviation parameters from the phase coefficients in GR introduced at each PN order as $(1 + \delta\hat{p}_q)p_{{\rm GR},q}$, where $p_{{\rm GR},\,q}$ is the coefficient in GR and $\delta\hat{p}_q$ is the deviation parameter at the q-th PN order [30–36]. The constraints on $\delta\hat{p}_q$ can be translated into the constraint on B_q through Eq. (2). For example, the constraint on B_{-1} corresponding to the dipole radiation and on B_0 corresponding to the quadrupole radiation were reported for two binary black hole events as [17]

$$B_0 \lesssim 10^{-1}, \quad B_{-1} \lesssim 10^{-2}. \tag{3}$$

Since vector or scalar modes are not allowed in GR, such additional polarization modes contribute to B_q, if they exist. Thus, the constraints (3) place upper bounds on the energy flux of additional polarization modes. However, since how the polarization modes are normalized is as yet unspecified, the constraint (3) does not directly place an upper bound on the amplitude of additional polarization modes, which are explored in the polarization tests. The observed GWs emitted from the inspiralling binaries did not show any signature of a large amount of energy leakage to extra modes. The possible energy carried by additional modes is, at most, that of the tensor modes as far as the observed phase evolution matches the GR prediction in terms of $\delta\hat{p}_q$.

3. Propagation process

In this section, we formulate the GW propagation process with general mixing of polarization modes. We show that the energy flux does not change in the slowly varying adiabatic background, although the amplitude of each polarization mode can change during GW propagation.

In general, any independently propagating modes in the $\hat{e}_z$ direction would be locally given by the linear combination of fundamental polarization modes,

$$h^I_{ij} = \sum_A h^I_A e^A_{ij}, \tag{4}$$

where I is the label for the canonically normalized propagating modes and $A \in \{+, \times, x, y, b, l\}$ is the label for the polarization modes. Lower-case Latin indices run over the spatial directions. In the synchronous gauge $h_{0\mu} = 0$, the polarization basis set $\{e^A_{ij}\}$ is given by [15]

$$e^+_{ij} = \hat{e}_{x,i}\hat{e}_{x,j} - \hat{e}_{y,i}\hat{e}_{y,j}\,, \quad e^\times_{ij} = \hat{e}_{x,i}\hat{e}_{y,j} + \hat{e}_{y,i}\hat{e}_{x,j}\,, \tag{5}$$

$$e^x_{ij} = \hat{e}_{x,i}\hat{e}_{z,j} + \hat{e}_{z,i}\hat{e}_{x,j}\,, \quad e^y_{ij} = \hat{e}_{y,i}\hat{e}_{z,j} + \hat{e}_{z,i}\hat{e}_{y,j}\,, \tag{6}$$

$$e^b_{ij} = \hat{e}_{x,i}\hat{e}_{x,j} + \hat{e}_{y,i}\hat{e}_{y,j}\,, \quad e^l_{ij} = \sqrt{2}\,\hat{e}_{z,i}\hat{e}_{z,j}\,. \tag{7}$$

Here, $\{\hat{e}_{x,i}, \hat{e}_{y,i}, \hat{e}_{z,i}\}$ are the mutually orthogonal unit basis vectors. We refer to $+$ (plus) and $\times$ (cross) modes as tensor modes, x (vector x) and y (vector y) modes as vector modes, and b (breathing) and l (longitudinal) modes as scalar modes. It is straightforward to generalize the propagation direction in Eqs. (5)–(7). Now, we describe the metric perturbation as

$$h_{ij} = \sum_I \phi_I h^I_{ij} + (\text{c.c.})\,, \tag{8}$$

where (c.c.) represents the complex conjugate. In the following, we assume slowly varying backgrounds and modes ϕ_I propagating at the speed of light.

When we have n real degrees of freedom, the quadratic action would be generally written as

$$S^{(2)} = \int d^4x \frac{\sqrt{-g}}{2}\left(-A^{\mu\nu}_{IJ}(x)\partial_\mu\phi^{I*}\partial_\nu\phi^J - B^\mu_{IJ}(x)\left\{(\partial_\mu\phi^{I*})\phi^J - \phi^{I*}(\partial_\mu\phi^J)\right\} - C_{IJ}(x)\phi^{I*}\phi^J\right)\,, \tag{9}$$

where we neglect the terms $\propto\phi^I\phi^J$, $\phi^{I*}\phi^{J*}$ that rapidly oscillate when we substitute $\phi^I = \hat{\phi}^I e^{-ik_\mu x^\mu}$ assuming that $\hat{\phi}^I$s are slowly varying functions. From the condition that the action should be real, we find that $A^{\mu\nu}_{IJ}$ is a symmetric spacetime tensor and C_{IJ} is a spacetime scalar, which are Hermitian matrices with respect to I, J-indices, while B^μ_{IJ} is a spacetime vector and an anti-Hermitian matrix with respect to I, J-indices. The Hermitian part $\tilde{B}^\mu_{IJ}\left\{(\partial_\mu\phi^{I*})\phi^J + \phi^{I*}(\partial_\mu\phi^J)\right\}$ can be absorbed by the C_{IJ}-term using integration by parts. We decompose $A^{\mu\nu}_{IJ}$ into the part proportional to $g^{\mu\nu}$ and the remainder as

$$A^{\mu\nu}_{IJ} = A^0_{IJ}g^{\mu\nu} + \delta A^{\mu\nu}_{IJ}\,. \tag{10}$$

Then, under the change of variables

$$\phi^I \to \phi'^I = (\Lambda^{-1})^I{}_J\phi^J\,, \tag{11}$$

the coefficients in the action transform as

$$A^{\mu\nu}_{IJ} \to A'^{\mu\nu}_{IJ} = A^{\mu\nu}_{KL}\Lambda^{*K}{}_{I}\Lambda^{L}{}_{J}\,,$$

$$B^{\mu}_{IJ} \to B'^{\mu}_{IJ} = \left[B^{\mu}_{KL} + \frac{A^{0}_{KM}\nabla^{\mu}\lambda^{M}_{L} + \delta A^{\mu\nu}_{KM}\nabla_{\nu}\lambda^{M}_{L} - (\text{h.c.})}{2}\right]\Lambda^{*K}{}_{I}\Lambda^{L}{}_{J}\,, \tag{12}$$

$$C_{IJ} \to C'_{IJ} = C_{KL}\Lambda^{*K}{}_{I}\Lambda^{L}{}_{J} + \left[B^{\mu}_{KL}\left(\nabla_{\mu}\Lambda^{K*}{}_{I}\right)\Lambda^{L}{}_{J} + (\text{h.c.})\right] + \frac{1}{2}\left[A^{\mu\nu}_{KL}\left(\nabla_{\mu}\Lambda^{K*}{}_{I}\right)\nabla_{\nu}\Lambda^{L}{}_{J} + (\text{h.c.})\right]\,, \tag{13}$$

where (h.c.) represents the Hermitian conjugate, and

$$\nabla^{\mu}\lambda^{M}_{L} := \left(\nabla^{\mu}\Lambda^{M}_{J}\right)\left(\Lambda^{-1}\right)^{J}_{L}\,. \tag{14}$$

$\nabla^{\mu}\lambda^{M}_{L}$ is a generic complex matrix when Λ^{M}_{L} is a generic complex matrix.

First, we transform the Hermitian matrix A^{0}_{IJ} to the identity matrix I_{IJ} by choosing the matrix $\Lambda^{I}{}_{J}$. Then, the equation of motion reduces to

$$\nabla^{\mu}\nabla_{\mu}\phi^{I} - \tilde{C}^{I}{}_{J}\phi^{J} = 0\,, \tag{15}$$

where

$$\tilde{C}_{IJ} = -k_{\mu}k_{\nu}\delta A^{\mu\nu}_{IJ} + 2ik_{\mu}B^{\mu}_{IJ} + C_{IJ}\,. \tag{16}$$

Here, we adopt the form satisfying the conditions $A^{0}_{IJ} = I_{IJ}$ and drop the terms that become higher order in $1/\omega L$, where ω is the angular velocity of the phase, and L is the coherence length of the spatial variation of $\tilde{C}_{IJ}$. Namely, we neglect the terms in which the derivative operators act on $\hat{\phi}^{I}$, except for the term coming from A^{0}_{IJ}. This can be justified if the background quantities $\delta A^{\mu\nu}_{IJ}$, B^{μ}_{IJ}, and C_{IJ} change slowly compared to the wavelength of the propagating waves. We note that $\tilde{C}_{IJ}$ is a Hermitian matrix.

Next, we solve the equation of motion perturbatively along the path of the GW. Let us choose the null coordinates u and v on the homogeneous and isotropic universe where u is the normalized coordinate along the path that satisfies $dx^{\mu}/du = k^{\mu}/\omega$ and v is the null coordinate pointing to the direction paired to u. We assume the form of the solution

$$\phi^{I} = \hat{\phi}^{I}e^{-ik_{\mu}x^{\mu}}\,, \tag{17}$$

where $\hat{\phi}^{I}$s are slowly varying functions given by

$$\hat{\phi}^{I} = \tilde{U}^{I}{}_{J}\hat{\phi}^{J}_{0}\,. \tag{18}$$

Here, $\hat{\phi}^{J}_{0}$ is an initial vector at the source position and $\tilde{U}^{I}{}_{J}$ is a certain matrix on the field space. We expand the transformation matrix $\tilde{U}_{IJ}$ as

$$\tilde{U}^{I}{}_{J} = U^{I}{}_{J} + U^{I}{}_{K}V^{K}{}_{J}\,. \tag{19}$$

By assuming that $\tilde{C}^{I}{}_{J}$ is small, we obtain the equation of motion for the leading order correction $U^{I}{}_{J}$ as

$$2i\omega\partial_{u}U = \tilde{C}U\,, \tag{20}$$

where we adopt the matrix notation, for brevity. The solution to Eq. (20) is formally given by

$$U := \text{P Exp}\left[\int_{u_0}^{u}\frac{\tilde{C}}{2i\omega}du'\right]. \tag{21}$$

Here, "P" represents the path ordered product and u_0 is the initial value of u corresponding to the source position. If we neglect $V^{K}{}_{J}$ in the second term in Eq. (19), $\tilde{U}^{I}{}_{J}(u)$ becomes a path ordered product of exponentiated anti-Hermitian matrices and hence a unitary matrix. Therefore, the norm of the vector $\hat{\phi}^I$ is conserved at this level of approximation.

Let us evaluate the phase shift caused by $U^I{}_J$, i.e. $\varphi := \left|\mathrm{Tr}\left[\mathrm{Log}(U^I{}_J)\right]\right|$. Here, we assume that the phase shift is given by the summation of the contributions from mutually independent domains whose number is of $O(d_L/L)$. Then, the magnitude of the phase shift can be estimated as $\varphi \sim \left|\sqrt{Ld_L}\tilde{C}^I{}_I/\omega\right|$, where d_L is the comoving distance from the source to the detector. The phase shifts that depend on the frequency as $\propto\omega$ and ω^{-1} modify the GW arrival time and the GW waveform, respectively. In the same manner as we give a constraint on the graviton mass using the GW observational data [9], the amplitude of these phase shifts is also observationally constrained as $\varphi \lesssim 1$. If this condition is largely violated, the detectability of extra polarization modes simultaneously based on the templates in GR would be significantly reduced. Since $\hat{\phi}_0^J$ does not have a specific form in general, the condition $\varphi \lesssim 1$ implies that the unitary matrix $U^I{}_J$ should be sufficiently close to $I^I{}_J$ even after the propagation over cosmological distances. Namely, all the rotation angles of the unitary matrix $U^I{}_J$ are less than $O(1)$. The only exceptions are the contributions from $\delta A^{\mu\nu}_{IJ}$ in case $\delta A^{\mu\nu}_{IJ} \propto I_{IJ}$ and B^{μ}_{IJ}, which are proportional to ω^2 and ω in $\tilde{C}_{IJ}$, respectively. The arrival times are shifted in the same way for all modes by the part of $\delta A^{\mu\nu}_{IJ}$ that is $\propto I_{IJ}$, which describes nothing but the standard gravitational lensing effect. The contribution from B^{μ}_{IJ} modifies the phase in a frequency-independent manner. Therefore, even if φ originating from B^{μ}_{IJ} is large, we can still detect the signal using GR templates.

Next, we consider the next to leading order correction, $V^I{}_J$. Substituting the ansatz (19) into the equation of motion, the equation to solve is reduced to

$$\nabla^\mu \nabla_\mu U + 2i\omega U(\partial_u V) = 0\,, \tag{22}$$

where we adopt the matrix notation, for brevity. $\nabla^\mu\nabla_\mu$ is decomposed into $\partial_u\partial_v$ and ${}^{(2)}\triangle$, which represents the Laplacian in the two spatial dimensions perpendicular to the propagation direction. We count the magnitude of each term assuming that $\partial_u \sim 1/d_L$ and the differentiation in the other directions $\sim 1/L$, when they act on U or V. Assuming $L/d_L \ll 1$, we find that the leading order of V is determined by solving

$$2i\omega(\partial_u V) \simeq -U^{-1\,(2)}\triangle U\,. \tag{23}$$

The equation can be solved formally as

$$\begin{aligned}
V \simeq{}& -\frac{1}{2i\omega}\Bigg[\int_{u_0}^{u} du' \int_{u_0}^{u'} du'' U^{-1}(u')U(u')U^{-1}(u'')\frac{{}^{(2)}\triangle\tilde{C}(u'')}{2i\omega}U(u'')U^{-1}(u_0) \\
&+ 2\int_{u_0}^{u} du' \int_{u_0}^{u'} du'' \int_{u_0}^{u''} du''' U^{-1}(u')U(u')U^{-1}(u'')\frac{{}^{(2)}\nabla^i\tilde{C}(u'')}{2i\omega}U(u'') \\
&\times U^{-1}(u''')\frac{{}^{(2)}\nabla_i\tilde{C}(u''')}{2i\omega}U(u''')U^{-1}(u_0)\Bigg] \\
\simeq{}& \frac{1}{4\omega^2}\left[\int_{u_0}^{u} du'(u-u')^{(2)}\triangle\mathcal{C}(u') - \frac{i}{\omega}\int_{u_0}^{u} du'(u-u')\int_{u_0}^{u'} du''^{(2)}\nabla^\mu\mathcal{C}(u')^{(2)}\nabla_\mu\mathcal{C}(u'')\right],
\end{aligned} \tag{24}$$

where we introduced $\mathcal{C} := U^{-1}\tilde{C}U$, which is analogous with the interaction picture field in the quantum field theory. The magnitudes of the two terms in the last line of Eq. (24) would be estimated as $O(d_L\varphi/\omega L^2)$ and $O(d_L\varphi^2/\omega L^2)$, respectively.

First, we consider the case in which $\varphi \lesssim 1$ is satisfied. In this case, we find that significant amplitude modification or phase shift due to V can occur only when $d_L/\omega L^2 \gg 1$. Namely, V can be neglected as long as

$$L \gtrsim 0.3\text{pc} \left(\frac{\omega/2\pi}{30\text{Hz}}\right)^{-1/2} \left(\frac{d_L}{3\text{Gpc}}\right)^{1/2} . \tag{25}$$

If the contribution of V can be neglected, the effect of $\mathcal{C}$ can be absorbed by the unitary rotation given by Eq. (18). Hence, we can safely assume the canonically normalized form of the quadratic action,

$$S = \int d^4x \frac{\sqrt{-g}}{2} \sum_I \left(-g^{\mu\nu}\partial_\mu \phi^{I*} \partial_\nu \phi^I\right) , \tag{26}$$

along the propagation path.

When the condition (25) is violated, we need to consider the wave optics to correctly take into account the effect of V. The effective size of extension of the observed waves during propagation ℓ would be estimated by the condition that the difference of the path length becomes $O(1)$. This means that $\ell \sim \sqrt{d_L/\omega}$. Therefore, when $d_L/\omega L^2 \gg 1$, L becomes much smaller than ℓ and the background is rapidly changing compared to the effective size of the beam width. In such situations, smearing over the effective size ℓ will eliminate the components in $\tilde{C}$ shorter than the length scale ℓ. Hence, even in this case any significant modification due to V would be unexpected.

When $\varphi \gg 1$, which can be realized in the case of $\delta A^{\mu\nu}_{IJ} \propto I_{IJ}$-term dominance or B-term dominance, amplification can happen. The amplitude change is related to the non-unitary nature of $\tilde{U}$. In the present perturbative analysis, this amplification is caused by the Hermitian component of V, which is identified as

$$\frac{V + V^\dagger}{2} \simeq \frac{1}{4\omega^2}\left(\int_{u_0}^{u}(u-u')du' {}^{(2)}\triangle\mathcal{C}(u') \right. \\ \left. - \frac{i}{\omega}\int_{u_0}^{u}(u-u')du' \int_{u_0}^{u'} du'' \left[{}^{(2)}\nabla^\mu \mathcal{C}(u'), {}^{(2)}\nabla_\mu \mathcal{C}(u'')\right]\right) . \tag{27}$$

The amplification due to the $\delta A^{\mu\nu}_{IJ} \propto I_{IJ}$-term is the result of the standard gravitational lensing amplifying all modes in the same way. On the other hand, since $\mathcal{C} \propto \omega$ in the case of B-term dominance, the magnification factor due to the B-term is frequency dependent, i.e. $\propto 1/\omega$. At the same time, the frequency-dependent phase shift also occurs owing to the contribution from the anti-commutator,

$$\frac{V - V^\dagger}{2} \simeq -\frac{i}{4\omega^3}\int_{u_0}^{u}(u-u')du' \int_{u_0}^{u'} du'' \left\{{}^{(2)}\nabla^\mu \mathcal{C}(u'), {}^{(2)}\nabla_\mu \mathcal{C}(u'')\right\} , \tag{28}$$

which leads to the modification of the waveform. Hence, the discussion above for the phase shift caused by $U^I{}_J$ applies and the B-term should be small enough to be consistent with the observed GW phase. When the $\delta A^{\mu\nu}_{IJ} \propto I_{IJ}$-term is large but the B-term is also present, we may need to worry about the cross term between these two. However, while the anti-commutator of these two can lead to the frequency-independent phase shift, the commutator related to the amplification vanishes. Thus, the cross term does not contribute to the amplification. Therefore, it seems difficult to have a large magnification of specific modes without changing the waveform.

To conclude, although the amplitude of h_{ij} may change during propagation, as long as the change of the background is adiabatic, the energy flux carried by the propagating waves cannot be altered without waveform or arrival time deformation, except for the effect of the usual redshift and gravitational lensing. Here, adiabatic means that the temporal and spatial scales of the gradient of the background fields are much longer than the wavelength of GWs. We should note that there is no change in the energy flux other than the effect due to the redshift, even for the case of well-known examples of amplitude variation during propagation, such as the Chern-Simons gravity [37] and a scalar field with a non-canonical kinetic term. This magnification or de-magnification is due to the difference between the forms of h^I_{ij} at the source and at the observer.

The above argument does not apply if the background is not smooth enough compared with the wavelength of the propagating modes, but such a situation seems to be quite unlikely under the condition that the propagation speed is very close to the speed of light. As one exceptional possibility, a rapid change of the background might be realized by the presence of a domain wall network in the universe [38], which we do not pursue here. Before closing this section, we would like to stress that we have established the conservation of the flux during the propagation, or equivalently the conservation of the amplitude of the canonically normalized field ϕ_I. We only use this fact in the succeeding discussion.

4. Observation process

In this section, we discuss the detectability of the scalar modes under two different settings regarding the number of polarization modes and the presence of an anisotropic background field. First, we consider the case in which an additional scalar mode propagates from the source to the detector along with the tensor modes. Here, we assume that the local background at the position of the observer is isotropic. Second, we consider the case in which we observe the apparent scalar mode induced by the anisotropies of the local background field, although there are only two propagating modes emitted from the GW source. In principle, we can consider the case in which the two mechanisms are simultaneously at work. However, since there does not seem to be any synergistic effect from the coexistence of the two mechanisms, the constraints on the magnitude of the scalar polarization would not be altered.

Before proceeding to the specific setups, we consider how polarization modes appear in gravitational-wave signals of interferometric detectors, which is common to both cases below. In order to maintain the weak equivalence principle, we assume the universal coupling between the metric perturbation and the effective matter stress energy tensor as

$$S_{\rm int} = \int d^4x \frac{\sqrt{-g}}{2} h_{\mu\nu} T^{\mu\nu} \,. \tag{29}$$

Under the coupling, the motion of a test mass is determined by the geodesic equation. Thus, the detector signal is given by

$$h(t) = d^{jk} h_{jk} \,, \tag{30}$$

when the wavelength of GWs is much longer than the arm length of the interferometric detector [11,39,40]. Here, d^{jk} is the detector tensor for an interferometric detector given by

$$d^{jk} := \frac{1}{2}(\hat{u}^j \hat{u}^k - \hat{v}^j \hat{v}^k) \,, \tag{31}$$

where $\hat{u}$ and $\hat{v}$ are unit vectors along the arms of the detector. In order to separate the observation process from the propagation process, we write the metric perturbation as

$$h_{ij} = \sum_I \psi_I h^I_{ij} + ({\rm c.c.}) \,, \tag{32}$$

instead of Eq. (8). In other words, we express the propagating modes during propagation as ϕ_I whereas those in the observation process or in the solar system are expressed as ψ_I. The new variables ψ_I take the same standard form of the action (26) as ϕ^I for small perturbation. After substituting Eq. (32) with Eq. (4) into Eq. (30), we obtain the gravitational-wave signal as

$$h(t) = \sum_A h_A F^A \,, \tag{33}$$

with the polarization modes given by

$$h_A = \sum_I \psi_I h^I{}_A \,. \tag{34}$$

Here, F^A are the antenna pattern functions defined by

$$F^A := d^{ij} e^A_{ij} \,, \tag{35}$$

which represent the angular dependence of the detector sensitivity to the polarization A. Note that since d^{jk} is traceless, $F^b = -F^l/\sqrt{2}$ holds which means that the responses of the detector to two scalar modes, b and l, are degenerate.

4.1. *Detectability of independently propagating scalar modes*

Here, we consider the possibility of detecting an additional scalar polarization mode propagating independently of the tensor modes at the location of observers. We assume that there are no background anisotropies in the solar system in the gravitational sector. In addition, we assume that there is only one additional massless propagating mode with scalar polarization components. Thus, we would be able to safely assume that h_{ij} can be expanded as

$$h_{ij} = \sum_{I=1,2,3} \psi_I h^I_{ij} + (\text{c.c.}) \,, \tag{36}$$

where each polarization basis is given by

$$\begin{aligned} h^1_{ij} &= e^+_{ij} \,, \qquad h^2_{ij} = e^\times_{ij} \,, \\ h^3_{ij} &= \zeta_b e^b_{ij} + \zeta_l e^l_{ij} \,, \end{aligned} \tag{37}$$

where ζ_b and ζ_l are some complex constants, which can depend on $k = |\vec{k}|$. If we have the other scalar degrees of freedom, we can generalize the model only by extending h^3_{ij}, ζ_b, and ζ_l, to h^I_{ij}, ζ^I_b, and ζ^I_l, respectively. The extension simply gives additional deviations from GR in the metric perturbation to make the model more inconsistent with the observations. There might be other massless propagating degrees of freedom, but we neglect their possible existence here. Including extra degrees of freedom will not change the following discussion basically.

In the universal coupling of Eq. (29), we assume the existence of the effective stress energy tensor that satisfies the conservation law with respect to the background Minkowski metric $T^{\mu\nu}_{\ \ ,\nu} = 0$, which means that $T^{\mu\nu}$ includes the contribution from the second-order gravitational perturbation. Then, we can calculate the metric perturbation in the synchronous gauge induced by a given stress energy tensor $T_{\mu\nu}$ as

$$\tilde{h}_{ij}(k) = \tilde{G}_{ijkl} \tilde{T}^{kl}(k) \,, \tag{38}$$

where the Green's function $\tilde{G}_{ijkl}$ is given by

$$\tilde{G}_{ijkl} = \frac{1}{\omega^2 - |\boldsymbol{k}|^2} \sum_I \left(h^{I*}_{ij} h^I_{kl} + (\text{c.c.}) \right) \,, \tag{39}$$

and the quantities associated with ~ represent Fourier components, defined by

$$\tilde{h}_{\mu\nu}(k) = \frac{1}{(2\pi)^4} \int d^4x\, e^{-ik_\mu x^\mu} h_{\mu\nu}(x)\,. \tag{40}$$

Thus, the metric perturbation would be given by

$$h_{ij}(x) = \int d^4k\, e^{ik_\mu x^\mu} \tilde{G}_{ijkl}(k) \tilde{T}^{kl}(k)$$
$$= \sum_I \int d^4k\, e^{ik_\mu x^\mu} \frac{h^{I*}_{ij} h^I_{kl} + (\text{c.c.})}{\omega^2 - |\boldsymbol{k}|^2} \tilde{T}^{kl}(k)\,. \tag{41}$$

To respect the rotational symmetry, $\tilde{G}_{ijkl}$ should be composed of the invariant tensors δ_{ij} and k^i. Imposing the symmetries $\tilde{G}_{ijkl} = \tilde{G}_{jikl},\ \tilde{G}_{ijkl} = \tilde{G}_{ijlk},\ \tilde{G}_{ijkl} = \tilde{G}_{klij}$, the form of $\tilde{G}_{ijkl}$ is restricted to

$$\tilde{G}_{ijkl}(k) = \frac{1}{(\omega^2 - |\boldsymbol{k}|^2)} \Big[a_1 \delta_{ij}\delta_{kl} + a_2 \delta_{i(k}\delta_{l)j} + a_3 \hat{k}_{(i}\delta_{j)(k}\hat{k}_{l)}$$
$$+ a_4 \left(\hat{k}_i \hat{k}_j \delta_{kl} + \delta_{ij} \hat{k}_k \hat{k}_l \right) + a_5 \hat{k}_i \hat{k}_j \hat{k}_k \hat{k}_l \Big], \tag{42}$$

where $\hat{k}_i = k_i/|\boldsymbol{k}|$ is the unit vector pointing at the wave propagation direction.

The Green's function obtained by summing over the + and × modes, $\tilde{G}^{(T)}_{ijkl} \propto e^+_{ij}e^+_{kl} + e^\times_{ij}e^\times_{kl}$, should satisfy $k^i \tilde{G}^{(T)}_{ijkl} = 0$ and $\delta^{ij}\tilde{G}^{(T)}_{ijkl} = 0$. Combined with the condition $\delta^{ik}\delta^{jl}\tilde{G}^T_{ijkl} = 2/(\omega^2 - |\boldsymbol{k}|^2)$, these conditions completely determine the coefficients as

$$\left(a^T_1, a^T_2, a^T_3, a^T_4, a^T_5\right) = (-1, 2, -4, 1, 1)\,. \tag{43}$$

The contribution to the Green's function due to the propagating mode with scalar modes h^3_{ij} would be given by[1]

$$\left(a^S_1, a^S_2, a^S_3, a^S_4, a^S_5\right) = \left(|\zeta_b|^2, 0, 0, \Re\left[\left(\sqrt{2}\zeta_l - \zeta_b\right)\zeta^*_b \right], \left|\sqrt{2}\zeta_l - \zeta_b\right|^2 \right)\,. \tag{45}$$

In the case with multiple massless degrees of freedom, the above formula can be simply extended to the sum of contributions from all modes.

Now, we shall consider the constraint on the polarization modes from the tests of gravity in the solar system. For this purpose, we observe how the linear perturbation around a static spherically symmetric object is reproduced. The above Green's function generates only *ij*-components of the metric perturbation, which are given in a different gauge from the standard Newtonian or PN gauge. Hence, a gauge transformation to a more familiar form of the metric is also addressed in the following discussion.

We start with the a_5 term, which turns out to give the dominant component of the Newtonian potential. Using the conservation law, we can replace the spatial component of the stress energy tensor T^{ij} with T_{00} as $k_i k_j \tilde{T}^{ij} = \omega^2 \tilde{T}^{00}$. Namely, we can calculate the contribution from the a_5

[1] In the same manner, we can also calculate the Green's function due to the propagating mode with vector modes such as $h^I_{ij} = \zeta_x e^x_{ij} + \zeta_y e^y_{ij}$,

$$\left(a^V_1, a^V_2, a^V_3, a^V_4, a^V_5\right) = \left(0, 0, 2\left(|\zeta_x|^2 + |\zeta_y|^2\right), 0, -2\left(|\zeta_x|^2 + |\zeta_y|^2\right)\right)\,. \tag{44}$$

term as

$$
\begin{aligned}
a_5 &\int d\omega\, d^3k \frac{k_i k_j k_l k_m}{|\boldsymbol{k}|^4(\omega^2 - |\boldsymbol{k}|^2)} \tilde{T}^{lm} e^{ik_\mu x^\mu} \\
&= -a_5 \partial_i \partial_j \int_{-\infty}^{\infty} d\omega \int d^3k \frac{\omega^2 \tilde{T}^{00}}{|\boldsymbol{k}|^4(\omega^2 - |\boldsymbol{k}|^2)} e^{-i\omega t + i\boldsymbol{k}\cdot\boldsymbol{x}} \\
&= -\frac{a_5}{2} \partial_i \partial_j \frac{(2\pi)^2}{r} \int_{-\infty}^{\infty} d\omega\, \omega^{-2} e^{-i\omega(t-r)} \tilde{T}_{00} \\
&= a_5 \partial_i \partial_j \frac{M t^2}{8\pi r} ,
\end{aligned} \tag{46}
$$

where in the third equality we replaced ω^{-1} with $-i\int dt$ and substituted $\tilde{T}^{00} = M\delta(\omega)/(2\pi)^3$, which is obtained from $T^{00} = M\delta^3(\boldsymbol{x})$.

The secular growth proportional to t^2, which originates from the factor ω^{-2} in the third line, can be eliminated by a gauge transformation. Under the infinitesimal coordinate transformation $x^\mu \to x^\mu + \xi^\mu$, the metric perturbation transforms as $h_{\mu\nu} \to h_{\mu\nu} - 2\xi_{(\mu,\,\nu)}$. By choosing ξ_μ as

$$
\xi_0 = -a_5 \frac{tM}{8\pi r} , \qquad \xi_j = a_5 \frac{t^2 M}{16\pi} \partial_j \frac{1}{r} , \tag{47}
$$

the contribution of the a_5 term to the metric perturbation becomes

$$
h_{00}^{(5)} = -\frac{a_5 M}{4\pi r} , \qquad h_{0i}^{(5)} = 0 , \quad h_{ij}^{(5)} = 0 . \tag{48}
$$

In the same way, we can compute the contribution of the a_4 term as

$$
a_4 \int d\omega\, d^3k \frac{k_i k_j \delta_{lm} + \delta_{ij} k_l k_m}{|\boldsymbol{k}|^2(\omega^2 - |\boldsymbol{k}|^2)} \tilde{T}^{lm} e^{ik_\mu x^\mu} = -a_4 \partial_i \partial_j \frac{\delta_{kl} q^{kl} t^2}{4\pi r} - a_4 \delta_{ij} \frac{M}{2\pi r} , \tag{49}
$$

where we define $q^{ij} := \int d^3x\, T^{ij}$. In the following, we neglect the contributions depending on q^{ij} because q^{ij} can be rewritten as

$$
q^{ij} = \int d^3x \frac{\partial x^i}{\partial x^k} \frac{\partial x^j}{\partial x^l} T^{kl} = \int d^3x\, x^i x^j \partial_t^2 T^{00} + (\text{boundary terms}) . \tag{50}
$$

Hence, only the boundary terms remain for a stationary source. The boundary terms are quadratic in the metric perturbation and hence suppressed by the smallness of the Newtonian potential. Therefore, we have $q^{ij} = O(\Phi)M$. Determination of the perturbation at this order is beyond the scope of the linear theory discussed here. Since the contributions of the a_1, a_2, and a_3 terms are proportional to q_{ij} or vanish, we do not discuss these contributions. As a result, we obtain

$$
h_{00} = -\frac{a_5 M}{4\pi r} , \qquad h_{0i} = 0 , \qquad h_{ij} = -\frac{a_4 M}{4\pi r} \delta_{ij} . \tag{51}
$$

Substituting the coefficients of the contributions from the tensor modes (43), we can recover the standard result in GR,

$$
h_{00}^{(T)} = \frac{M}{8\pi r} , \qquad h_{0i}^{(T)} = 0 , \qquad h_{ij}^{(T)} = \frac{M}{8\pi r} \delta_{ij} . \tag{52}
$$

The deviation from GR in the relative magnitude between the scalar potentials $h_{00} = 2\Phi$ and $h_{ij} = 2\Psi\delta_{ij}$ is denoted by γ [39], which is defined by

$$
\frac{\gamma - 1}{2} := \frac{\Psi}{\Phi} - 1 , \tag{53}
$$

in the context of the parameterized post-Newtonian (PPN) formalism. $\gamma = 1$ corresponds to the GR case. In the present case we have

$$\begin{aligned}\frac{\gamma-1}{2} &= \frac{a_4}{a_5} - 1 \\ &\approx -2\Re\left[\left(\zeta_b - \frac{1}{\sqrt{2}}\zeta_l\right)\left(\zeta_b^* - \sqrt{2}\zeta_l^*\right)\right],\end{aligned} \tag{54}$$

where a_4 and a_5 are given as the sum of Eqs. (43) and (45), and we assume that the ζ_b and ζ_l are small and the second equality is approximated up to the second order with respect to them. Since the additional polarization modes couple to the matter, the new force mediated by the additional modes, which is often called the fifth force, would be present. The strength of the fifth force is constrained by various experiments [29]. The fifth force mediated by the massless field is most strongly constrained by the measurement of the Shapiro time delay by the Cassini satellites [41]. The constraint is given by

$$\frac{\gamma-1}{2} < 2.1 \times 10^{-5}. \tag{55}$$

After translating the bound (55) to the bound on the coupling parameter from Eq. (54) and combining it with the bounds on the energy flux (3), we can constrain the detector amplitude of the signal of additional polarization modes, as we shall see below.

Since ψ_I takes the same standard form as given for ϕ_I in Eq. (26), ψ_I must be related to ϕ_I by a transformation

$$\psi_I = \mathcal{U}^J{}_I \phi_J\,, \tag{56}$$

with a unitary matrix $\mathcal{U}$. Suppose that ϕ_1 and ϕ_2 are the results of propagation of the standard two tensor modes evaluated at the position of the observer. An additional mode ϕ_3 may or may not be excited at the source and propagate in the same way as the other two modes.

When additional propagating mode ϕ_3 is excited, the amplitude $|\phi_3|$ relative to the two tensor modes $|\phi_1|$ and $|\phi_2|$ is directly given by the fractional deviation of the energy loss in Eq. (1) as

$$\frac{|\phi_3|^2}{|\phi_1|^2 + |\phi_2|^2} \lesssim B_q v^{2q}\,. \tag{57}$$

On the other hand, as we do not have a generic argument that constrains the form of U^J_I, in principle, U^J_I can be an arbitrary unitary matrix. Hence, Eq. (57) does not constrain the relative magnitude of the extra mode when expanded in terms of the observer's basis, $|\psi_3|/\sqrt{|\psi_1|^2 + |\psi_2|^2}$. However, it is impossible to choose the unitary matrix U such that $|\psi_3|$ becomes much larger than $\sqrt{|\psi_1|^2 + |\psi_2|^2}$, when the power is almost equally distributed among two or three modes. If we have two independent scalar modes, two tensor amplitudes might be completely transferred to the two scalar modes. Even in that case, we should notice that the interchange between tensor modes and scalar modes can occur only under the influence of an anisotropic background. As long as we consider a homogeneous isotropic background, there can be no mixing between different helicity modes. Therefore, the maximum amplitude of the energy flux of the scalar mode that we can expect would be the one estimated under the assumption of equipartition among all propagating modes. Thus, we conclude that the amplitude of the scalar polarization mode $|\psi_3|$ is, at most, of the same order as the tensor modes $\sim \sqrt{|\psi_1|^2 + |\psi_2|^2}$, i.e.

$$\frac{|\psi_3|^2}{|\psi_1|^2 + |\psi_2|^2} \lesssim 1. \tag{58}$$

Then, the amplitude of scalar polarization measured by GW detectors would be suppressed by the factor $\zeta_b - \sqrt{2}\zeta_l$ from Eq. (37) compared with the tensor polarization because the antenna pattern functions of the interferometric detectors of the breathing mode and the longitudinal mode are identical except for a factor of $-\sqrt{2}$ as mentioned below Eq. (35). This factor is constrained by the constraint on the PPN parameter γ, Eq. (55), through Eq. (54) unless the parameters ζ_b and ζ_l are tuned. Therefore, the upper bound on the detector amplitude of the scalar polarization relative to the tensor polarization can be estimated as $O(10^{-2.5})$ from the constraint on the coupling factor with the constraint on the energy flux of Eq. (58).

On the other hand, this constraint could be evaded if the parameters are tuned so as to vanish the right-hand side of Eq. (54). This is possible if $|\zeta_l|$ is in the range between $\sqrt{2}|\zeta_b|$ and $|\zeta_b|/\sqrt{2}$ and its phase is chosen appropriately. The possible choice is $\zeta_b = \sqrt{2}\zeta_l$ or $\zeta_l = \sqrt{2}\zeta_b$. Then, the value of $\gamma - 1$ would be dominated by the contribution coming from q^{ij}, which becomes $O(\zeta_b^2\Phi)$, and can be sufficiently suppressed even if $|\zeta_b|$ and $|\zeta_l|$ are $O(1)$. In the case with $\zeta_b = \sqrt{2}\zeta_l$, the sensitivity of the GW detectors to the scalar polarization vanishes. Hence, what we are interested in here would be the cases close to the opposite boundary with $\zeta_l = \sqrt{2}\zeta_b$. The fine-tuning condition becomes more complicated in the case of multiple massless scalar degrees of freedom.

Even in such fine-tuned models, in general, the second-order correction to the Newtonian potential Φ will have $O(\zeta_b^2\Phi^2)$ deviation from the prediction of GR. The coefficient of the deviation proportional to Φ^2 is often denoted by $\beta - 1$ [39]. In terms of β, h_{00} to the second order is expressed as $2\Phi - 2\beta\Phi^2$, and $\beta = 1$ corresponds to the GR case. This β parameter is slightly less tightly constrained compared with $\gamma - 1$, from the measurement of the perihelion advance as $|\beta - 1| < 8 \times 10^{-5}$ [39]. The constraint on the detector amplitude is relaxed to the one set by the observation of $\beta - 1$, which is $O(10^{-2})$, although we do not have any concrete model that can realize such a tuning.

4.2. *Detectability of apparent scalar mode induced by background anisotropies*

In the previous subsection, we gave an upper bound on the detector amplitude of the scalar polarization when the observational environment in the solar system can be approximated by a homogeneous and isotropic background. In this subsection, we consider the possibility of detecting apparent scalar polarization induced by hypothetical background anisotropies at the stage of the observation. Here, we assume the background anisotropies to be in the solar system, but we do not assume any additional polarization modes propagating from the GW source.

Hence, we start with only two tensor propagating polarization modes at hand. We consider that the polarization basis tensors of the two modes would be perturbatively modified by the anisotropic background from the original ones, $e_{ij}^{(+)}$ and $e_{ij}^{(\times)}$. The unperturbed $+$ and $\times$ polarization basis tensors have an ambiguity corresponding to the degree of freedom for the rotation around the axis pointing at k^i. The final result must be independent of the way of fixing this arbitrariness. In order to maintain this basic symmetry, the modified polarization tensors should be generated by applying the same operator to the original polarization tensors, $e_{ij}^{(+)}$ and $e_{ij}^{(\times)}$. Note that the Green's function constructed from the modified polarization tensors violates the rotation symmetry reflecting the background anisotropies.

To describe the homogeneous but anisotropic background, we introduce a background homogeneous vector W^i, or tensor X^{ij}. We restrict our attention to the perturbation linear in W^i or X^{ij}. Then, we can use $\hat{k}^i$, k, and δ_{ij} as well as W^i and X^{ij} to generate the modified polarization tensors. Here, we claim that we need to associate k^0, k, or k^2 to W^i and X^{ij}, according to the number of indices contracted with $\hat{k}^i$. When the index of $\hat{k}^i$ is not contracted with W^i or X^{ij}, k^i in $\hat{k}^i$ can be converted to ω in the end after the manipulations described in the preceding subsection. Then, $\hat{k}^i$ does not produce any extra negative power of ω. However, once $\hat{k}^i$ is contracted with W^i or X^{ij}, k^i in $\hat{k}^i$ is replaced with the spatial derivative and the factor $1/k$, which produces a negative power of ω. The extra negative power of ω results in the secular growth of metric perturbation in time that is not eliminated by the gauge transformation. Such an extra negative power of ω can be avoided simply by associating an appropriate power of k, whose order is determined by the number of indices in W^i and X^{ij} contracted with $\hat{k}^i$. We can multiply higher powers of k more than the minimal requirement to avoid the secular growth. However, extra k^2 would be transformed to the Laplacian operator, giving a vanishing contribution when it acts on $1/r$ or its derivatives. Thus, we find that the terms that can appear at the linear order in W^i and X^{ij} generated from the tensor modes are given by

$$\text{(a) } kW^k e^{(T)}_{k(i}\hat{k}_{j)}\,, \quad kW^k\hat{k}_k e^{(T)}_{ij}\,, \quad \text{(b) } k^2X^{kl}e^{(T)}_{kl}\hat{k}_i\hat{k}_j\,, \quad k^2X^{kl}\hat{k}_k\hat{k}_l e^{(T)}_{ij}\,, \quad k^2\hat{k}_l X^{kl}e^{(T)}_{k(i}\hat{k}_{j)}\,,$$
$$\text{(c) } k^2X^{kl}e^{(T)}_{kl}\delta_{ij}\,, \quad \text{(d) } kX^k_{\ (i}e^{(T)}_{j)k}\,, \quad \text{(e) } X^k_{\ k}e^{(T)}_{ij} \tag{59}$$

where $e^{(T)}_{ij}$ represents the pair of basis tensors, $e^{(+)}_{ij}$ and $e^{(\times)}_{ij}$.

The contribution of each term can be identified by noticing that the products of these two original basis tensors after the summation over + and × modes are given by the form in the square brackets in Eq. (42) with the coefficients from Eq. (43). Thus, we find that the (a) terms contribute to the Newtonian potential in the form of

$$\propto W^k\partial_k\frac{M}{r}\,. \tag{60}$$

The correction to the other components of the metric perturbation is higher order in the slow motion dynamics. The contribution of the (a) terms can be understood as an anomalous dipole moment of the gravitational source. The dipole perturbation in the direction specified by the constant vector W^i, caused by the (a) term, can be interpreted as a systematic shift of the position of the center of mass. Here, we do not discuss this term in details because it does not produce any contamination to the scalar polarization detected by GW detectors. In fact, one can easily find that

$$e^{(b)ij}\times kW^k e^{(T)}_{k(i}\hat{k}_{j)} = 0\,, \qquad e^{(l)ij}\times kW^k e^{(T)}_{k(i}\hat{k}_{j)} = 0\,, \tag{61}$$

and

$$e^{(b)ij}\times kW^k\hat{k}_k e^{(T)}_{ij} = 0\,, \qquad e^{(l)ij}\times kW^k\hat{k}_k e^{(T)}_{ij} = 0\,. \tag{62}$$

The (b) terms contribute to the Newtonian potential in the form of

$$\propto X^{kl}\partial_k\partial_l\frac{M}{r}\,. \tag{63}$$

This contribution can be interpreted as an anomalous quadrupole moment of the gravitational source. Since this effect should cause correction to the orbital motion of planets and satellites, it is observationally constrained by experiments in the solar system. We estimate the bound on the magnitude of the extra contribution to the Newtonian potential, Eq. (63), based on the experimental results by the GRACE satellites [42], which are monitoring the Earth's gravitational potential. The length scale of the orbit, $R_\oplus \sim 10^9$cm, is comparable to the wavelength of GWs detected by the ground-based GW detectors. The modification to the relative velocity between the two satellites can be estimated by the deviation from the Newtonian potential in GR given as $\sim |X| r_{\rm sep}/R_\oplus^3$, where $|X|$ is the magnitude of X^{kl} and $r_{\rm sep} \sim 2.2 \times 10^7$ cm is the separation between the two GRACE satellites. The observed power spectrum of the relative velocity error $\lesssim 10^{-10}$ gives a rough estimate for the constraint on the magnitude, $|X| \lesssim 10^{-10} R_\oplus^3/r_{\rm sep}$. Then, we can estimate the bound on the relative amplitude of the induced scalar mode as

$$\frac{|h_s|}{|h_T|} \sim |X|\omega^2 < 10^{-10} \times \frac{R_\oplus^3 \omega^2}{r_{\rm sep}} \sim 10^{-7}\,, \tag{64}$$

where $|h_s|$ and $|h_T|$ are, respectively, the strain amplitudes of the scalar and tensor modes, and we set the GW frequency ω to the typical value, 200 Hz, corresponding to the observation band of the ground-based GW detectors.

The (c) term does not provide correction to the Newtonian potential. Hence, it will not be severely constrained by the solar system gravity test, but the (c) term does not give the detectable scalar modes, either. This is because the waves induced by the (c) term are proportional to δ^{ij}, whereas the detector tensor for the interferometric GW detectors is traceless.

The contribution of the (d) term to the gravitational field is very much suppressed by the slow motion parameter, since it appears only in the $0i$ component of the metric perturbation. Thus, the observational constraint would be weaker by the factor $\beta \sim 10^{-4}$, which is the typical value of the velocity in the solar system. The waves induced by the (d) term contain the breathing mode, as can be seen from

$$e^{(b)ij} \times kX^k_{(i} e^{(T)}_{j)k} = kX^{kl} e^{(T)}_{kl} \neq 0\,, \tag{65}$$

but no longitudinal mode. We perform an order estimate similar to the case of the (b) term. The magnitude of the metric perturbation h_{0i} relative to the Newtonian potential is $|X|/R_\oplus$. The effect of this perturbation on the satellite motion is suppressed by β. Hence, the modification to the relative velocity between the two satellites is estimated as $\sim \beta |X| r_{\rm sep}/R_\oplus^2$. Then, we can estimate the bound on the relative amplitude of the induced breathing mode as

$$\frac{|h_s|}{|h_T|} \sim |X|\omega < 10^{-10} \times \frac{R_\oplus^2 \omega}{r_{\rm sep}} \sim 10^{-3}\,. \tag{66}$$

Since the (e) term is proportional to the original tensor basis, the discussion does not change essentially from the case of an isotropic background. Trivially, the term does not possess any scalar modes.

To conclude, extra scalar GW polarization modes induced by a hypothetical local anisotropic background would hardly be detected in all of these cases.

5. Discussions and conclusion

We discussed the detectability of scalar polarization modes in GWs from compact binary coalescences. We consistently considered the whole processes of generation, propagation, and observation in a general framework that includes all polarization modes, without relying on specific models.

First, we claimed that the energy flux that can be attributed to additional modes is, at most, comparable to that of the ordinary tensor modes, so as to be consistent with the observed GW phase evolution. Next, we constructed a linear model of GW propagation with full mixing among various polarization modes. We showed that the amplitude of each polarization mode may change during propagation, but the energy flux of the propagating modes cannot change without frequency-dependent waveform deformations as far as the background is smoothly varying and the propagation speed is very close to the speed of light. If the dispersion relation is modified significantly, we cannot use the GR waveform as the search templates and need to construct a waveform model considering the propagation process. If we allow these conditions to be violated, there might be some unknown conversion mechanism, which may cause the enhancement of the energy fluxes of the propagating modes. Practically, the only possibility would be violating the smoothness of the background. Anyway, it seems very challenging to find a mechanism that realizes the selective amplification of the energy flux of some particular mode without modifying the dispersion relation.

In our general framework, the mode conversion is not prohibited. Hence, the maximum of the energy flux of the scalar mode is given by the one estimated by assuming equipartition among all polarization modes. Therefore, the maximum energy flux for the scalar modes that we can expect could be as high as the tensor modes, although we did not present any example of such an efficient conversion mechanism. Together with the constraint on the energy flux based on the consideration in the generation process described above, the maximum amplitude of the energy flux of scalar modes at the position of observers would be $O(1)$ relative to that of the tensor modes.

On the other hand, the deviation from the PN gravity in GR has been tightly constrained by several experiments in the solar system. These constraints are translated into the constraints on the coupling between the scalar mode and GW detectors or the anisotropies of the background field in the solar system. As for the observation process, we considered two scenarios: (i) additional scalar modes exist but no background anisotropies exist, and (ii) some background anisotropies exist but no scalar modes are generated and propagate. In scenario (i), we used the constraint that the energy flux of the scalar modes is, at most, of $O(1)$ relative to that of the tensor modes, to give an upper bound on the amplitude of the scalar polarization detected by GW detectors. We showed that the scalar polarization modes in the GW signal should be smaller by the suppression factor of $O(10^{-2.5})$ than the tensor polarization modes, based on the experiments in the solar system. In scenario (ii), the suppression factor was estimated to be as small as $O(10^{-3})$.

According to Refs. [15,43], the detection limit to the additional polarization amplitude is roughly given by the inverse of the signal-to-noise ratio. The detection limit for a single compact binary coalescence event with the ground-based GW detectors would be expected as $\lesssim 10^{-2}$. Therefore, these suppression factors indicate that it is difficult to detect the scalar polarization modes with a single compact binary coalescence event. The detectability of scalar polarization modes with ground-based GW telescopes in a single event is severely restricted. However, the detection limit using an ideal stacking of multiple events is estimated as $\sim 10^{-3}$ [43], assuming observations of expected multiple events with third-generation GW detectors such as the Einstein Telescope [44] and Cosmic Explore [45]. Hence, we might be able to obtain meaningful constraints on the amplitude of scalar polarization in the future. For this purpose, it is required to develop some efficient stacking methods.

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