Eur. Phys. J. C (2024) 84:1203
https://doi.org/10.1140/epjc/s10052-024-13591-y

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article

Theoretical Physics

Anisotropic extension of the Kohler–Chao–Tikekar cosmological solution with like Wyman IIa complexity factor

B. Siza[1], **J. Andrade**[1,a], **D. Santana**[1,b], **Tayyab Naseer**[2,3,c]

[1] Escuela Superior Politécnica de Chimborazo, (ESPOCH), 060155 Riobamba, Ecuador
[2] Department of Mathematics and Statistics, The University of Lahore, 1-KM Defence Road, Lahore 54000, Pakistan
[3] Research Center of Astrophysics and Cosmology, Khazar University, 41 Mehseti Street, Baku AZ1096, Azerbaijan

Received: 10 September 2024 / Accepted: 4 November 2024

Abstract We construct a new stellar compact object model in the regime of anisotropic pressure using the framework of gravitational decoupling via minimal geometric deformation, with the particularity that the seed solution used is the known Kohler–Chao–Tikekar cosmological solution. As an extra condition to close the Einstein's field equations resulting for this construction, we use a generalised complexity factor for self-gravitating spheres to the well-known Wyman IIa solution. The resulting model fulfils the fundamental physical acceptability stellar conditions for a compactness factor of a pulsar SMC X-1. The stability of the model is also investigated.

1 Introduction

The construction of new stellar compact object models is not an easy task since we need to solve Einstein's Field Equations (EFE) and figure out their non-linearity. We can not avoid this particularity since the Newtonian theory of gravitation is obsolete in the study of these stellar remnants, so we have to deal with the EFE. The extreme conditions in the interior of these astrophysical objects are so significant that the use of advanced tools of general relativity is mandatory; even other physics branches such as plasma physics, nuclear physics, quantum mechanics, and others are needed, but at the same time the characteristics of these compact objects make them into very interesting systems of study [1–5].

Since K. Schwarzschild published the first interior solution of EFE for a homogeneous perfect fluid sphere in 1916 [6], interest in developing new stellar interior models has grown with time until the actuality. In fact, it is an active area of research within the general relativity and astrophysics of stellar objects. In the first approximation, the interior of the stellar remnants was considered isotropic fluids; however, there are several phenomena that make this assumption inadequate for modelling real stellar compact objects; between them we can mention high density ($> 10^5$ g/cm^3) [7–9], viscosity [10–12], the existence of a superfluid core [13–15], strong magnetic fields [16–26], several types of phase transitions [27–29], pion condensation [30–32], as so on [33–46]. It is demonstrated in [47] that any initially isotropic arrangement can become anisotropic due to dissipative flows, energy density inhomogeneities, or the emergence of shear in a relativistic fluid. Furthermore, anisotropic fluid distributions have to exist throughout the last stages of star evolution, since dissipative flows are expected to accompany these events.

The assumption of local anisotropy inside stellar objects dates back to 1922 in [48], where it is suggested that anisotropy can be important in the stellar structure, as well as suggested by Lemaitre in 1933 [49]. After, in 1974, Bowers and Liang presented a seminal work in studying analytically the influence of anisotropic in compact objects [50]. In this regard, several works have been developed since this pioneering work, aiming to generate methods for creating new models of stellar objects as well as to study their main characteristics from the perspective of general relativity [51–122]. Among these methods, a novel framework known as Gravitational Decoupling (GD) has been developed to generate new interior solutions in the regime of anisotropic pressure, using an existing isotropic solution as a starting point (called "seed" solution into de GD) [123–125]. This method has a particularity of solving the problem of EFE with multiple

[a] e-mail: julio.andrade@espoch.edu.ec (corresponding author)
[b] e-mail: diego.santana@espoch.edu.ec
[c] e-mail: tayyabnaseer48@yahoo.com, tayyab.naseer@khazar.org, tayyab.naseer@math.uol.edu.pk

Published online: 22 November 2024

gravitational sources in an analytic way; in specific, when two sources are considered in EFE (one known and other unknown), it is widely used in order to obtain new models through what is called minimal geometric deformation (MGD) [126,127], as well as extended minimal geometric deformation (MGDe) [128]. Examples of use of this method with such purpose are [129–163].

Furthermore, the GD has been applied alongside the novel concept of complexity for self-gravitating spheres, developed by L. Herrera [164–166] in order to solve the EFE and obtain new stellar models. This concept is based on the existence of a scalar of structure defined through the matter sector of an interior solution, which permits defining the simplest system as an isotropic fluid and others as more complex, which deviate from the simplest one since they have anisotropy and inhomogeneity in energy density. The simplest system has the assignment of vanishing complexity, but there are systems that are no perfect fluid but also have vanishing complexity. It happens since isotropy balances with the energy density. When this factor is zero (vanishing complexity) is used as an equation of state in order to obtain new interior solutions, examples of research that use this vanishing condition are [167–181].

However, the option to explore the possibility to construct new stellar models with a non-vanishing complexity factor is also plausible. In fact, some stellar models have been constructed with this characteristic; this idea was first successfully developed by J. Andrade and E. Contreras in [182], where a family of stable stellar compact models were constructed with a complexity as a generalisation of the well-known Tolman IV solution [183] through the use of GD via MGD. After that, the stability of such solutions was analysed in [184] using the gravitational cracking concept [185]. Subsequently, the seminal work mentioned before, an interesting ultracompact anisotropic star with a polynomial complexity factor, was constructed also in the framework of GD via MGD by M. Carrasco and E. Contreras [186]. Moreover, M. Zubair presented two new families of anisotropic solutions for static spherically symmetric stellar systems with a polynomial complexity factor in [187], as well as a new analogue of the Durgapal-Fuloria model [71] under the condition of a complexity factor as a generalisation of the complexity of the same Durgapal-Fuloria model in [188]. Recently, a new family of stellar interior solutions in the anisotropic regime of pressure using the framework of GD via MGD with a complexity factor as a generalization of complexity of the Wyman IIa solution [189] were constructed in [190]. Even an energetic interaction between Einstein's universe and a generic gravitational source like Tolman IV complexity factor has been studied in [191].

Given the limited number of these kinds of stellar models, it can open a new way to construct new stellar models that can have an interesting behaviour since they are more complex than those that are constructed with the vanishing complexity condition. Now, in this manuscript, we construct a new model with a generalisation of complexity like Wyman IIa solution (with $n = 1$) using the framework of GD via MGD with the particularity that the seed solution is the Kohler–Chao–Tikekar cosmological solution [192,193]. We construct this new interior solution in order to extend the solutions with non-vanishing complexity, as well as show another solution obtained through the use of GD framework from a cosmological solution as a seed, which is not usual because the seed solution habitually is an isotropic solution. Throughout this manuscript, we will be using natural units where $G = c = 1$.

2 Gravitational decoupling via minimal geometric deformation

The GD solves EFE

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi T_{\mu\nu}, \tag{1}$$

for an effective static and spherically symmetric system whose space-time is given by the metric

$$ds^2 = e^{\nu}dt^2 - e^{\lambda}dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{2}$$

with the particularity that

$$T_{\mu\nu} = T^{(s)}_{\mu\nu} + \alpha\Theta_{\mu\nu}, \tag{3}$$

where ν and λ are functions depending only on the radial coordinate r, $T^{\nu(s)}_{\mu} = diag\left[\rho^{(s)}, -p^{(s)}_r, -p^{(s)}_t, -p^{(s)}_t\right]$ is a known gravitational source (called seed source), $\Theta_{\mu\nu}$ is unknown source (additional source), and α is a dimensionless constant that measures the influence of the additional source on the first one (for detailed explanation of GD via MGD see Ref. [126]).

Now using Eq. (2) in EFE (1), the following matter sector is obtained:

$$8\pi T^0_0 = \frac{1}{r^2} - e^{-\lambda}\left(\frac{1}{r^2} - \frac{\lambda'}{r}\right), \tag{4}$$

$$8\pi T^1_1 = \frac{1}{r^2} - e^{-\lambda}\left(\frac{1}{r^2} + \frac{\nu'}{r}\right), \tag{5}$$

$$8\pi T^2_2 = -\frac{e^{-\lambda}}{4}\left(2\nu'' + \nu'^2 - \lambda'\nu' + 2\frac{\nu' - \lambda'}{r}\right). \tag{6}$$

So we can identify the effective density

$$T^0_0 = \rho = \rho^{(s)} + \Theta^0_0, \tag{7}$$

the effective radial pressure

$$T^1_1 = p_r = p^{(s)}_r - \Theta^1_1, \tag{8}$$

and the effective tangential pressure

$$T^2_2 = T^3_3 = p_t = p^{(s)}_t - \Theta^2_2, \tag{9}$$

which clearly leads to the effective anisotropy

$$\Pi = p_r - p_t. \tag{10}$$

Now, the solution of EFE (1) when $T_{\mu\nu} = T^{(s)}_{\mu\nu}$ (namely when $\alpha = 0$) is given by

$$ds^2 = e^{\xi}dt^2 - e^{\mu}dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{11}$$

which should deform by the presence of the source $\Theta_{\mu\nu}$ in the following way

$$\nu = \xi + \alpha g, \tag{12}$$

$$e^{-\lambda} = e^{-\mu} + \alpha f, \tag{13}$$

where f and g are functions only of radial coordinate r called "deformations functions". In this case, we shall consider $f \neq 0$ and $g = 0$ (MGD case). Deforming the seed metric given by Eq. (11) through f deformation and exploring the consequences of the effect of $\Theta_{\mu\nu}$ on $T^{(s)}_{\mu\nu}$ is the key of idea of GD through MGD.

So if we use Eq. (13) on the system (4)–(6), we obtain two subsystems of differential equations:

One related with $T^{(s)}_{\mu\nu}$

$$8\pi\rho^{(s)} = \frac{1}{r^2} - e^{-\mu}\left(\frac{1}{r^2} - \frac{\mu'}{r}\right), \tag{14}$$

$$8\pi p_r^{(s)} = -\frac{1}{r^2} + e^{-\mu}\left(\frac{1}{r^2} + \frac{\nu'}{r}\right), \tag{15}$$

$$8\pi p_t^{(s)} = \frac{e^{-\mu}}{4}\left(2\nu'' + \nu'^2 - \mu'\nu' + 2\frac{\nu' - \mu'}{r}\right), \tag{16}$$

and the second one related with $\Theta_{\mu\nu}$

$$8\pi\Theta^0_0 = -\frac{f}{r^2} - \frac{f'}{r}, \tag{17}$$

$$8\pi\Theta^1_1 = -f\left(\frac{1}{r^2} + \frac{\nu'}{r}\right), \tag{18}$$

$$8\pi\Theta^2_2 = -\frac{f}{4}\left(2\nu'' + \nu'^2 + 2\frac{\nu'}{r}\right) - \frac{f'}{4}\left(\nu' + \frac{2}{r}\right). \tag{19}$$

Furthermore, the contracted Bianchi identities ensure that the Einstein tensor $G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}$ is divergence-free. Then, by Eq. (1), we can derive the covariant conservation of the energy–momentum tensor as follows:

$$\nabla_\mu T^{\mu\nu} = 0, \tag{20}$$

which in turn in its explicit form is the well-known Tolman–Oppenheimer–Volkoff (TOV) equation

$$\frac{dp_r}{dr} = -\frac{1}{2}\nu'(\rho + p_r) + \frac{2}{r}(p_r - p_t). \tag{21}$$

This equation is valuable as it describes stellar equilibrium by balancing radial pressure, gravitational forces, and the contribution of anisotropy.

Therefore, we can decouple the EFE system (4)–(6) into two systems analytically, hence the name of the GD method. It is easy to see the problem translated now as found f, for which it is necessary to give some extra condition, such as an equation of state or some geometric condition. In this work, we will provide the system with a non-vanishing complexity factor as an extra condition to solve the problem.

3 Complexity of self-gravitating spheres

In general, the concept of complexity in physical systems is not entirely straightforward. Complex systems are often characterised by time-dependent interactions among their numerous components, leading to behaviour that is intricate, non-trivial, and often surprising [194–197]. In such a sense, the idea of complexity is not an easy task to define and measure for a particular system. Particularly, in this work, we will employ a concept of complexity recently proposed by L. Herrera for static and spherically symmetric self-gravitational systems [164–166]. The idea of such a complexity concept is based on the existence of a particular structure scalar (called $\mathcal{Y}_{TF}$) that appears in the orthogonal splitting of the Riemann tensor [198,199]. This factor is explained by physical quantities that define the internal structure of the physical system from a relativistic point of view in the following way:

$$\mathcal{Y}_{TF} = 8\pi\Pi - \frac{4\pi}{r^3}\int_0^r \eta^3\rho'(\eta)d\eta, \tag{22}$$

namely, it is defined through the inhomogeneity and anisotropy ($\Pi = p_r - p_t$) of the stellar system. Moreover, this quantity called "complexity factor" in certain ways is like an indicator of "complexity" for these stellar systems since it is zero for the most simplest system of isotropic fluid distribution (with ρ = constant and $\Pi = 0$), and it can increase for the systems that deviate from the simplest configuration. In fact, this scalar can be considered a relative variable that indicates the degree of ambiguity in our comprehension of the self-gravitating system [191,200]. Also, it is advantageous to express this complexity factor in terms of the interior solution metric as

$$\mathcal{Y}_{TF} = \frac{e^{-\lambda}}{4r}\left(\nu'(2 + r\lambda' - r\nu') - 2r\nu''\right), \tag{23}$$

which can be found using Eqs. (4)–(6) in (22).

It is worth noting that Eq. (22) reveals that there may exist systems that may have vanishing complexity (namely, $\mathcal{Y}_{TF} = 0$) when the inhomogeneity of the energy density is compensated by anisotropy in the following way

$$2\Pi = \frac{1}{r^3}\int_0^r \eta^3\rho'(\eta)d\eta, \tag{24}$$

which reveals that there are systems with this condition of zero complexity but that are not necessarily isotropic fluids. In fact, recently three intriguing formalisms have been created in order to construct solutions with this particular property [201–203], which makes it a novel and useful tool for building new interior solutions. However, this is not the only way in which the concept of complexity is useful for constructing new interior solutions, but rather it is also possible to assign particular values of non-zero complexity factors, as, for example, generalisations of already known interior solutions.

4 New stellar model

In this section, we will apply the GD through the MGD, starting from the cosmological solution of Kohler–Chao–Tikekar as a seed solution and also making use of an extra condition of the Wyman IIa type complexity factor to find a new anisotropic interior solution.

First, the metric of the cosmological solution of Kohler–Chao–Tikekar is given by

$$e^{\nu} = A + Br^2, \tag{25}$$
$$e^{\mu} = \frac{A + 2Br^2}{A + Br^2}, \tag{26}$$

where A and B are constants. After that, we will obtain the deformation function f through the use of Eq. (13) in (23) giving us the following differential equation:

$$\nu' f' + 2\left(\nu'' - \frac{\nu'}{r} + \frac{\nu'^2}{r}\right) f + 2\frac{e^{-\mu}}{\alpha} \times \left(\nu'' - \frac{\nu'}{r} + \frac{\nu'^2}{r} - \frac{\mu'\nu'}{2}\right) + \frac{4}{\alpha}\mathcal{Y}_{TF} = 0, \tag{27}$$

which can be solved if we know the functions ν, μ and the value of $\mathcal{Y}_{TF}$. The values of ν and μ are done by the seed metric (25) and (26), and the value of $\mathcal{Y}_{TF}$ is obtained as a generalisation of the complexity factor of the known solution of Wyman IIa (with $n = 1$) given by the following metric components

$$e^{\nu} = (a_0 - b_0 r^2)^2, \tag{28}$$
$$e^{-\lambda} = 1 + c_0 r^2 (a_0 - 3b_0 r^2)^{-3/2}, \tag{29}$$

where a_0, b_0 and c_0 are constants. So using these metrics in (23), the following complexity factor is obtained:

$$\mathcal{Y}_{TF} = \frac{2Bcr^2}{(A - 3Br^2)^{5/3}}, \tag{30}$$

which can generalise easily as

$$\mathcal{Y}_{TF} = \frac{a_1 r^2}{\left(a_2 + a_3 r^2\right)^{5/3}}, \tag{31}$$

where a_1 is an arbitrary constant with dimension of length^{-4}, a_2 is an arbitrary dimensionless constant, and a_3 is an arbitrary constant with dimension of length^{-2}. Therefore, using this value of $\mathcal{Y}_{TF}$ given by (31) in Eq. (27), we obtain

$$f = \left(A + Br^2\right) \times \left[\frac{3a_1}{2a_3 B\alpha \left(a_2 + a_3 r^2\right)^{2/3}} - \frac{1}{\left(A + 2Br^2\right)\alpha} + c_0\right], \tag{32}$$

where c_0 is an integration constant, which for purposes of regularity of the effective matter sector, it has to be

$$c_0 = \frac{2a_2^{2/3} a_3 B - 3a_1 A}{2\alpha a_2^{2/3} a_3 AB}. \tag{33}$$

Now, using the values of (32) and (26) in Eq. (13), we obtain the effective space-time of our new stellar model

$$e^{\nu} = A + Br^2, \tag{34}$$
$$e^{-\lambda} = \frac{e^{\nu}\left[\frac{\left(a_2 + a_3 r^2\right)^{2/3}\left(2a_2^{2/3} a_3 B - 3Aa_1\right)}{Aa_2^{2/3}} + 3a_1\right]}{2a_3 B\left(a_2 + a_3 r^2\right)^{2/3}}, \tag{35}$$

and with these metric components in EFE (4)–(6), the effective matter sector is obtained as

$$\rho = -\frac{\frac{a_1 \varrho_2(r)}{a_2^{2/3} a_3 Br^2 \varrho_1(r)^{5/3}} + \frac{6B}{A}}{16\pi}, \tag{36}$$
$$p_r = -\frac{3\left(\frac{a_1 A\varrho_3(r)}{a_3 B} - \frac{2Br^2}{A} + \frac{3a_1 r^2 \varrho_3(r)}{a_3}\right)}{16\pi r^2}, \tag{37}$$
$$p_t = \frac{a_1\left(\frac{5Br^2 - 2A}{B\varrho_1(r)^{5/3}} - \frac{9}{a_2^{2/3} a_3} + \frac{9a_2}{a_3 \varrho_1(r)^{5/3}}\right) + \frac{6B}{A}}{16\pi}, \tag{38}$$

where $\varrho_1(r) = a_2 + a_3 r^2$, $\varrho_2(r) = -Aa_2^{2/3} a_3 r^2 + 3Aa_2^{5/3} - 3A\varrho_1(r)^{5/3} + 5a_2^{2/3} a_3 Br^4 + 9a_2^{5/3} Br^2 - 9Br^2 \varrho_1(r)^{5/3}$ and $\varrho_3(r) = \frac{1}{a_2^{2/3}} - \frac{1}{\varrho_1(r)^{2/3}}$.

Furthermore, it is necessary to ensure the continuity of the space-time of our system on the surface of the stellar compact object, namely, the space-time given by (34) and (35) must match smoothly with the Schwarzschild exterior solution

$$ds^2 = \left(1 - \frac{2M}{r}\right) dt^2 - \left(1 - \frac{2M}{r}\right)^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\phi^2, \tag{39}$$

where M is the total mass of the stellar compact object. Also, we have to ensure that the effective radial pressure matches with zero pressure outside the stellar object, namely

$$p_r|_{r=R} = 0, \tag{40}$$

where R is the radius of the compact star. So applying these conditions to our model we obtain

$$a_1 = \frac{2a_2^{2/3} a_3 \left(a_2 + a_3 R^2\right)^{2/3} M^2 R^{-4}}{\left(1 - \frac{3M}{R}\right)\left(1 - \frac{6M}{R}\right)\left(\left(a_2 + a_3 R^2\right)^{2/3} - a_2^{2/3}\right)}, \quad (41)$$

$$A = 1 - \frac{3M}{R}, \quad (42)$$

$$B = \frac{M}{R^3}. \quad (43)$$

From the above, it follows that $\frac{M}{R} \neq \frac{1}{3}$ and $\frac{M}{R} \neq \frac{1}{6}$.

5 Physical analysis of the obtained model

In this section, we analyse the physical behaviour of our model using the compactness factor of the pulsar SMC X-1 $\left(u = \frac{M}{R} = 0.19803\right)$ [204] as a reference. The parameter a_2 is held constant at 0.50, while the parameter a_3 is varied. Specifically, we consider the following values for $a_3 = 0.70$ (black line), 0.71 (blue line), 0.72 (red line), and 0.73 (violet line). These constants have been calibrated to yield a physically acceptable interior solution, though they were not initially assigned based on specific physical reasoning. However, this flexibility allows readers to experiment with alternative values, including varying compactness, making the model highly adaptable and applicable to systems beyond SMC X-1. This analysis is necessary to determine the plausibility of the new model, as it behaves like a compact star supported by the anisotropic fluid. Therefore, we need to check whether the model satisfies the physical acceptability conditions detailed in [205].

In this regard, we plot the behaviour of both metric components (34) and (35) in Figs. 1 and 2. We can observe that both potentials are regular and positive definite inside the stellar object. Moreover, the temporal component e^{ν} is a monotonically increasing function of the radial coordinate r, while $e^{-\lambda}$ decreases monotonically with r. Also, we verify that $e^{\nu}|_{r=0}$ = constant and $e^{-\lambda}|_{r=0} = 1$. This behaviour of the metric coefficients is consistent with the interior space-time of a real stellar object.

After this, we plot the profile of the matter sector of our new model $\{\rho, p_r, p_t\}$ in Figs. 3, 4, and 5. We observe that these physical quantities are regular and positive in the interior of the stellar object. They are monotonically decreasing functions of r, having their maximum values at the stellar centre. Note that the radial pressure vanishes at the stellar surface, which is consistent with the fact that there is nothing outside the stellar system. Also, we checked that $p_t > p_r$ inside the stellar system. However, both pressures are equal just at the stellar centre (see Fig. 6). This behaviour of the matter sector is exactly what would be expected inside a realistic stellar object.

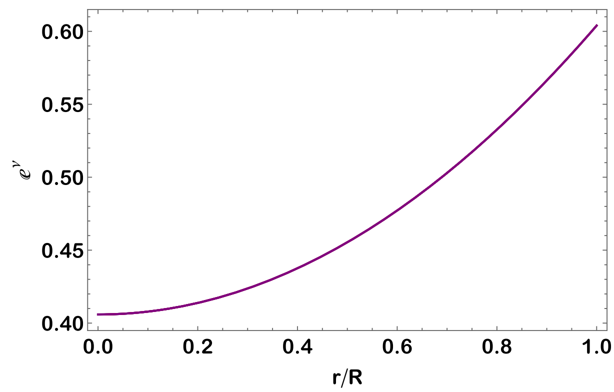

Fig. 1 Temporal component of the metric

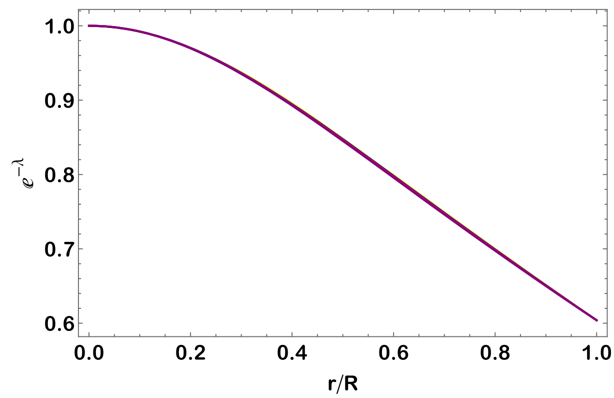

Fig. 2 Radial component of the metric profile

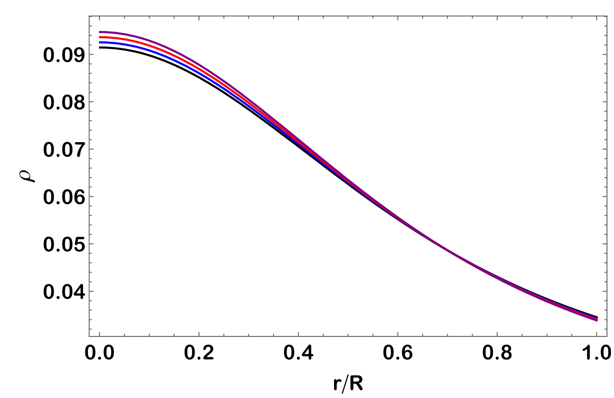

Fig. 3 Density energy profile

On the other hand, from an energetic point of view, it is confirmed that our interior solution satisfies the dominant energy conditions (DEC)

$$\rho > p_r \text{ and } \rho > p_t, \quad (44)$$

and as well the strong energy condition (SEC) (see Figs. 7, 8, 9)

$$\rho > p_r + 2p_t. \quad (45)$$

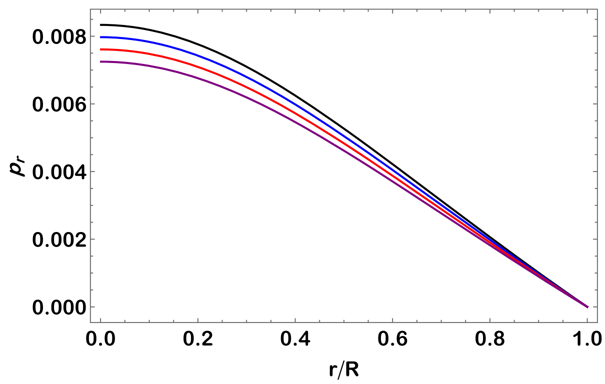

Fig. 4 Radial pressure profile

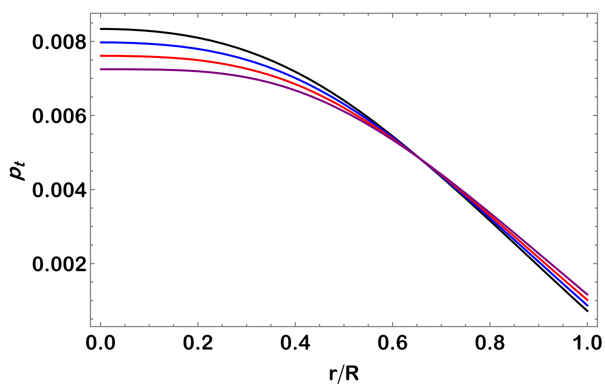

Fig. 5 Tangential pressure profile

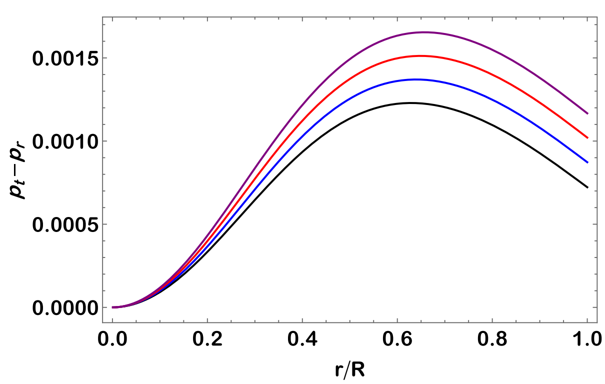

Fig. 6 Local anisotropy profile

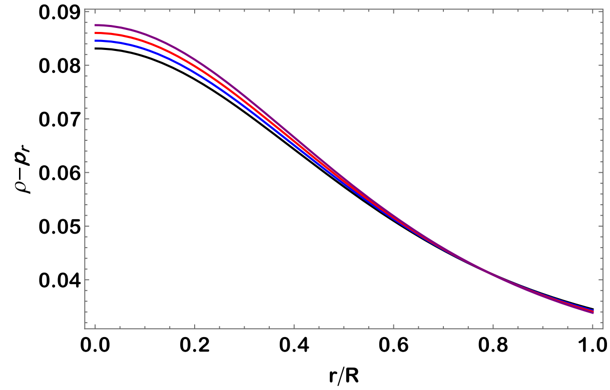

Fig. 7 $\rho - p_r$ profile

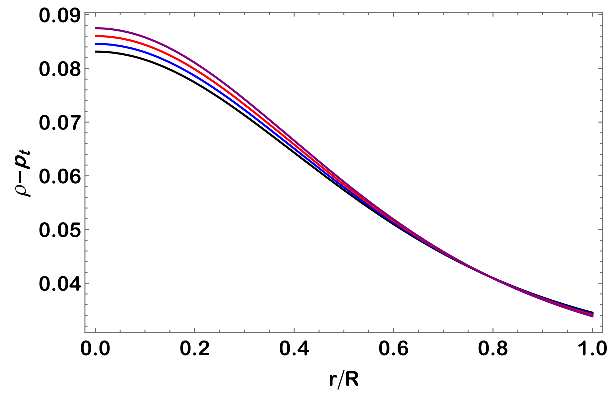

Fig. 8 $\rho - p_t$ profile

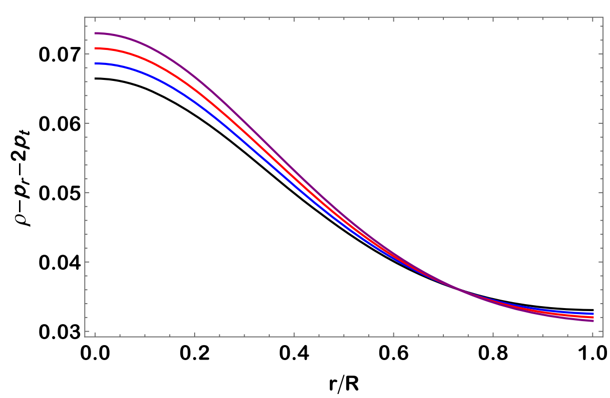

Fig. 9 $\rho - p_r - 2p_t$ profile

Thus, our interior solution effectively models a compact star supported by an energetically realistic anisotropic fluid.

Also, we verify if the sound velocities inside the stellar object do not surpass the causal limit of light velocity in vacuum ($c = 1$), namely

$$0 \leq v_r = \sqrt{\frac{dp_r}{d\rho}} < 1 \text{ and } 0 \leq v_t = \sqrt{\frac{dp_t}{d\rho}} < 1. \qquad (46)$$

It is clearly verified in Figs. 10 and 11. Also, we verify that the profile for redshift function $z = g_{tt}^{-1/2} - 1$ of our model is a monotonously decreasing function of radial coordinate r (see Fig. 12). Likewise, we can observe that the redshift value on the surface is far from exceeding the universal limit of $z_{bound} = 5.211$ [206].

With all the acceptability conditions verified for our model, we can confidently infer that it is a physically relevant model capable of representing the compact star system SMC X-1. While our analysis focused on the SMC X-1 system, the developed model is adaptable to other compact systems with

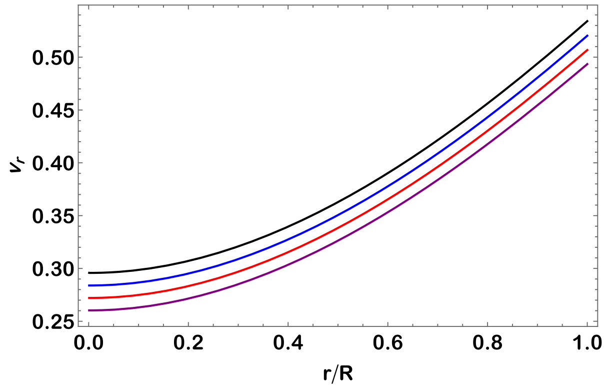

Fig. 10 Radial sound velocity profile

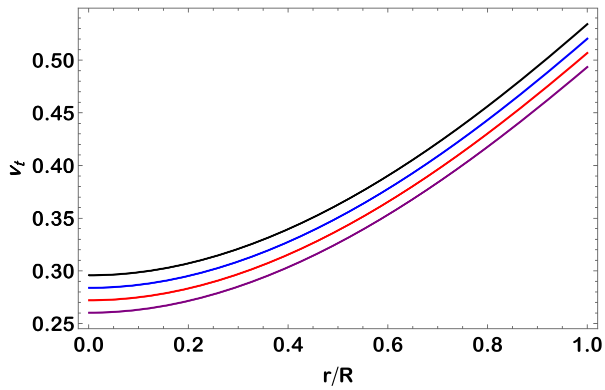

Fig. 11 Tangential sound velocity profile

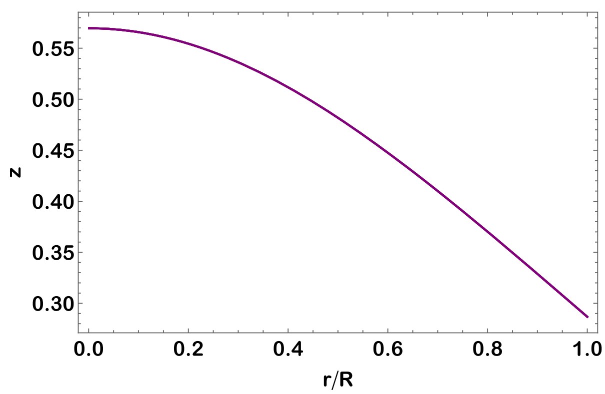

Fig. 12 Redshift profile

different compactness factors. This flexibility enhances the applicability of our model to a broad range of stellar systems.

Additionally, it is important to study the stability of the model against various criteria, such as convective motion, gravitational anti-collapse, and gravitational cracking, in order to assess its overall stability.

Thus, in Fig. 13, we plot the profile of ρ'' in order to analyse the response of the stellar object in the presence of the convective fluid movement in its interior. We must then consider the criterion that, for the fluid inside the stellar object to

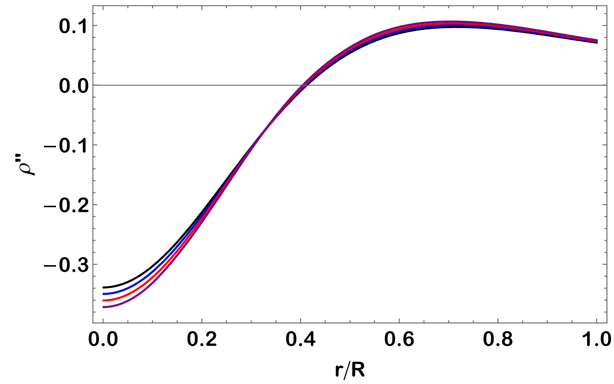

Fig. 13 ρ'' profile

satisfy the buoyancy principle, it should meet the following condition [207]:

$$\rho'' < 0. \tag{47}$$

By satisfying the above condition, we ensure that any fluid element will tend to return to its original position, indicating stability against convection. As a result, the fluid element does become buoyant and does not continues to sink. Similarly, Fig. 13 illustrates the contrast between the system's highly stable inner layers and the less stable outer layers. This fluid behaviour is crucial for preventing mass accumulation at the centre of the stellar system, thereby avoiding a potential gravitational collapse caused by the buildup of mass that does not fulfil the buoyancy principle.

Also, we plot the profile of the adiabatic index $\Gamma = \frac{\rho+p_r}{p_r} v_r^2$ in Fig. 14 in order to analyse its behaviour, and determine if the model is stable against gravitational collapse. For such purpose, it is necessary that the adiabatic index Γ should be greater than a critical value $\Gamma_c = \frac{4}{3} + \frac{19}{21}u$, namely

$$\Gamma \geq \Gamma_c. \tag{48}$$

This essential value is contingent upon the internal structure and force balance of the stellar compact object. Otherwise, a minor disturbance can develop exponentially and cause gravitational collapse if Γ falls below the critical threshold Γ_c. This is because there is insufficient pressure response to compression to resist gravitational forces (for detailed information about this condition see Refs. [208,209]). Thus, we have checked that for the values of parameters a_2 and a_3 set in this work, our model is unstable against gravitational collapse in the inner regions of the stellar compact object.

Furthermore, it is worth determining the potential zones within the stellar compact object that are stable against gravitational cracking, that are areas where the appearance of fractures is not likely. These are given by the fulfilment of [210]

$$-1 \leq v_t^2 - v_r^2 \leq 0. \tag{49}$$

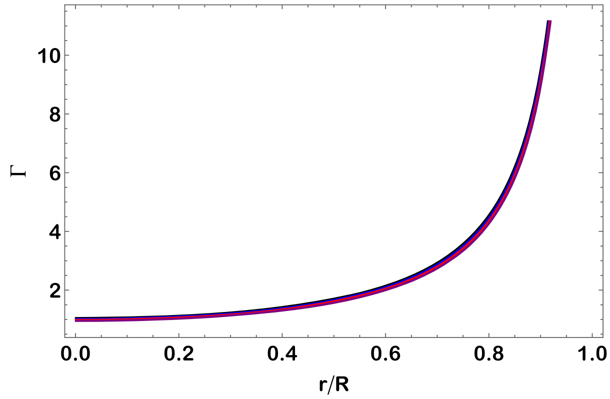

Fig. 14 Γ profile

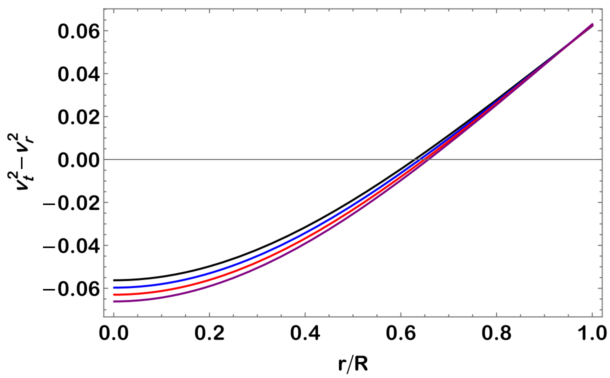

Fig. 15 $v_t^2 - v_r^2$ profile

Thus, we plot the profile of $v_t^2 - v_r^2$ in Fig. 15, from where it is easy to check that the stable zones coincide with the inner shells of the stellar compact object. This suggests that the outer layers of the stellar compact object are prone to fractures, while the inner layers may indicate the presence of a solid core surrounded by a less rigid crust.

6 Discussion and conclusions

The complexity of the EFE poses challenges in finding analytical solutions for static, spherically symmetric space-times. This has driven extensive research in theoretical stellar astrophysics to develop novel stellar models. Several methods have been proposed, with the GD approach being a significant breakthrough. This approach enables the determination of exact solutions for the interior structure of stellar configurations using a particular linear transformation that converts the EFE into a new reference frame.

The MGD approach provides a powerful technique to derive new interior solutions from known seed solutions. By applying MGD to a static spherical space-time, the EFE can be separated into two distinct sectors corresponding to the original and additional gravitationally coupled fluid sources. At this moment, we need to brief the GD approach before discussing the current research work. This approach was initially proposed to extend the domain of an isotropic gravitational source, which could be extended by adding the Lagrangian density corresponding to any other fluid source into the action function. The only point we need to highlight here is that both the original and additional fluid sources must be gravitationally coupled with each other. Many such works have been done in the literature where the isotropic solutions are extended to the anisotropic domain. The resulting extended solution exhibits modifications to the original seed solution, which can be analysed to understand the impact of the additional gravitational sector and the decoupling parameter on the behaviour of the effective system. However, in the current study, the seed source is not taken to be isotropic interior fluid; rather, it is an isotropic fluid cosmological configuration.

The significance of anisotropy in interior solutions cannot be minimized, as extensive cosmological studies over the past two decades have increasingly indicated that our universe exhibits anisotropic characteristics. Observations, such as the subtle deviations from isotropy identified in inhomogeneous Supernova Ia studies, support this notion [211–213]. In addition, various inconsistencies have emerged from different observational data, including findings related to radio sources [214], gamma-ray bursts [215], etc., which collectively reinforce the idea that isotropy may not be a valid assumption in cosmological and interior models. Observational evidence, along with various theoretical models, strongly indicates that the universe exhibits anisotropic properties. This realisation emphasises the necessity of developing anisotropic solutions, which are crucial for enhancing our understanding of various cosmic evolutionary stages.

The complexity factor proposed by Herrera [164–166] represents a significant advancement in understanding the behaviour of self-gravitating systems within the framework of general relativity. Introduced in 2018, this concept quantifies the complexity of a static, spherically symmetric system by assessing the relationship between energy density inhomogeneity and pressure anisotropy. The complexity factor is derived from the orthogonal splitting of the Riemann tensor, specifically utilising a traced-free scalar known as $\mathcal{Y}_{TF}$. This scalar serves as an auxiliary condition to evaluate the deformation function of stellar configurations, establishing that simpler systems with uniform energy density and isotropic pressure have a zero complexity factor. In contrast, systems with non-zero complexity factors indicate a departure from this simplicity, reflecting the intricate interplay of local anisotropies and inhomogeneities. Herrera's framework not only enhances the theoretical understanding of compact stellar objects but also provides a robust tool for analysing their structure and evolution. By facilitating the exploration of various gravitational scenarios, including those involving dissipative fluids, the complexity factor opens new avenues for research in astrophysics, allowing for a more nuanced examination of gravitational phenomena and the stability of stellar configurations under diverse conditions. This innovative approach underscores the importance of complexity in gravitational studies, offering a fresh perspective on the intricate nature of self-gravitating systems and their underlying physical principles.

Following the above discussion, we have derived an anisotropic extension of a particular cosmological seed solution within the context of the vanishing complexity factor. For this, we have considered a static spherical spacetime filled with an isotropic cosmological fluid distribution. We have then added an extra gravitational source $\Theta_{\mu\nu}$ and formulated EFE that describe the overall fluid configuration, encompassing both the seed and additional sources. Subsequently, we have divided EFE into two separate sets utilising the MGD technique. Each set of equations effectively corresponded to its respective parent source. For the initial set that pertains to the seed source, the Kohler–Chao–Tikekar cosmological solution has been considered. Afterwards, we have formulated the deformation function using the complexity factor of the known solution of Wyman IIa. Once the function f has been obtained, we determined the effective matter variables through the system (7)–(9). The Schwarzschild exterior solution has also been utilised to find the constants associated with the considered seed solution.

For graphical analysis of the developed model, we have adopted the compactness of a pulsar SMC X-1. The temporal and inverse radial metric components have shown increasing and decreasing profiles, respectively (Figs. 1, 2). They are also consistent with their required condition at the centre, i.e., $e^{\nu}|_{r=0}$ = constant and $e^{-\lambda}|_{r=0} = 1$. The energy density and both components of pressure have been found to be maximum in the core of the considered pulsar and monotonically decreasing outwards (Figs. 3, 4, 5). Also, the radial pressure is observed to vanish at the spherical boundary. Since both principal pressures are equal at the centre, their difference, known as anisotropy, is null at this point and positive otherwise, which indicates that there is enough pressure to counterbalance the gravitational force (Fig. 6). The DEC and SEC have been analysed and found to be positive in the whole domain, thus we have claimed our model to be viable (Figs. 7, 8, 9). Both sound speeds have been lied in the acceptable range (Figs. 10, 11). We have also observed the gravitational redshift, admitting maximum value in the core and decreasing towards the boundary (Fig. 12). The factor ρ'' is shown to be negative at the center and increasing outwards (Fig. 13). Finally, we have also analyzed the stability using adiabatic index and cracking approach, and found the resulting interior model to be stable in the inner shells, however it is susceptible to experience a possible gravitational collapse (Figs. 14, 15).

It is worth mentioning that since the local anisotropy in Fig. 6 is not a monotonically increasing function, it is possible to associate it as a symptom of instability. Examples where a similar situation occurs have been found in [160,162,170,216]), where the anisotropy is indeed not a completely monotonically increasing function, and which at the same time is directly related to the stability of the system. It is expected that the anisotropy plays an important role since it appears explicitly in the stellar equilibrium Eq. (21).

The above idea is reinforced since the anisotropy of our model experiences a decrease around $r/R = 0.6$, which coincides precisely with the place where there is a distinction between stable internal zones and unstable external zones regarding the convective motion inside the stellar object (see Fig. 6). Likewise, this is related to the $v_t^2 - v_r^2$ profile (Fig. 13), where we observe that the stable zones regarding gravitational cracking coincide with the internal layers, while around the same radius, $r/R = 0.6$, the external layers of the stellar model are prone to fractures; that is, their configuration tends to break and compress due to radial forces directed inwards. Therefore, we can conclude that the external regions of the model exhibit instability, the internal regions remain stable. This balance of stability in the core makes the model suitable for describing the pulsar SMC X-1 that can after collapse of the outer layers, the core of the progenitor star compresses into an extremely dense object. Such core can consist of exotic states of matter, including neutrons, protons, possibly hyperons, and even more exotic particles at extremely high densities [14,217–219]. Despite the challenges in the outer layers, the model's ability to maintain stability in the inner regions suggests it can effectively capture the key physical properties of this pulsar. However, we must not forget that the model presents instability with regard to gravitational collapse, so that even though the stellar compact object has a very solid and stable core, it is also prone to instability in the face of strong radial oscillations, which could presumably generate a possible collapse.

Expanding the boundaries of our scientific understanding often requires looking beyond the conventional and embracing novel approaches. In the realm of theoretical astrophysics, the concept of GD has shown great promise, and its potential applications extend far beyond the realm of typical solutions. By delving into cosmological solutions under complete geometric deformation, we can uncover new avenues for scientific inquiry and push the limits of our current knowledge. This approach not only broadens our horizons but also opens up the possibility of discovering groundbreaking insights that could revolutionise our understanding of the universe and the fundamental forces that govern its evolution.

Funding The authors declare that they have received no funding from any institutional projects for the publication of this article.

Data Availability Statement This manuscript has no associated data. [Authors' comment: In the present study, no datasets are generated or analyzed.]

Code Availability Statement The manuscript has no associated code/software. [Author's comment: The code/software generated during and/or analysed during the current study is available from the corresponding author on reasonable request.]

Funded by SCOAP^3.

References

1. M. Camenzind, *Compact Objects in Astrophysics* (Springer, Berlin, 2007)
2. R. Kippenhahn, A. Weigert, A. Weiss, *Stellar Structure and Evolution*, vol. 192 (Springer, Berlin, 1990)
3. J. Schaffner-Bielich, *Compact Star Physics* (Cambridge University Press, Cambridge, 2020)
4. C.J. Hansen, S.D. Kawaler, V. Trimble, *Stellar Interiors: Physical Principles, Structure, and Evolution* (Springer Science & Business Media, Berlin, 2012)
5. S.E. Woosley, A. Heger, T.A. Weaver, Rev. Mod. Phys. **74**(4), 1015 (2002)
6. K. Schwarzschild, Sitzungsber. K. Preuss, Akad. Wiss. Berlin, 424–434 (1916)
7. M. Ruderman, Astro. Astrophys. **10**, 427 (1972)
8. V. Canuto, Annu. Rev. Astron. Astrophys. **12**, 167 (1974)
9. S. Sadiq, A. Shafiq, A. Ikram, Eur. Phys. J. Plus **139**(2), 1–12 (2024)
10. W. Barreto, S. Rojas, Astrophys. Space Sci. **193**, 201–215 (1992)
11. P.B. Jones, Phys. Rev. D **64**, 084003 (2001)
12. E.N.E. van Dalen, A.E.L. Dieperink, Phys. Rev. C **69**, 025802 (2004)
13. N. Anderson, G. Comer, K. Glampedakis, Nucl. Phys. A **763**, 212 (2005)
14. N.K. Glendenning, *Compact Stars: Nuclear Physics, Particle Physics, and General Relativity* (Springer, New York, 1997)
15. H. Heiselberg, M. Hjorth-Jensen, Phys. Rep. **328**, 237 (2000)
16. A.P. Martinez, R.G. Felipe, D.M. Paret, Int. J. Mod. Phys. D **19**, 1511 (2010)
17. D. Reimers, S. Jordan, D. Koester, N. Bade, T. Kohler, L. Wisotzki, Astron. Astrophys. **311**, 572–578 (1996)
18. G.H. Bordbar, M. Karami, Eur. Phys. J. C **82**(1), 74 (2022)
19. E.J. Ferrer, V. de la Incera, J.P. Keith, I. Portillo, P.L. Springsteen, Phys. Rev. C **82**, 065802 (2010)
20. G.D. Schmidt, P.S. Schmidt, Astrophys. J. **448**, 305 (1995)
21. J.C. Kemp, J.B. Swedlund, J.D. Landstreet, J.R.P. Angel, Astrophys. J. **161**, L77 (1970)
22. A. Putney, ApJL **451**, 67 (1995)
23. A. Perez Martinez, H. Perez Rojas, H.J. Mosquera Cuesta, Eur. Phys. J. C **29**, 111 (2003)
24. S.S. Yazadjiev, Phys. Rev. D **85**, 044030 (2012)
25. C.Y. Cardall, M. Prakash, J.M. Lattimer, Astrophys. J. **554**, 322 (2001)
26. N. Bucciantini, A.G. Pili, L. Del Zanna, Mon. Not. R. Astron. Soc. **447**, 3278 (2015)
27. N. Chamel, A.F. Fantina, J.M. Pearson, S. Goriely, Astron. Astrophys. **553**, A22 (2013)
28. A.I. Sokolov, Sov. Phys. JETP **52**, 575–576 (1980)
29. S. Nelmes, B.M. Piette, Phys. Rev. D **85**(12), 123004 (2012)
30. R.F. Sawyer, Phys. Rev. Lett. **29**, 382–385 (1972)
31. J.B. Hartle, R. Sawyer, D. Scalapino, Astrophys. J. **199**, 471 (1975)
32. A.B. Migdal, Phys. Lett. B **45**(5), 448–450 (1973)
33. K.P. Reddy, M. Govender, S.D. Maharaj, Gen. Relativ. Gravit. **47**, 35 (2015)
34. M. Esculpi, M. Malaver, E. Aloma, Gen. Relativ. Gravit. **39**, 633–652 (2007)
35. L. Herrera, A. Di Prisco, J. Martin, J. Ospino, N.O. Santos, O. Troconis, Phys. Rev. D **69**, 084026 (2004)
36. V.B. Thapa, M. Sinha, Phys. Rev. D **102**(12), 123007 (2020)
37. E.E. Kolomeitsev, K.A. Maslov, D.N. Voskresensky, Nucl. Phys. A **970**, 291–315 (2018)
38. L. Herrera, N.O. Santos, Phys. Rep. **286**(2), 53–130 (1997)
39. F. Weber, J. Phys. G Nucl. Part. Phys. **95**, 9 (1999)
40. S. Gedela, N. Pant, J. Upreti, R.P. Pant, Eur. Phys. J. C **79**, 1–12 (2019)
41. M. Kalam, A.A. Usmani, F. Rahaman, S.M. Hossein, I. Karar, R. Sharma, Int. J. Theor. Phys. **52**, 3319–3328 (2013)
42. D. Horvat, S. Ilijić, A. Marunović, Class. Quantum Gravity **28**(2), 025009 (2010)
43. J.A. Pons, D. Viganò, Living Rev. Comput. Astrophys. **5**, 1–65 (2019)
44. D.N. Aguilera, V. Cirigliano, J.A. Pons, S. Reddy, R. Sharma, Phys. Rev. Lett. **102**(9), 091101 (2009)
45. R. Ruffini, S. Bonazzola, Phys. Rev. **187**, 1767 (1969)
46. S. Aktar, K.N. Singh, P. Bhar, F. Rahaman, P. Bhandakkar, Chin. J. Phys. **89**, 1188–1197 (2024)
47. L. Herrera, Phys. Rev. D **101**(10), 104024 (2020)
48. J.H. Jeans, Mon. Not. R. Astron. Soc. **82**, 122–132 (1922)
49. G. Lemaitre, Ann. Soc. Sci. Brux. A **53**, 51 (1933)
50. R.L. Bowers, E.P.T. Liang, Class. Astrophys. J. **188**, 657 (1974)
51. M. Cosenza, L. Herrera, M. Esculpi, L. Witten, J. Math. Phys. **22**, 118 (1981)
52. A.A. Isayev, Phys. Rev. D **96**, 083007 (2017)
53. H. Heintzmann, W. Hillebrandt, Astron. Astrophys. **38**, 51 (1975)
54. L. Herrera, W. Barreto, Phys. Rev. D **88**, 084022 (2013)
55. B. Biswas, S. Bose, Phys. Rev. D **99**, 104002 (2019)
56. A. Rahmansyah, A. Sulaksono, A.B. Wahidin, A.M. Setiawan, Eur. Phys. J. C **80**, 769 (2020)
57. R. Nasheeha, S. Thirukkanesh, F. Ragel, Eur. Phys. J. C **80**, 6 (2020)
58. R. Nasheeha, S. Thirukkanesh, F. Ragel, Eur. Phys. J. Plus **136**, 132 (2021)
59. L.M. Becerra, E.A. Becerra-Vergara, F.D. Lora-Clavijo, Phys. Rev. D **109**, 043025 (2024)
60. D. Deb, B. Mukhopadhyay, F. Weber, Astrophys. J. **922**, 149 (2021)
61. L. Baskey, S. Das, F. Rahaman, Mod. Phys. Lett. A **36**, 2150028 (2021)
62. M. Bocquet, S. Bonazzola, E. Gourgoulhon, J. Novak, Astron. Astrophys. **301**, 757 (1995)
63. M. Pattersons, A. Sulaksono, Eur. Phys. J. C **81**, 698 (2021)
64. J. Kumar, P. Bharti, New Astron. Rev. **95**, 101662 (2022)
65. M. Sharif, T. Naseer, Int. J. Mod. Phys. D **31**(11), 2240017 (2022)

66. M. Sharif, T. Naseer, Phys. Scr. **97**(12), 125016 (2022)
67. M. Sharif, T. Naseer, Chin. J. Phys. **86**, 596–615 (2023)
68. Z. Roupas, Astrophys. Space Sci. **366**, 9 (2021)
69. S.K. Maurya, A. Banerjee, S. Hansraj, Phys. Rev. D **97**, 044022 (2018)
70. K. Lake, Phys. Rev. D **80**, 064039 (2009)
71. M. Durgapal, R. Fuloria, Gen. Relativ. Gravit. **17**, 671 (1985)
72. M. Alqahtani, M. Nopoush, M. Strickland, Prog. Part. Nucl. Phys. **101**, 204 (2018)
73. P. Boonserm, T. Ngampitipan, M. Visser, Int. J. Mod. Phys. D **25**, 1650019 (2016)
74. K. Ioka, M. Sasaki, Astrophys. J. **600**, 296 (2004)
75. M. Mak, T. Harko, Proc. R. Soc. Lond. A Math. Phys. Eng. Sci. **459**(2030), 393–408 (2003)
76. J.M. Sunzu, S.D. Maharaj, S. Ray, Astrophys. Space Sci. **352**, 719–727 (2014)
77. K. Dev, M. Gleiser, Gen. Relativ. Gravit. **34**, 1793–1818 (2002)
78. F. Tello-Ortiz, M. Malaver, Á. Rincón, Y. Gomez-Leyton, Eur. Phys. J. Plus C **80**(5), 371 (2020)
79. O. Bustos, E. Fuenmayor, P. León, E. Contreras, Eur. Phys. J. Plus **138**(9), 790 (2023)
80. O.F. Bustos, J. Phys. Conf. Ser. **2796**, 012006 (2024)
81. M. Sharif, T. Naseer, Eur. Phys. J. Plus **137**, 1304 (2022)
82. M. Sharif, T. Naseer, Class. Quantum Gravity **40**, 035009 (2023)
83. T. Naseer, M. Sharif, Fortschr. Phys. **71**, 2300004 (2023)
84. M. Sharif, T. Naseer, Eur. Phys. J. Plus **139**(3), 1–16 (2024)
85. S.K. Maurya et al., Mon. Not. R. Astron. Soc. **519**(3), 4303–4324 (2023)
86. R. Tamta, P. Fuloria, P. Tamta, Indian J. Phys. 1–14 (2024)
87. S. Gedela, N. Pant, J. Upreti, R.P. Pant, Mod. Phys. Lett. A **36**(08), 2150055 (2021)
88. T. Naseer, M. Sharif, Commun. Theor. Phys. **76**, 095407 (2024)
89. T. Naseer, M. Sharif, Phys. Scr. **99**, 075012 (2024)
90. D. Santana, E. Fuenmayor, E. Contreras, Eur. Phys. J. C **82**(8), 703 (2022)
91. G. Abellán, P. Bargueño, E. Contreras, E. Fuenmayor, Int. J. Mod. Phys. D **29**(12), 2050082 (2020)
92. P. León, E. Fuenmayor, E. Contreras, Phys. Rev. D **107**(6), 064010 (2023)
93. G. Abellán, E. Fuenmayor, E. Contreras, L. Herrera, Phys. Dark Universe **30**, 100632 (2020)
94. L. Herrera, A.D. Prisco, J. Ospino, Fuenmayor, J. Math. Phys. **42**(5), 2129–2143 (2001)
95. L. Herrera, J. Ospino, A. Di Prisco, Phys. Rev. D **77**(2), 027502 (2008)
96. L. Herrera, A. Di Prisco, J. Martin, J. Ospino, N.O. Santos, O. Troconis, Phys. Rev. D **69**(8), 084026 (2004)
97. M. Sharif, T. Naseer, Phys. Scr. **98**, 115012 (2023)
98. M. Sharif, T. Naseer, Eur. Phys. J. Plus **139**, 86 (2024)
99. K.N. Singh, N. Pant, N. Pradhan, Astrophys. Space Sci. **361**, 1–8 (2016)
100. K.N. Singh, N. Pant, A.M. Govender, Chin. Phys. C. **41**(1), 015103 (2017)
101. S. Kaur, S.K. Maurya, S. Shukla, R. Nag Chin, J. Phys. **77**, 2854–2870 (2022)
102. S. Kaur, S.K. Maurya, S. Shukla, B. Dayanandan, New Astron. **110**, 102230 (2024)
103. S.K. Maurya, S.K. Singh, A. Aziz, S. Ray, G. Mon, Not. R. Astron. Soc. **527**(3), 5192–5205 (2024)
104. N. Sarkar, S. Sarkar, F. Rahaman, K.N. Singh, Mod. Phys. Lett. A **36**(26), 2150190 (2021)
105. S.K. Maurya, Y.K. Gupta, S. Ray, D. Deb, Eur. Phys. J. C **76**, 1–12 (2016)
106. B. Dayanandan, S.K. Maurya, Eur. Phys. J. A **53**(6), 141 (2017)
107. A. Errehymy, Y. Khedif, M. Daoud, Eur. Phys. J. C **81**(3), 1–16 (2021)
108. K.N. Singh, S.K. Maurya, P. Bhar, F. Rahaman, Phys. Scr. **95**(11), 115301 (2020)
109. K.N. Singh, F. Rahaman, N. Pradhan, N. Pant, Indian J. Phys. **95**, 1271–1281 (2021)
110. R. Sharma, S. Das, S. Thirukkanesh, Astrophys. Space Sci. **362**, 1–11 (2017)
111. S. Das, K.N. Singh, L. Baskey, F. Rahaman, A.K. Aria, Gen. Relativ. Gravit. **53**, 1–32 (2021)
112. C.R. Ghezzi, Astrophys. Space Sci. **333**, 437–447 (2011)
113. J.M. Sunzu, A.K. Mathias, S.D. Maharaj, J. Astrophys. Astron. **40**, 1–9 (2019)
114. L. Herrera, A. Di Prisco, J. Ospino, Symmetry **16**(5), 562 (2024)
115. S. Gedela, R.K. Bisht, Eur. Phys. J. C **84**(8), 877 (2024)
116. M. Sharif, T. Naseer, Gen. Relativ. Gravit. **55**, 87 (2023)
117. M. Sharif, T. Naseer, Phys. Dark Universe **42**, 101324 (2023)
118. T. Naseer, M. Sharif, Phys. Scr. **99**, 035001 (2024)
119. T. Naseer, Phys. Dark Universe **46**, 101663 (2024)
120. J.M. Pretel, T. Tangphati, A. Banerjee, A. Pradhan, Phys. Lett. B **848**, 138375 (2024)
121. Y. Feng, T. Naseer, A. Ashraf, D. Sofuoglu, Phys. Scr. **99**, 085034 (2024)
122. B. Das, S. Das, B.C. Paul, Astrophys. Space Sci. **368**(11), 98 (2023)
123. J. Ovalle, Mod. Phys. Lett. A **23**(38), 3247–3263 (2008)
124. J. Ovalle, R. Casadio, J. Ovalle, R. Casadio, A generalization of the minimal geometric deformation. Beyond Einstein Gravity: The Minimal Geometric Deformation Approach in the Brane-World, (SpringerBriefs in Physics, 2020), p. 77-93
125. J. Ovalle, R. Casadio, Beyond Einstein gravity: the minimal geometric deformation approach in the brane-world (Springer Nature, 2020)
126. J. Ovalle, Phys. Rev. D **95**(10), 104019 (2017)
127. J. Ovalle, R. Casadio, R. Da Rocha, A. Sotomayor, Eur. Phys. J. C **78**, 1–11 (2018)
128. J. Ovalle, Phys. Lett. B **788**, 213–218 (2019)
129. R. Casadio, J. Ovalle, R. Da Rocha, Class. Quantum Gravity **32**(21), 215020 (2015)
130. R. Casadio, R. da Rocha, Eur. Phys. J. C **83**(6), 1–15 (2023)
131. J. Ovalle, C. Posada, Z. Stuchlík, Class. Quantum Gravity **36**(20), 205010 (2019)
132. B. Dayanandan, S.K. Maurya, T.T. Smitha, J.P. Maurya, Chin. J. Phys. **82**, 155–170 (2023)
133. K. Pant, P. Fuloria, Int. J. Mod. Phys. D **32**(15), 2350093–182 (2023)
134. P. Tamta, P. Fuloria, Phys. Scr. **96**(9), 095003 (2021)
135. E. Contreras, F. Tello-Ortiz, S.K. Maurya, Class. Quantum Gravity **37**(15), 155002 (2020)
136. N. Pant, S. Gedela, R.K. Bisht, Chin. J. Phys. **72**, 530–546 (2021)
137. F. Tello-Ortiz, Eur. Phys. J. C **80**(5), 413 (2020)
138. N. Pant, S. Gedela, S. Ray, K.G. Sagar, Mod. Phys. Lett. A **37**(14), 2250072 (2022)
139. S. Gedela, N. Pant, M. Govender, Braz. J. Phys. **51**(5), 1371–1405 (2021)
140. G. Abellán, Á. Rincón, E. Fuenmayor, E. Contreras, Eur. Phys. J. Plus **135**, 1–12 (2020)
141. S.K. Maurya, A. Al Saadi, W. Al Amri, S. Al Hosni, R. Al Sharyani, Phys. Scr. **97**(10), 105002 (2022)
142. M. Zubair, M. Amin, H. Azmat, Phys. Scr. **96**(12), 125008 (2021)
143. M. Al Hadhrami, S.K. Maurya, Z. Al Amri, N. Al Hadifi, A. Al Buraidi, H. Al Wardi, R. Nag, Pramana, **97**(1), 13 (2022)
144. M. Estrada, R. Prado, Eur. Phys. J. Plus **134**, 1–10 (2019)
145. S.K. Maurya, Eur. Phys. J. C **79**(11), 958 (2019)
146. S.K. Maurya, K.N. Singh, B. Dayanandan, Eur. Phys. J. C **80**(5), 448 (2020)
147. B. Dayanandan, T.T. Smitha, S.K. Maurya, Phys. Scr. **96**(12), 125041 (2021)

148. L. Gabbanelli, J. Ovalle, A. Sotomayor, Z. Stuchlik, R. Casadio, Eur. Phys. J. C **79**, 1–9 (2019)
149. C. Las Heras, P. León, Fortschr. Phys. **66**(7), 1800036 (2018)
150. V. Vertogradov, M. Misyura, Universe **8**(11), 567 (2022)
151. L. Gabbanelli, Á. Rincón, C. Rubio, Eur. Phys. J. C **78**(5), 370 (2018)
152. K.N. Singh, S.K. Maurya, M.K. Jasim, F. Rahaman, Eur. Phys. J. C **79**(10), 851 (2019)
153. V.A. Torres-Sánchez, E. Contreras, Eur. Phys. J. C **79**(10), 829 (2019)
154. S.K. Maurya, Eur. Phys. J. C **80**(5), 429 (2020)
155. R. da Rocha, Eur. Phys. J. C **81**(9), 1–17 (2021)
156. R. da Rocha, Eur. Phys. J. C **82**(1), 34 (2022)
157. S.K. Maurya, F. Tello-Ortiz, Eur. Phys. J. C **79**, 1–14 (2019)
158. R.P. Graterol, Eur. Phys. J. Plus **133**(6), 244 (2018)
159. E. Morales, F. Tello-Ortiz, Eur. Phys. J. C **78**, 1–12 (2018)
160. S. Hensh, Z. Stuchlík, Eur. Phys. J. C **79**(10), 834 (2019)
161. E. Morales, F. Tello-Ortiz, Eur. Phys. J. C **78**, 1–17 (2018)
162. J. Andrade, K.Y. Ortega, W.N. Klínger, R.C. Copa, S.S. Medina, J.D. Cruz, Eur. Phys. J. C **83**(11), 1–11 (2023)
163. J. Andrade, S. Carrera, D. Santana, Eur. Phys. J. C **84**(5), 533 (2024)
164. L. Herrera, A. Di Prisco, J. Ospino, Phys. Rev. D **98**(10), 104059 (2018)
165. L. Herrera, Entropy **23**(7), 802 (2021)
166. L. Herrera, Complexity and simplicity of self-gravitating fluids, in *New Frontiers in Gravitational Collapse and Spacetime Singularities* (Singapore: Springer Nature Singapore, 2024), p. 199–253
167. R. Casadio, E. Contreras, J. Ovalle, A. Sotomayor, Z. Stuchlik, Eur. Phys. J. C **79**, 1–8 (2019)
168. C. Arias, E. Contreras, E. Fuenmayor, A. Ramos, Ann. Phys. **436**, 168671 (2022)
169. T. Naseer, M. Sharif, Eur. Phys. J. C **84**(6), 1–19 (2024)
170. J. Andrade, D. Santana, Eur. Phys. J. C **83**(6), 1–16 (2023)
171. S.K. Maurya, A. Errehymy, B. Dayanandan, S. Ray, N. Al-Harbi, A.H. Abdel-Aty, Eur. Phys. J. C **83**(6), 1–16 (2023)
172. S.K. Maurya, M. Govender, G. Mustafa, R. Nag, Eur. Phys. J. C **82**(11), 1006 (2022)
173. S. Kaur, S.K. Maurya, S. Shukla, Phys. Scr. **98**(10), 105304 (2023)
174. S. Gedela, R.K. Bisht, Eur. Phys. J. C **83**(9), 1–12 (2023)
175. E. Contreras, E. Fuenmayor, G. Abellán, Eur. Phys. J. C **82**(3), 187 (2022)
176. S. Das, N. Sarkar, A. Das, S.K. Pal, Eur. Phys. J. C **84**(8), 817 (2024)
177. T. Naseer, M. Sharif, Phys. Dark Universe **46**, 101595 (2024)
178. T. Naseer, J.L. Said, Eur. Phys. J. C **84**, 808 (2024)
179. G. Panotopoulos, Á. Rincón, I. Lopes, Phys. Lett. B 138901 (2024)
180. Á. Rincón, G. Panotopoulos, I. Lopes, Universe **9**(2), 72 (2023)
181. P. Rej, R.S. Bogadi, M. Govender, Chin. J. Phys. **87**, 608–619 (2024)
182. J. Andrade, E. Contreras, Eur. Phys. J. C **81**, 1–12 (2021)
183. R.C. Tolman, Phys. Rev. **55**(4), 364 (1939)
184. J. Andrade, E. Fuenmayor, E. Contreras, Int. J. Mod. Phys. D **31**(12), 2250093 (2022)
185. L. Herrera, Phys. Lett. A **165**, 206 (1992)
186. M. Carrasco-Hidalgo, E. Contreras, Eur. Phys. J. C **81**(8), 757 (2021)
187. M. Zubair, Eur. Phys. J. C **82**(11), 984 (2022)
188. M. Zubair, H. Jameel, H. Azmat, Eur. Phys. J. C **83**(7), 604 (2023)
189. M. Wyman, Phys. Rev. **75**(12), 1930 (1949)
190. J. Andrade, D. Andrade, J. Phys. Conf. Ser. **2796**, 012007 (2024)
191. J. Andrade, D. Santana, Int. J. Theor. Phys. **63**(5), 134 (2024)
192. M. Kohler, K.L. Chao, Z. Naturforsch Ser. **20**, 1537 (1965)
193. R.S. Tikekar, Curr. Sci. **39**(20), 460–461 (1970)
194. J. Kwapień, S. Drożdż, Phys. Rep. **515**(3–4), 115–226 (2012)
195. Y. Holovatch, R. Kenna, S. Thurner, Eur. J. Phys. **38**(2), 023002 (2017)
196. S. Thurner, R. Hanel, P. Klimek, *Introduction to the Theory of Complex Systems* (Oxford University Press, Oxford, 2018)
197. S.H. Strogatz, *Nonlinear Dynamics and Chaos with Student Solutions Manual: With Applications to Physics, Biology, Chemistry, and Engineering* (CRC Press, Boca Raton, 2018)
198. L. Bel, Ann. Inst. Henri Poincare **17**, 37 (1961)
199. A.G.P. Gómez-Lobo, Class. Quantum Gravity **25**, 015006 (2007)
200. N.P. Suh, *Complexity: Theory and Applications* (Oxford University Press, Oxford, 2005)
201. E. Contreras, Z. Stuchlik, Eur. Phys. J. C **82**, 706 (2022)
202. S.K. Maurya, A. Errehymy, M.K. Jasim et al., Eur. Phys. J. C **82**, 1173 (2022)
203. P. León, C. Las Heras, Eur. Phys. J. C **83**(3), 1–15 (2023)
204. M.L. Rawls et al., Astrophys. J. **730**(1), 25 (2011)
205. B.V. Ivanov, Eur. Phys. J. C **77**(11), 1–12 (2017)
206. B.V. Ivanov, Phys. Rev. D **65**(10), 104011 (2002)
207. H. Hernández, L. Núñez, A. Vázques, Eur. Phys. J. C **78**, 883 (2018)
208. S. Chandrasekhar, Phys. Rev. Lett. **12**(4), 114 (1964)
209. S. Chandrasekhar, Astrophys. J. **140**, 1342 (1964)
210. H. Abreu, H. Hernández, L.A. Núnez, Class. Quantum Gravity **24**(18), 4631 (2007)
211. D.J. Schwarz, B. Weinhorst, Astron. Astrophys. **474**(3), 717 (2007)
212. J. Colin, R. Mohayaee, S. Sarkar, A. Shafieloo, Mon. Not. R. Soc. **414**(1), 264 (2011)
213. B. Javanmardi, C. Porciani, P. Kroupa, J. Pflamm-Altenburg, Astrophys. J. **810**(1), 47 (2015)
214. P. Tiwari, A. Nusser, J. Cosmol. Astropart. Phys. **2016**(03), 062 (2016)
215. J. Rípa, A. Shafieloo, Astrophys. J. **851**(1), 15 (2017)
216. J. Andrade, Eur. Phys. J. C **82**, 617 (2022)
217. S.L. Shapiro, S.A. Teukolsky, *Black Holes, White Dwarfs, and Neutron Stars: The Physics of Compact Objects* (Wiley-Interscience, New York, 1983)
218. P. Haensel, A.Y. Potekhin, D.G. Yakovlev (eds.), *Neutron Stars 1* (Springer, New York, 2007)
219. A.W. Steiner, M. Hempel, T. Fischer, ApJ **774**(1), 17 (2013)