Standard Model with Free Initial Data of Higgs Field

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Abstract

In order to remove vacuum energy of the Higgs field in the Standard Model, we consider the version of the model, where the fundamental dimensional parameter C in the Higgs potential $\lambda(\phi^2 - C^2)^2$ is replaced by the zeroth Fourier harmonic of the Higgs field ϕ. In this case all masses in SM are determined by the initial data of the potential free equation of this harmonic. By consideration the extremum of the quantum Coleman–Weinberg effective potential obtained from the unit vacuum-vacuum transition amplitude, we get the prediction of the lowest Higgs boson mass: $m_h = 138$ GeV, which corresponds to renormalization constant value of the order of the weak boson masses, i.e. the scale where all the relevant quantities in the equation are well defined. This result is in qualitative agreement with the result of Ref. [18], $m_h > 128 \pm 33$, obtained from the Higgs potential stability condition.

There can be two types of parameters in the Standard Model (SM) [1] like in any dynamic theory, fundamental parameters of the potential and the initial data of the equations of motion. In this paper, a new version [2] of the Higgs effect [3, 4] in the SM is considered, where the constant parameter C of the potential is replaced by an initial datum of the zeroth Fourier harmonic [5] $\langle\phi\rangle = \frac{1}{V_0}\int d^3x\phi$ as the averaging over the coordinate volume $V_0 = \int d^3x$:

$$\mathsf{V}_{\mathsf{Higgs}}(\phi) = \lambda\left[\phi^2 - C^2\right]^2 \longrightarrow \mathsf{V}_{\mathsf{inertial}}(\phi) = \lambda\left[\phi^2 - \langle\phi\rangle^2\right]^2. \tag{1}$$

This model corresponds to the zero value of the zeroth mode potential $\mathsf{V}_{\mathsf{inertial}}(\langle\phi\rangle) \equiv 0$ and the potential free (*i.e.* inertial) equations

$$\langle\ddot{\phi}\rangle = 0. \tag{2}$$

There is a solution of these vacuum equations

$$\langle\phi\rangle = C_{(\text{initial datum})} = \phi_I, \qquad \langle\dot{\phi}\rangle = 0 \tag{3}$$

that does not contradict the relativistic invariance of masses. Note that the zero-potential constraint is applied to renormalized quantities. So quantum corrections at all orders are already included. Such a choice of the potential might look rather artificial. But to our mind it is motivated by two serious reasons: minimization of the vacuum energy and embedding the Higgs field into cosmology. Moreover, contrary to the SM case, where the symmetry between the components of the Higgs field is broken spontaneously due to the special form of the potential, we break the symmetry by the initial condition (3). On the one hand our model correspond to a special extreme choice of parameters in the Standard Model, while on the other hand it has particular features.

The present paper is devoted to physical consequences of this model. We claim that this model is compatible with the present experimental data in high energy physics and cosmological observations. Moreover, the constraint on the potential leads to a prediction for the Higgs boson mass.

The action of the Standard Model with the choice of the Higgs field potential (1) can be written in the form

$$S_{\mathrm{SM}} = \int d^4x \mathcal{L}_{\mathrm{SM}} = \int d^4x \left[\mathcal{L}_{\mathrm{Ind}} + \mathcal{L}_{\mathrm{Higgs}}\right], \tag{4}$$

where $\mathcal{L}_{\mathrm{Ind}}$ is the standard part of the Lagrangian independent of the Higgs field and

$$\mathcal{L}_{\mathrm{Higgs}} = \partial_\mu\phi\partial^\mu\phi - \phi\sum_f g_f\bar{f}f + \frac{\phi^2}{4}\sum_{\mathrm{v}} g_{\mathrm{v}}^2 V^2 - \underbrace{\lambda\left[\phi^2 - \langle\phi\rangle^2\right]^2}_{\mathrm{V}_{\mathrm{inertial}}(\phi)} \tag{5}$$

is the Higgs field dependent part. Here $\sum_f g_f\bar{f}f \equiv \sum_{f=f_1,f_2} g_f\left[\bar{f}_{fR}f_{fL} + \bar{f}_{fL}f_{fR}\right]$ and

$$\frac{1}{4}\sum_{\mathrm{v}} g_{\mathrm{v}}^2 V^2 \equiv \frac{1}{4}\sum_{\mathrm{v}=\mathrm{W}_1,\mathrm{W}_2,\mathrm{Z}} g_{\mathrm{v}}^2 V^2 = \frac{g^2}{4}W_\mu^+ W^{-\mu} + \frac{g^2+g'^2}{4}Z_\mu Z^\mu \tag{6}$$

are the mass-like terms of fermions and vector bosons coupled with the Higgs field; g and g' are the Weinberg coupling constants, and measurable gauge bosons W_μ^+, W_μ^-, Z_μ are defined by the usual relations: $W_\mu^\pm \equiv A_\mu^1 \pm A_\mu^2 = W_\mu^1 \pm W_\mu^2$, $Z_\mu \equiv -B_\mu \sin\theta_W + A_\mu^3\cos\theta_W$, $\tan\theta_W = g'/g$, where θ_W is the Weinberg angle.

After the separation of the zeroth mode $\langle\phi\rangle$ the bilinear part of the Higgs Lagrangian takes the form

$$\mathcal{L}_{\mathrm{Higgs}}^{\mathrm{bilinear}} = \frac{1}{2}\partial_\mu h\partial^\mu h - \langle\phi\rangle\sum_f g_f\bar{f}f + \frac{\langle\phi\rangle^2}{4}\sum_{\mathrm{v}} g_{\mathrm{v}}^2V^2 - 2\lambda\langle\phi\rangle^2h^2. \tag{7}$$

In the lowest order in the coupling constant, the bilinear Lagrangian of the sum of all fields arises with the masses of vector (W, Z), fermion (f), and Higgs (h) particles:

$$M_W = \frac{\langle\phi\rangle}{\sqrt{2}}g, \quad M_Z = \frac{\langle\phi\rangle}{\sqrt{2}}\sqrt{g^2+g'^2}, \quad m_f = \langle\phi\rangle g_f, \quad m_h = 2\sqrt{\lambda}\langle\phi\rangle \tag{8}$$

according to the definition of the masses of vector (V) and fermion (f) particles $\mathcal{L}_{\mathrm{mass}} = \frac{M_V^2}{2}V_\mu V^\mu - m_f\bar{f}f$.

The sum of all vacuum-vacuum transition amplitude diagrams of the theory is known as the Coleman – Weinberg potential [6]

$$\mathrm{V}_{CW}(\langle\phi\rangle) = -i\mathrm{Tr}\log <0|0>_{(\langle\phi\rangle)} = -i\mathrm{Tr}\log\prod_F G_F^{-A_F}[\langle\phi\rangle]G_F^{A_F}[\phi_{\mathrm{I}}], \tag{9}$$

where $G_F^{-A_F}$ are the Green-function operators with $A_F = 1/2$ for bosons and $A_F = -1$ for fermions. In this case, the unit vacuum-vacuum transition amplitude $<0|0>\Big|_{\langle\phi\rangle=\phi_{\mathrm{I}}} = 1$ means that $\mathrm{V}_{CW}(\phi_{\mathrm{I}}) = 0$, where ϕ_{I} is a solution of the variation equation

$$\partial_0^2\langle\phi\rangle + \left.\frac{d\mathrm{V}_{CW}(\langle\phi\rangle)}{d\langle\phi\rangle}\right|_{\langle\phi\rangle=\phi_{\mathrm{I}}} = \tag{10}$$

$$= \partial_0^2\langle\phi\rangle + \sum_f g_f <\bar{f}f> - \frac{\langle\phi\rangle}{2}\sum_{\rm v} g_{\rm v}^2 <V^2> + 4\lambda\langle\phi\rangle <h^2> = 0.$$

where $< V^2 >$, $< \bar{f}f >$, and $< h^2 >$ are the condensates determined by the Green functions in [7]:

$$< V^2 > = \langle V_\mu(x)V_\mu(x)\rangle, \qquad < \bar{f}f > = \langle\bar{\psi}_\alpha(x)\psi_\alpha(x)\rangle, \qquad < h^2 > = \langle h(x)h(x)\rangle. \quad (11)$$

Finally, using the definitions of the condensates and masses (8) we obtain the equation of motion

$$\langle\phi\rangle\partial_0^2\langle\phi\rangle = -\sum_f m_f < \bar{f}f > + \sum_{\rm v} M_{\rm v}^2 < V^2 > + m_h^2 < h^2 > . \quad (12)$$

In the class of constant solutions of $\partial_0^2\langle\phi\rangle \equiv 0$ all masses (8) are defined by the initial data of this equation

$$\langle\phi\rangle = \phi_I. \quad (13)$$

The nonzero solution means that there is the Gell-Mann–Oakes–Renner type relation

$$-\sum_f m_f < \bar{f}f > + \sum_b m_b^2 < bb > = 0, \quad (14)$$

where b stands for bosons.

If we suppose that the condensates are defined by the subtraction procedure associated with the renormalization of masses and wave functions leading to the finite value for bosons (b) and fermions (f) respectively

$$\begin{aligned} < bb >_R (m_{Rb}^2) &= < bb > (m_{Rb}^2) - < bb > (\Lambda^2) - (m_{Rb}^2 - \Lambda^2)\frac{d}{d\Lambda^2} < bb > (\Lambda^2) = \\ &= \frac{m_{Rb}^2}{(4\pi)^2}\left[\log\frac{m_{Rb}^2}{\Lambda^2} - \frac{5}{6}\right], \quad (15) \\ < \bar{f}f >_R (m_{Rf}^2) &= -\frac{2m_{Rf}^2}{(4\pi)^2}\left[\log\frac{m_{Rf}^3}{\Lambda^2} - \frac{3}{2}\right], \quad (16) \end{aligned}$$

where Λ is a subtraction constant. In this case, the sum rule (14) takes the form

$$< h^2 > m_h^2 = -\sum_f < \bar{f}f > m_f + 2M_W < W_\mu W^\mu > + M_Z^2 < Z_\mu Z^\mu > . \quad (17)$$

We substitute the experimental data by the values of masses of bosons $M_W = 80.403 \pm 0.029$ GeV, $M_Z = 91.1876 \pm 0.00021$ GeV [9], and t-quark $m_t = 170.9 \pm 1.8$ GeV [10]. In the minimal SM [8], the three color t-quark dominates $\sum_f m_f^2 \simeq 3m_t^2$ because contributions of other fermions $\sum_{f\neq t} m_f^2/2m_t \sim 0.17$ GeV are very small.

In Fig. 1 the solution of the above equation is plotted for the range 0.3 GeV $< \Lambda <$ 100 GeV. There is no solution of Eq. (17) for 84 GeV $< \Lambda <$ 370 GeV. The lower right point of the plot corresponds to $m_h = 138$ GeV, which is the best choice, since taking renormalization constant value of the order of the weak boson masses corresponds to the scale where all the relevant quantities in the equation are well defined. This result is in qualitative agreement with the result of Ref. [18], $m_h > 128 \pm 33$, obtained from the

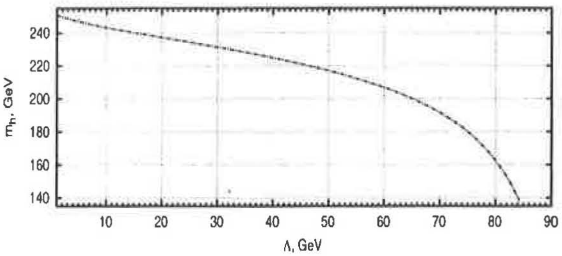

Figure 1: Value of the Higgs mass from Eq. (17) with the condensates defined by Eq. (15) as a function of Λ.

Higgs potential stability condition. Note that the stability boundary of the potential just corresponds to the point, we are looking at.

In general radiative corrections to the quantities in this relation within the Standard Model are not small, first of all due to a large coupling constant of Higgs with top quark. The corrections can be treated in the usual way, *e.g.* following Refs. [17, 18], since our model formally is a partial case of the Standard Model. Note that the zero value of the Higgs potential and relation (17) in our model should be valid in all orders of the perturbation theory. Implementation of higher order effects can reduce the renormalization scale dependence. This problem will be considered elsewhere.

The choice of the parameters in the inertial Higgs potential in our model can be motivated by the cosmological reasons. Even so that the resulting Lagrangian of the model is practically the same as the on of SM, we get a prediction for value of the the Higgs boson mass to be in the range $215 \div 255$ GeV. In this range of m_h the width of the Higgs particle is between 5 and 10 GeV. Here the main decay modes are $W \to ZZ$ and $H \to WW$ (since $M_Z < m_h < 2m_t$), which are quite convenient for experimental studies [4]. The so-called "gold-plated" channel $H \to 4\mu$ should allow a rather accurate measurement of m_h with at least 0.1% relative error [11]. So it is important to provide adequately precise theoretical predictions for this quantity. As concerns the production mechanism, the sub-process with gluon-gluon fusion dominates [12] for the given range of m_h and the corresponding cross section of about 10^4 fb provides a good possibility to discover the Higgs boson at the high-luminosity LHC machine.

In this way the potential free Higgs mechanism gives the possibility to solve the question about a consistence of the nonzero vacuum value of the scalar field with the zero vacuum cosmological energy as a consequence of the unit vacuum-vacuum transition amplitude. The inertial motion of a scalar field corresponds to the dominance of the most singular rigid state at the epoch of the intensive vacuum creation of the primordial bosons [13]. As it was shown, it can be compatible with energy budget of the universe and the Supernova data [14–16].

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