Neutrino predictions in a modified Babu-Ma-Valle model

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Abstract.   The measurement of the last neutrino mixing angle θ_{13} giving a a non zero value has established a new challenge for the theoretical understanding of the lepton mixing. The use of discrete symmetries to successfully explain that mixing is still possible. As an example, we have modified the so called Babu-Ma-Valle model in such a way that we account for the current neutrino mixing values at $3\,\sigma$. In particular, we have obtained not only compatibility with a $\theta_{13} \neq 0$, but also a non maximal θ_{23}.

1. Introduction

Oscillation mechanism is nowdays an accurate description for the neutrino flavor transitions. Three active neutrino oscillations globally explain almost all neutrino data [1]. Neutrino oscillations implies massive neutrinos and then it becomes one of the evidences of the standard model (SM) limitations. Unfortunately, a theoretical explanation for the origin of the neutrino masses is still elusive. The see-saw mechanism offers an explanation for the smallness of the neutrino mass opening a theoretical framework to study massive neutrinos.

Neutrino oscillations also imply a mixing in the neutrino sector. That mixing pattern, before the determination of the last mixing angle θ_{13}, was well described using some discrete symmetries [2]. After the determination of a non zero mixing angle θ_{13} from reactor experiments, models with discrete symmetries get challenged to take into account the non zero result for the reactor mixing angle without spoiling the predictions for the other two mixing angles.

The Babu-Ma-Valle (BMV) model [3] is a supersymmetrical extension of the SM with an A_4 discrete symmetry that described the neutrino phenomenology before the θ_{13} determination. The goal of this work is to show that a slight modification of the BMV model successfully describes the current neutrino data [1].

The work is organized in the following way: in section 2 we introduce the BMV model and in sections 3 we explore the possible modifications. The numerical results are shown in section 4 and finally we conclude in section 5.

2. BMV model

The Babu-Ma-Valle model (BMV) [3] is a supersymmetric model with A_4 discrete symmetry, where A_4 is broken at some high scale.

A_4 is a discrete non-Abelian group of even permutations of 4 objects, it has $\underline{1}, \underline{1}', \underline{1}''$ and $\underline{3}$ irreducible representation (irrep) and it is the smallest finite group with triplet irrep [2].

The usual quark $\hat{Q}_i = (\hat{u}_i, \hat{d}_i)$, lepton $\hat{L}_i = (\hat{\nu}_i, \hat{e}_i)$, and Higgs $\hat{\phi}_i$ transforms under A_4 as follows:

	$\hat{Q}$	$\hat{L}$	$\hat{u}_1^c, \hat{d}_1^c, \hat{e}_1^c$	$\hat{u}_2^c, \hat{d}_2^c, \hat{e}_2^c$	$\hat{u}_3^c, \hat{d}_3^c, \hat{e}_3^c$	$\hat{\phi}_{1,2}$
A_4	3	3	1	1'	1"	1
Z_3	1	1	ω^2	ω^2	ω^2	1

Then the following heavy quark, lepton, and Higgs **superfields are added**: which are all $SU(2)$

	$\hat{U}$	$\hat{U}^c$	$\hat{D}$	$\hat{D}^c$	$\hat{E}$	$\hat{E}^c$	$\hat{N}^c$	$\hat{\chi}$
A_4	3	3	3	3	3	3	3	3
Z_3	1	1	1	1	1	1	1	ω

singlets and with $\omega = \exp i\, 2\pi/3$.

The superpotential of the BMV model is then given by:

$$\begin{aligned}
\hat{W} = \quad & M_U \hat{U}_i \hat{U}_i^c + f_u \hat{Q}_i \hat{U}_i^c \hat{\phi}_2 + h_{ijk}^u \hat{U}_i \hat{u}_i^c \hat{\chi}_k + M_D \hat{D}_i \hat{D}_i^c + f_d \hat{Q}_i \hat{D}_i^c \hat{\phi}_2 + h_{ijk}^d \hat{D}_i \hat{d}_i^c \hat{\chi}_k \\
& + M_E \hat{E}_i \hat{E}_i^c + f_e \hat{L}_i \hat{E}_i^c \hat{\phi}_1 + h_{ijk}^e \hat{E}_i \hat{e}_j^c \hat{\chi}_k + \frac{1}{2} M_N \hat{N}_i^c \hat{N}_i^c + f_N \hat{L}_i \hat{N}_i^c \hat{\phi}_2 + \mu \hat{\phi}_1 \hat{\phi}_2 \\
& + \frac{1}{2} M_\chi \hat{\chi}_i \hat{\chi}_i + h_\chi \hat{\chi}_1 \hat{\chi}_2 \hat{\chi}_3
\end{aligned} \qquad (1)$$

The scalar potential involving χ_i is given by:

$$V = |M_\chi \chi_1 + h_\chi \chi_2 \chi_3|^2 + |M_\chi \chi_2 + h_\chi \chi_3 \chi_1|^2 + |M_\chi \chi_3 + h_\chi \chi_1 \chi_2|^2, \qquad (2)$$

which have the supersymmetric solution $(V = 0)$

$$\langle \chi_1 \rangle = \langle \chi_2 \rangle = \langle \chi_3 \rangle = u \qquad (3)$$

Considering now the Dirac mass matrix linking (e_i, E_i) to (e_j^c, E_j^c), we have:

$$\mathcal{M}_{eE} = \begin{pmatrix}
0 & 0 & 0 & f_e v_1 & 0 & 0 \\
0 & 0 & 0 & 0 & f_e v_1 & 0 \\
0 & 0 & 0 & 0 & 0 & f_e v_1 \\
h_1^e u & h_2^e u & h_3^e u & M_E & 0 & 0 \\
h_1^e u & h_2^e u\omega & h_3^e u\omega^2 & 0 & M_E & 0 \\
h_1^e u & h_2^e u\omega^2 & h_3^e u\omega & 0 & 0 & M_E
\end{pmatrix} \equiv \begin{pmatrix} 0 & X_1^I \\ X_2 & Y^I \end{pmatrix} \qquad (4)$$

where $v_1 = \langle \phi_1^0 \rangle$, with similar forms for the quark mass matrices. After block diagonalization of Eq. (4), the reduced 3×3 Dirac mass matrix for the charged leptons is diagonalized by the

magic matrix U_ω:

$$U_\omega = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} \tag{5}$$

where for $f_e v_1 \ll h_i\, u \ll M_E$ the charge lepton masses are:

$$\tilde{m}_i^2 \simeq \frac{3 f_e^2 v_1^2}{M_E^2} \frac{h_i^{e\,2} u^2}{1 + 3(h_i^e u)^2/M_E^2} \tag{6}$$

In the neutrino sector, after going to the basis where the charged leptons are diagonal, the Majorana mass matrix for spanning (ν_i, N_i^c) is given by:

$$\mathcal{M}_{\nu N} = \begin{pmatrix} 0 & f_N v_2\, U_\omega \\ f_N v_2\, U_\omega^T & M_N \end{pmatrix} \tag{7}$$

where $v_2 = \langle \phi_2^0 \rangle$. Hence, the see-saw mass matrix for ν_i becomes:

$$\mathcal{M}_\nu = \frac{f_N^2 v_2^2}{M_N} U_\omega^T U_\omega = m_o \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} = m_o\, \lambda \tag{8}$$

and the neutrino masses are degenerate at this stage.

Coming down to the electroweak scale, Eq. (8) is corrected by the wave function renormalizations of ν_i, as well as the corresponding vertex renormalizations [3]. Given the structure of the λ_{ij} at the high scale (Eq. (8)), its form at low scale is necessarily fixed to first order as:

$$\lambda = \begin{pmatrix} 1 + 2\delta_{ee} & \delta_{e\mu} + \delta_{e\tau} & \delta_{e\mu} + \delta_{e\tau} \\ \delta_{e\mu} + \delta_{e\tau} & 2\delta_{\mu\tau} & 1 + \delta_{\mu\mu} + \delta_{\tau\tau} \\ \delta_{e\mu} + \delta_{e\tau} & 1 + \delta_{\mu\mu} + \delta_{\tau\tau} & 2\delta_{\mu\tau} \end{pmatrix} \tag{9}$$

where all parameters are assumed to be real [3].

Rewriting Eq. (8) with $\delta_0 \equiv \delta_{\mu\mu} + \delta_{\tau\tau} - 2\delta_{\mu\tau}$, $\delta \equiv 2\delta_{\mu\tau}$, $\delta' \equiv \delta_{ee} - \delta_{\mu\mu}/2 - \delta_{\tau\tau}/2$, and $\delta'' \equiv \delta_{e\mu} + \delta_{e\tau}$. Then

$$\begin{pmatrix} 1 + \delta_0 + 2\delta + 2\delta' & \delta'' & \delta'' \\ \delta'' & \delta & 1 + \delta_0 + \delta \\ \delta'' & 1 + \delta_0 + \delta & \delta \end{pmatrix}, \tag{10}$$

and eigenvectors and eigenvalues are computed *exactly*. The effective neutrino mixing matrix is given by:

$$U_\nu(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta/\sqrt{2} & \cos\theta/\sqrt{2} & -1/\sqrt{2} \\ \sin\theta/\sqrt{2} & \cos\theta/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}, \tag{11}$$

with the eigenvalues:

$$\begin{aligned} \lambda_1 &= 1 + \delta_0 + 2\delta + \delta' - \sqrt{\delta'^2 + 2\delta''^2} \\ \lambda_2 &= 1 + \delta_0 + 2\delta + \delta' + \sqrt{\delta'^2 + 2\delta''^2} \\ \lambda_3 &= -1 - \delta_0 \end{aligned} \tag{12}$$

The neutrino predictions of the BMV model are:

$$\begin{aligned} \tan^2\theta_{12} &= \frac{\delta''^2}{\delta''^2 + \delta'^2 - \delta'\sqrt{\delta'^2 + 2\delta''^2}} \\ \sin^2\theta_{13} &= 0 \\ \tan^2\theta_{23} &= 1 \Rightarrow \text{maximal} \end{aligned} \tag{13}$$

for the mixing angles. With $\delta' < 0$ and $|\delta''/\delta'| = 1.7$ the BMV predicted the neutrino mixing before, Daya Bay [4] and RENO results [5] for θ_{13}. For the mass square differences, assuming $\delta', \delta'' \ll \delta$, we have:

$$\Delta m_{31}^2 \simeq \Delta m_{32}^2 \simeq 4\delta m_0^2$$
$$\Delta m_{21}^2 \simeq 4\sqrt{\delta'^2 + 2\delta''^2}\, m_0^2 \tag{14}$$

The radiative corrections are then responsible for the solar prediction and the splitting of the neutrino mass degeneracy.

The main goal of this letter is to modify the BMV to accommodate the current neutrino data [1]. In general, the mixing in the leptonic sector is given by:

$$K = (U_\omega)^\dagger U_\nu(\theta) = U_\nu(\theta) \tag{15}$$

when we go to the basis were the charged lepton mass matrix is diagonal. New predictions in the neutrino sector will result through modifications of the mixing in the charged sector $U_\omega^c = U_\omega . \hat{\delta}$, in such a way that:

$$K^c = \hat{\delta}^\dagger U_\nu(\theta), \tag{16}$$

where $\hat{\delta}$ matrix parameterize the modifications to the mixing respect to the BMV predictions.

3. Modifications to the BMV model

3.1. Unitarity Violation

Relaxing the condition used to obtained the charged lepton masses Eq. (6), i.e, allowing M_E scale (see Eq. (4)) to be at TeV will produce unitarity violation and therefore, next to the leading order terms in the diagonalization becomes important. The non unitarity effect will enhance the charge lepton flavor violating processes and we will have to carefully fulfill the current bounds.

We used the Schechter-Valle procedure [6] for the block diagonalization:

$$U = \mathcal{U} \cdot V = \exp(iH) \cdot V \qquad H = \begin{pmatrix} 0 & S \\ S^\dagger & 0 \end{pmatrix}, \qquad V = \begin{pmatrix} V_1 & 0 \\ 0 & V_2 \end{pmatrix}, \tag{17}$$

where H is an anti-hermitian operator and V_i are unitary matrices which diagonalized each block. The S matrix is determined at the first order form the diagonalization condition $U^\dagger M U = \text{Diag}\{m_i\}$ for a given hermitian matrix M.

For a general hermitian matrix M:

$$M = \begin{pmatrix} m_1 & m_2 \\ m_2^\dagger & m_3 \end{pmatrix} = \begin{pmatrix} (f_e v_1)^2\, I & M_E\, f_e v_1\, I \\ M_E\, f_e v_1\, I & U_\omega(\text{Diag}\{3(h_i^e u)^2\})U_\omega^\dagger + M_E^2\, I \end{pmatrix} \tag{18}$$

the S is given by:

$$i\, S = -m_2(m_1 - m_3)^{-1} = U_\omega \, \text{diag}\{-M_E\, f_e v_1[(f_e v_1)^2 - 3(h_i^e u)^2 - M_E^2]^{-1}\}\, U_\omega^\dagger \tag{19}$$

where the second terms in Eq (18) and Eq (19) correspond to our specific case.

To calculate the next to the leading order terms, we have to expand the exponential in Eq. (17) around S. The next to the leading order terms are combinations of the Identity and products of $S/, S^\dagger$ and S. Given the structure of the S matrix in Eq. (19) is clear that even if we go to higher orders in the expansion, the effective charged lepton mass will always be diagonalized by the magic matrix U_ω. The origin of the structure of the S matrix in Eq. (19) comes from the fact that in the BMV model, the matrices in the upper right corner and the lower right corner in Eq. (4) are proportional to the identity. At the end, allowing for unitarity violation in the charge sector does not changes the lepton mixing. Somehow a remnant symmetry of the A_4 ($\mu - \tau$ symmetry) is still present and therefore, there are no corrections or equivalently in Eq. 16 $\hat{\delta} = I$ and we end with the same predictions of the BMV model.

3.2. Adding a Scalar Singlet $1'$ *of* A_4

To break the remnant symmetry present in the charged sector, we add [7] an scalar singlet $1'$ of A_4 (flavon) ζ, end we still allow the M_E scale be decreased until the TeV. Therefore, we add the following term to the superpotential:

$$+\zeta_{ii} E_i E_i^c \qquad (20)$$

where we will parameterize the flavon scale as $\langle \zeta \rangle = \beta\, M_E$.

The new mass matrix for the lower right corner of Eq. (4) has now the structure:

$$\hat{Y}_D = M_E \times I + \beta M_E \times \mathrm{Diag}\{1,\omega,\omega^2\}, \qquad (21)$$

and the M matrix in Eq. (18) is therefore given by:

$$M = \begin{pmatrix} (f_e v_1)^2\, I & f_e v_1\, \hat{Y}_D^\dagger \\ f_e v_1\, \hat{Y}_D & U_\omega(\mathrm{Diag}\{3(h_i^e u)^2\})U_\omega^\dagger + \hat{Y}_D \hat{Y}_D^\dagger \end{pmatrix} \qquad (22)$$

where $\hat{Y}_D$ can not be diagonalized by the magic matrix, breaking the remnant symmetry, and changing structure of the S matrix in Eq. (19). As we will see in the next section, the total effect will be a change in the lepton mixing.

4. Numerical results

Using the modifications to the BMV model explained in section 3.2, we numerically diagonalized the charge mixing matrix in Eq. (22) therefore, including all the higher order contributions to the effective charged lepton mass.

After fitting the three lepton masses, we finally calculated the leptonic mixing in Eq. (15) and then re-calculated the new predictions for the neutrino mixing:

$$\tan\theta_{12} = |K_{1,2}^c(\theta)|/|K_{1,1}^c(\theta)|$$
$$\sin\theta_{13} = |K_{1,3}^c(\theta)| \qquad (23)$$
$$\tan\theta_{23} = |K_{2,3}^c(\theta)|/|K_{3,3}^c(\theta)|$$

where the free parameter θ have been varied randomly as the scales M_E and $f_e v_1$.

The results are presented in Fig. 1. As we can see from the right plot of Fig. 1, there is a correlation between the magnitude and the phase of the β parameter. In order to generate points compatible with the current neutrino values for the θ_{13} and θ_{23} mixing angles (compatible points), β must be complex and different from zero.

We have generated not only θ_{13} different from zero, but also a non-maximal θ_{23}. The region for those compatible points is bigger for θ_{23} in the second octant than for the first one, as is shown in the left plot of Fig. 1.

5. Conclusions

We have showed that decreasing the M_E scale to the TeV in the BMV model and therefore adding NLO terms in the diagonalization of charged sector does not change the lepton mixing pattern.

To break the remnant symmetry we have introduced a scalar singlet keeping the M_E scale to the TeV. The final effect was a good description of the current neutrino mixing angles at $3\,\sigma$ for some values of θ and β. We still have to check if the corrections in the neutrino sector (δ' and δ'') can be obtained in some specific model.

Our results are preliminary. We still have to check that our results are compatible with lepton flavor violating bounds [7].
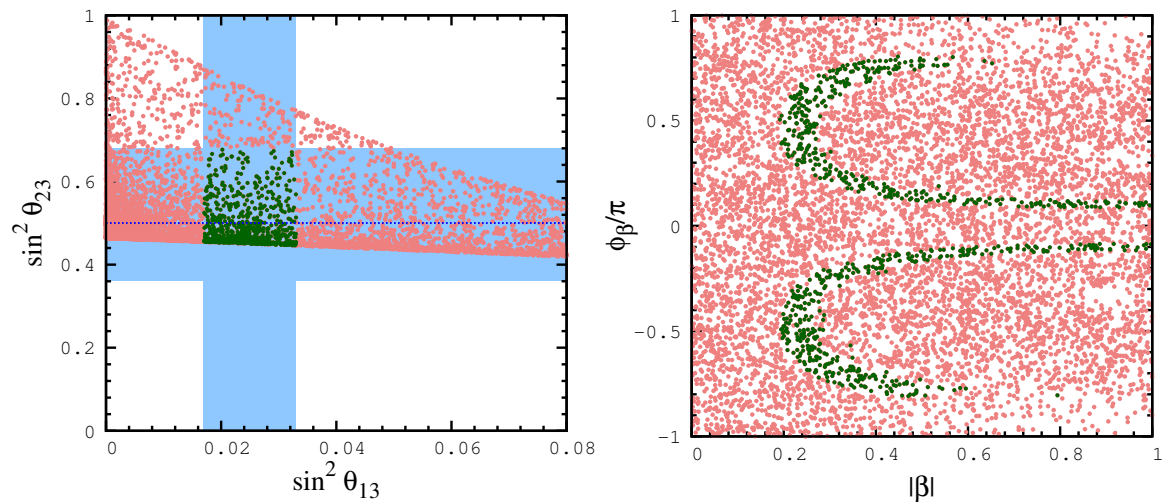
Figure 1. Vertical(horizontal) bands are the current values for $\sin^2 \theta_{13}$ ($\sin^2 \theta_{23}$) at $3\,\sigma$. The horizontal dotted line represents maximal θ_{23}. All points are compatible with the current $3\,\sigma$ range of the solar angle θ_{12}, but only the green ones are compatible with the θ_{13} and θ_{23} range at $3\,\sigma$. (Left) Prediction of the model according to values for β parameter.(Right) Random values for β parameter, phase ϕ_β against magnitude $|\beta|$.

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