LANDAU-POMERANCHUK-MIGDAL EFFECT
THEORY AND EXPERIMENT

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Abstract

On donne une explication théorique d'une conduite «inattendu» d'un spectre d'irradiation d'électrons dans un fin couche de substance, qui a été observé dans la récente expérience dans SLAC (E-146), en étudiant l'effet de Landau-Pomerantchuk-Migdal.

1. Introduction. The radiation process of high energy electrons develops in large spatial area along particle momentum, which has the name of coherent length or formation zone of radiation process [1,2]. If in limits of this area electron collides with large number of atoms, the interaction with them can occur not the same way as with separate atoms, carried far apart.

The Landau and Pomeranchuk have shown [3], that in amorphous medium at rather large energies multiple scattering of electrons leads to reduction of spectral density of radiation in the field of small frequencies in comparison with Bethe and Heitler's result [4].

The quantitative theory of multiple scattering effect on electron bremsstrahlung in amorphous medium was advanced by Migdal [5] on the basis of the kinetic equation method. Now the effect of influence of multiple scattering on electron bremsstrahlung in amorphous medium is called the Landau-Pomeranchuk-Migdal effect (the LPM effect) [6-12].

The detailed experimental study of the LPM effect was carried out only recently on the SLAC accelerator at electron energies up to 25 GeV [6]. In this experiment an electron radiation spectrum in the region of photon energies from a parts of MeV up to several hundred of MeV was investigated. The measurements are carried out for several elements from carbon up to uranium. The thickness of targets varied from a parts of percent up to several percents of radiation length. The comparison of obtained data with results of accounts, carried out with the help of the Monte-Carlo method by the Migdal formula, has shown their good accordance for rather thick targets. For relatively thin targets in some cases a significant divergence with predictions of the Migdal theory was observed. Especially significant this divergence was fixed for gold target by thickness 0.7 % of radiation length in the photon energies region of smaller of 30 MeV.

The experiment [6] has caused significant interest and stimulated development of the new approaches to research of the LPM effect [7-11]. The interest to the given problem is also connected with searches of analogues of the LPM effect in quantum chromodynamics and in other fields of physics [7-14].

In the present work the analysis of experimental data marked above which is not concordance with predictions of Migdal's theory of the LPM effect will be carried out. We show, that the rejection from predictions of the Migdal theory, observed in [6], takes place in a case, when the coherent length of radiation process is great in comparison with target thickness ($l_c >> L$), that is when applicability conditions of the basic results of the Migdal theory are not carried out. The case $l_c >> L$ was considered by us earlier in Ref. [15], where it is shown, that the radiation spectrum of electrons in a thin target can essentially differ from a spectrum, determined both the Migdal formula and the Bethe and Heitler formula. In [15] the asymptotic formulas for radiation spectrum of ultra high energy electrons, when the effect is significant, were also obtained. (Similar asymptotic formulas were obtained also in Ref. [16] on the basis of the kinetic equation method). However, direct application of these asymptotic formulas to the analysis of experimental data [6] leads to absurd results, as at the SLAC energies it is not yet enough to use asymptotic formulas from [15,16]. Therefore for the analysis of experimental data [6] development of the quantitative theory of radiation process of relativistic electrons in a thin layer of amorphous substance is required. The present work is devoted to this problem.

2. LPM effect. Spectral density of electron radiation, driven in external field on trajectory $\vec{r}(t)$, is determined in classical electrodynamics by the formula [1,2]

$$d\varepsilon/\,d\omega = (e^2/4\pi^2)\int do\left|\vec{k}\times\vec{I}\right|^2, \quad \text{where} \quad \vec{I} = \int_{-\infty}^{\infty} dt\,\vec{v}(t)e^{i(\omega t-\vec{k}\vec{r}(t))}, \qquad (1)$$

$\vec{k}$ and ω are wave vector and frequency of radiated wave, $\left|\vec{k}\right| = \omega$, do is element of solid angle in direction of radiation.

In amorphous medium the trajectory of a particle is random due to multiple scattering by atoms, therefore the formula (1) should be averaged out from various electron trajectories. On the basis of qualitative estimations Landau and Pomeranchuk have shown [3], that if meansquare angle of multiple scattering of electron in the frame of a coherent length $l_c = 2\gamma^2/\omega$ (where γ is the Lorenz-factor of an electron) exceeds characteristic angle of relativistic electron radiation $\vartheta_\gamma \sim \gamma^{-1}$, then electron bremsstrahlung in substance will be suppressed in comparison with appropriate result of the Bethe and Heitler theory [4].

The first quantitative results, relating to this effect, were obtained by Migdal on the basis of average procedure offered by him for the formula (1), using the kinetic equation for distribution function of particles in matter on scattering angles. He managed to carry out the average procedure for boundless amorphous medium in a small scattering angles approximation, when the process of multiple scattering can be considered as the Gaussian process. Thus it was shown [5], that

$$\langle d\varepsilon/d\omega\rangle_{LPM} = (2e^2/3\pi)\cdot\gamma^2 qL\cdot\Phi_M(s), \qquad (2)$$

where $s = (1/4\gamma^2)\sqrt{\omega/q}$, q is average square angle of multiple scattering per unit of length and $\Phi_M(s) = 24s^2(\int_0^{\infty} dx\,cthxe^{-2sx}\sin 2sx - \pi/4)$ is Migdal's function.

The value s^2 represents the relation of a square of characteristic angle of relativistic electron radiation $\vartheta_\gamma^2 \sim \gamma^{-2}$ to average square angle of electron multiple scattering in the frame of coherent length $\overline{\vartheta}^2 = q\cdot l_c$.

At $s > 1$ formula (2) coincides with logarithmic accuracy with the appropriate result of Bethe and Heitler

$$\langle d\varepsilon/d\omega\rangle_{BH} = 4L/3L_R, \qquad (3)$$

where L_R is radiation length.

If $s \ll 1$, according to (2), we have reduction effect of radiation spectral density of electron in substance in comparison with the appropriate Bethe and Heitler's result $\langle d\varepsilon/d\omega\rangle_{LPM} \ll \langle d\varepsilon/d\omega\rangle_{BH}$ (see Fig. 1). This effect now has the name of the LPM effect.

The formula (2) is fair for boundless medium. More precisely, it is required, that target thickness L should be great than length, at which radiation process develops: $L \gg l_c$.

The formula (2) is obtained with logarithmic accuracy. Such accuracy is caused by the fact that at deriving (2) it was supposed, that q is a constant and does not depend on length L, at which there is scattering. On small sites of a way the value q logarithmically depends on L .

We shall notice, that formula (2) can be also obtained on the basis of the functional integration method [17].

3. Radiation in a thin layer of substance. The stated above theory of LPM effect is fair, if a condition L >> l_c is carried out. We shall consider now opposite case, when the process of radiation develops on length of a target considerably superior thickness (l_c >> L). By effect of medium polarization on radiation and transition radiation we shall neglect, that it is fair, if $\omega \gg \gamma\omega_p$, where ω_p is the plasma frequency of radiator.

In Refs.[15] it was shown, that spectral density of relativistic electron radiation in a thin target (L << l_c) has follows form

$$\frac{d\varepsilon}{d\omega} = \frac{2e^2}{\pi}\left[\frac{2\xi^2+1}{\xi\sqrt{\xi^2+1}}\ln(\xi+\sqrt{\xi^2+1})-1\right], \tag{4}$$

where $\xi = \gamma\vartheta/2$ and ϑ is scattering angle of electron by target ($\vartheta \ll 1$).

The scattering angles $\vec{\vartheta}$ for various particles in target are different, therefore the formula (4) should be averaged out from their angles

$$\langle d\varepsilon / d\omega\rangle = \int d^2\vartheta \cdot f(\vec{\vartheta})\cdot(d\varepsilon\{\vec{\vartheta}\}/d\omega). \tag{5}$$

It is possible to establish general regularity of particle radiation in thin targets, caused by asymptotes of (4). Namely, replacing in this formula ϑ^2 by average square angle of particle scattering by a target $\overline{\vartheta^2}$, we come to following estimations for average meaning of radiation spectral density

$$\left\langle\frac{d\varepsilon}{d\omega}\right\rangle \sim \frac{2e^2}{3\pi}\begin{cases}\gamma^2\overline{\vartheta^2} & \gamma^2\overline{\vartheta^2} \ll 1\\ 3[\ln\gamma^2\overline{\vartheta^2}-1] & \gamma^2\overline{\vartheta^2} \gg 1\end{cases} \tag{6}$$

Eqs. (6) show, that if the value $\overline{\vartheta^2}$ grows with target thickness (in amorphous medium $\overline{\vartheta^2}$ ~ L), at $\gamma^2\overline{\vartheta^2} \sim 1$ there is a change of character of particle radiation in substance. Thus the linear dependence of radiation spectrum from $\overline{\vartheta^2}$ (or from L) is replaced by weaker logarithmic dependence. The condition $\gamma^2\overline{\vartheta^2} \sim 1$, at which there is the change of character of particle radiation in a thin layer of substance, has the same form, as condition of occurrence of the LPM effect (the suppression of radiation in a thick target). At the same time, the formulas, describing influence of multiple scattering on radiation in thick and thin targets are rather different. So for the LPM effect the radiation spectrum depends on frequency of a radiated photon, whereas the formulas (6) don't contain dependence from ω (see also Fig.1).

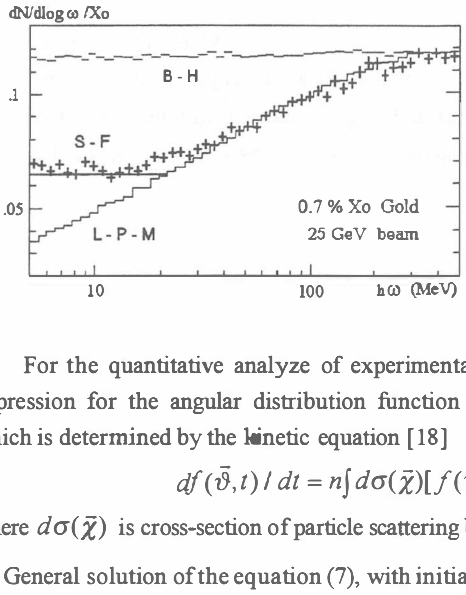

Fig. 1. Spectral density of the radiation from 25 GeV electrons in a gold target with a thickness of 0.7% of the radiation length. The crosses represent the experimental data [6]; the L-P-M and B-H histograms represent the Monte Carlo calculations based on the Landau-Pomeranchuk-Migdal and Bethe-Heitler theories presented in Ref. [6]; the solid line S-F is the result of our calculations.

For the quantitative analyze of experimental data [6] it is necessary to know exact expression for the angular distribution function of fast particles in amorphous medium, which is determined by the kinetic equation [18]

$$df(\vec{\vartheta},t)/dt = n\int d\sigma(\vec{\chi})[f(\vec{\vartheta}-\vec{\chi},t) - f(\vec{\vartheta},t)], \qquad (7)$$

where $d\sigma(\vec{\chi})$ is cross-section of particle scattering by separate atom of medium for angle $\vec{\chi}$.

General solution of the equation (7), with initial condition $f(\vec{\vartheta},0) = \delta(\vec{\vartheta})$, is

$$f(\vec{\vartheta},t) = (1/4\pi)\cdot\int d^2\eta \exp\{i\vec{\vartheta}\vec{\eta} - nt\int d\sigma(\vec{\chi})(1-e^{i\vec{\chi}\vec{\eta}})\}. \qquad (8)$$

This formula for $f(\vec{\vartheta},t)$ describes both single and multiple scattering of particle in a medium. At $t \to 0$, that is when we deal only with a single scattering, in (8) it is possible to carry out expansion over t. In the first approximation of it we find $f(\vec{\vartheta}) = ntd\sigma(\vec{\vartheta})$. Substituting this distribution function to (5), after integration over scattering angle ϑ we shall obtain radiation spectrum, that coincides with corresponding result of Bethe and Heitler (3).

At t increasing the multiple scattering of particle on various atoms of matter becomes essential. In general distribution function $f(\vec{\vartheta},t)$ can be found by numerical integration of expressions (8). The results of these calculations immediately for electron radiation spectrum, determining by formulas (4) and (5), and corresponding to the conditions of experiment [6] are presented as solid curve S-F on Fig.1. They demonstrate a good agreement with the experimental data.

4. Conclusion. Thus, in experiment [6] alongside with confirmation of Landau-Pomeranchuk-Migdal effect (see Fig.1, 20MeV<ω<300Mev), the confirmation of suppression effect of electron bremsstrahlung in a thin layer of substance, predicted in Ref. [14], is also given (see Fig.1, ω<20Mev).

The condition $l_c \approx L$ determines the boundary between two different regimes of suppression of high energy electron radiation in a «thick» (LPM effect) or in a «thin» target. For 25 GeV electrons and for 0.7 % of the radiation length gold target it means $\omega_b \approx 20\,MeV$. The area of photon energies and target thickness, in which stated effect is fair, increase rapidly with electron energy increasing. So, for 250 GeV electrons there will be $\omega_b \approx 2GeV$ for the same target.

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