General Relativity and Gravitation (2024) 56:105
https://doi.org/10.1007/s10714-024-03293-x

REVIEW

Cosmic strings and gravitational waves

Lara Sousa[1,2]

Received: 18 July 2024 / Accepted: 26 August 2024
© The Author(s) 2024

Abstract

Cosmic string networks are expected to generate a characteristic stochastic gravitational wave background that may be within the reach of current and upcoming gravitational wave detectors. A detection of this spectrum would provide invaluable information about the physics of the early universe, as it would allow us to probe the sequence of phase transitions that happened in the distant past. Here, I review the emission of gravitational waves by Nambu–Goto cosmic strings—thin cosmic strings that couple strongly to gravity only—and by superconducting strings—strings that carry electromagnetic currents. A comparison between the stochastic gravitational wave background predicted in these two very distinct string-forming scenarios reveals that this spectrum may have signatures that may allows us to discriminate between them observationally. The stochastic gravitational wave background generated by cosmic string networks may then enable us to uncover not only the energy-scale of the string-forming phase transition, but the underlying particle physics scenario as well.

Keywords Cosmic string · Topological defects · Gravitational waves · Early universe

Contents

✉ Lara Sousa
lara.sousa@astro.up.pt

[1] Instituto de Astrofísica e Ciências do Espaço, Centro de Astrofísica da Universidade do Porto, Rua das Estrelas, 4150-762 Porto, Portugal

[2] Departamento de Física e Astronomia, Faculdade de Ciências, Rua do Campo Alegre 687, 4169-007 Porto, Portugal

Published online: 14 September 2024

1 Introduction

A plethora of particles physics scenarios—including supersymmetric grand unified theories, hybrid and brane inflation, the seesaw mechanism for neutrino mass generation, axion models, and many others [1–11] (see also [12, 13] for a review)—predict the production of line-like topological defects known as cosmic strings as remnants of symmetry-breaking phase transitions in the early universe. These cosmic strings are expected to form networks and are generally expected to survive throughout cosmic history, while preserving the original unbroken symmetries in their cores.

The mass per unit length of cosmic strings, μ, is determined by the energy scale η of the phase transition that originates them: $G\mu \sim \eta^2$, where G is the gravitational constant. This quantity determines the strength of cosmic string gravitational interactions and, in the simplest string models, μ is also a measure of cosmic string tension. A detection of cosmic strings would then help us pinpoint the energy scale of the string forming phase transition and to reconstruct the series of phase transitions that happened in the early universe, enabling us then to use the universe as a particle physics laboratory.

Although cosmic strings have evaded detection in standard observational probes [13], with the onset of Gravitational Wave Astronomy, we may be about to detect them for the very first time through their stochastic gravitational wave background. Compellingly, pulsar timing arrays recently announced the detection of a stochastic gravitational wave background in the nanohertz frequency band [14–17] and cosmic strings were identified as potential contributors to the signal [18, 19] (although supermassive black hole binaries remain the most likely explanation). The upcoming years will bring increases in the sensitivity in this frequency range, with the Square Kilometer Array [20], and a promising new band to explore, once the Laser Interferometer Space Antenna (LISA) [21] starts to operate. Understanding the stochastic gravitational wave background generated by different cosmic strings models is then essential to explore the upcoming gravitational wave data to its full potential as a probe of the physics of the early universe.

Cosmic strings are expected to frequently collide and self-intersect as they evolve and, when this happens, closed (sub-horizon) loops of string may form. These loops then detach from the Hubble flow and start to evolve under the effect of their tension, oscillating periodically with relativistic speeds. They are expected to decay by emitting gravitational radiation, progressively shrinking until they eventually evaporate. The frequency of the gravitational waves emitted by a cosmic string loop, as measured by an observer at the present time t_0, is determined by harmonics of the length of the loop ℓ at the time of emission t:

$$f = \frac{2j}{\ell(t)}\frac{a(t)}{a_0}, \quad (1)$$

where j is the harmonic mode of emission, $a(t)$ is the cosmological scale factor, $a_0 = a(t_0)$, $\ell = E_\ell/\mu$, and E_ℓ is the energy of the loop. These loops, then, generate transient gravitational wave signals and, at any moment in cosmic history, many such loops, emitting gravitational waves in different directions, should exist. The superposition of their emissions is therefore expected to give rise to a stochastic gravitational wave background [22, 23].

The stochastic gravitational wave background generated by cosmic string loops has, at any given frequency f, contributions from all loops that have emitted, at any moment, gravitational waves that have a frequency f at the present time. The spectral density of the stochastic gravitational wave background generated by cosmic strings, in units of critical density ρ_c, may then be expressed as follows [13, 24–26]:

$$\Omega_{\rm gw}(f) \equiv \frac{1}{\rho_c}\frac{d\rho_{\rm gw}}{d\log f} = \frac{16\pi}{3}\left(\frac{G\mu}{H_0}\right)^2 \frac{1}{f}\int_{t_i}^{t_0} dt' \sum_{j=1}^{+\infty} j\Gamma_j n(\ell_j(t'), t')\left(\frac{a(t')}{a_0}\right)^5, \tag{2}$$

where $\rho_c = 3H_0^2/(8\pi G)$, H_0 is the Hubble constant, $\rho_{\rm gw}$ is the gravitational radiation energy density and t_i is the time at which significant gravitational wave emission by cosmic string loops begins.[1] Here, $\ell_j \equiv (2ja(t')/(fa(t_0))$ is the length that a loop should have at a time t' to emit, in the j-th harmonic mode of emission, gravitational waves that have a frequency f at $t = t_0$ and Γ_j is the power emitted in the j-th harmonic mode (in $G\mu^2$ units). Moreover, $n(\ell, t)d\ell$ represents the number density of loops with lengths between ℓ and $\ell + d\ell$ that exist at a time t. To compute the stochastic gravitational wave background generated by cosmic string loops one then needs not only to characterize the number of loops that are created throughout cosmic history, but also to know how their length evolves as they decay as a result of gravitational wave emission. In this article, I review the characterization of these two key ingredients for the computation of the stochastic gravitational wave background—the loop number density $n(\ell, t)$ and the spectrum of emission Γ_j—and the main properties of the background generated by Nambu–Goto strings. I will briefly discuss the case superconducting strings as well, to illustrate the potential of using this background to help discriminate observationally between different string-forming scenarios.

This paper is organized as follows. In Sect. 2, the emission of gravitational waves by Nambu–Goto strings is reviewed. I start by outlining the characterization of the gravitational wave bursts emitted by cosmic string loops in Sect. 2.1, then review the computation of the loop number density in Sect. 2.2 and summarize the properties of the stochastic gravitational wave background in Sect. 2.3. In Sect. 3, I review the case of superconducting strings, discussing the impact of current on the gravitational wave emission of loops (3.1), on loop production (3.2) and on the stochastic gravitational wave background (3.3). I then conclude in Sect. 4.

Throughout this article, greek indices run over spacetime coordinates (from 0 to 3). Latin indices i, k, l, m run over space dimensions, while $a, b = 0, 1$ run through worldsheet coordinates. Natural units, with $c = \hbar = 1$—where c is the speed of

[1] In the early universe, cosmic strings interact frequently with the particles of the surrounding plasma (see e.g. [12, 13]), which results in a frictional force that damps the strings. In the literature, although there is loop production during this friction-dominated era, it is generally assumed that significant gravitational wave emission only starts after the universe has rarefied enough for friction to become irrelevant for cosmic strings dynamics. It was, however, recently shown in [27] that loops created during the friction-dominated era may give rise to a prominent signature in ultra-high-frequency range of the stochastic gravitational wave background.

light in the vacuum and $\hbar$ is the reduced Planck constant—are used throughout. The subscript '0' is used to indicate that we are considering the values of the corresponding quantities at the present time. Moreover, the cosmological parameters are determined by Planck 2018 data [28]: the density parameters for radiation, matter and for the cosmological constant at the present time are respectively given by $\Omega_{\rm r} = 9.1476 \cdot 10^{-5}$, $\Omega_{\rm m} = 0.308$, $\Omega_\Lambda = 1 - \Omega_{\rm r} - \Omega_{\rm m}$ and the Hubble constant by $H_0 = 2.13 \cdot h \cdot 10^{-33}$ eV, with $h = 0.678$.

2 Nambu–Goto strings and gravitational waves

Cosmic strings are vortex-like field configurations and their properties generally depend on the nature of the fields that constitute the strings. However, in many situations of interest in cosmology, the energy of the strings is concentrated in a thin region of space and their dynamics and evolution is expected to be mostly independent from what happens within their cores. In this case, the emission of gravitational radiation is expected to be the main decay mechanism for cosmic strings [29] and they may be regarded as infinitely-thin and featureless objects. These strings describe, in their movement, a $1 + 1$ dimensional worldsheet in spacetime, represented by:

$$X^\mu = X^\mu(\sigma^0, \sigma^1), \tag{3}$$

where σ^0 and σ^1 are respectively a timelike and spacelike parameter. Their dynamics may then be described by the Nambu–Goto action:

$$S = -\mu \int \sqrt{-\gamma} d^2\sigma, \tag{4}$$

where $\gamma = \det(\gamma_{ab})$, $\gamma_{ab} = g_{\mu\nu} x^\mu_{,a} x^\nu_{,b}$ is the worldsheet metric, $g_{\mu\nu}$ the spacetime metric, and $_{,a}$ represents a derivative with respect to σ^a. By varying this action with respect to X^μ, one finds that the equation of motion for cosmic strings is of the form:

$$\left(\sqrt{-\gamma}\gamma^{ab} X^\mu_{,b}\right)_{,a} + \sqrt{-\gamma}\, \Gamma^\mu_{\lambda\nu} X^\lambda_{,a} X^{\nu,a} = 0. \tag{5}$$

2.1 Gravitational wave emission by cosmic string loops

Cosmic string loops, after creation, detach from the Hubble flow and we may therefore neglect the impact of the expansion of the background on their dynamics. To study their gravitational wave emission, one may therefore consider a Minkowski background and, in this case, the second term in Eq. (5) vanishes. Since the Nambu–Goto action is invariant under worldsheet reparametrization, one may choose the timelike worldsheet coordinate to coincide with physical time, so that $\sigma^0 \equiv t$ and

$$X^\mu = [t, \mathbf{x}(t, \sigma)], \tag{6}$$

where $\sigma \equiv \sigma^1$ and $\mathbf{x}$ is a 3-vector that represents the trajectory of the string. Moreover, it is also convenient to choose the transverse gauge, in which

$$\dot{\mathbf{x}} \cdot \mathbf{x}' = 0, \tag{7}$$

—here and for the remainder of this section a dot and a prime represent respectively a derivative with respect to t and σ—and to impose a conformal gauge condition:

$$\dot{\mathbf{x}}^2 + \mathbf{x}'^2 = 1. \tag{8}$$

With these choices, $\dot{\mathbf{x}}$ and $\mathbf{x}'$ have a clear physical meaning [13]: $\dot{\mathbf{x}}$ is perpendicular to the string and then represents its observable velocity and σ may be regarded as a measure of the energy of the string. The equation of motion for the strings then reduces to a wave equation:

$$\ddot{\mathbf{x}} - \mathbf{x}'' = 0\,, \tag{9}$$

and the acceleration of a string at a given point is proportional to its curvature. Strings, due to their tension, have then a tendency to straighten.

Closed loops of string describe periodical trajectories with a period $T = \ell/2$, where ℓ is the length of the loop. General loop solutions in Minkowski space may then be expressed in terms of left- and right-moving functions $\mathbf{x}_+$ and $\mathbf{x}_-$ as follows

$$\mathbf{x} = \frac{T}{2\pi}\left[\mathbf{x}_+(\sigma_+) + \mathbf{x}_-(\sigma_-)\right], \tag{10}$$

where $\sigma_\pm = (\sigma \pm t)\pi/T$. Note that left- and right-movers are also periodical and, as a result of the gauge conditions, satisfy $\left|\mathbf{x}'_\pm\right| = 1$ (note that, when the subscript $+$ or $-$ is also present, a prime represents a derivative with respect to σ_+ or σ_-, respectively). This latter aspect has interesting consequences for cosmic string loops: $\mathbf{x}'_+$ and $-\mathbf{x}'_-$ may be regarded as curves on a sphere of radius unity with centroids at the sphere's center. So, quite generally, these curves should be expected to intersect. When this happens $\mathbf{x}'_+ = -\mathbf{x}'_-$ and therefore $\dot{\mathbf{x}}^2 = 1$ and $\mathbf{x}' = 0$. In these points [30], which are known as cusps, strings curl upon themselves and move instantaneously at the speed of light.

The existence of discontinuities in the string tangent, known as kinks, may however prevent the formation of cusps (but not necessarily). These kinks are naturally created as a result of string collisions and intercommutation—the same phenomena that leads to the creation of loops—and therefore they are expected to also be a general feature of cosmic string loops. Kinks are generally created in pairs and travel along the string in opposite directions at the speed of light.

Cusps and kinks on oscillating cosmic string loops are both expected to generate strong bursts of gravitational radiation. The energy-momentum tensor of a cosmic string may be found by varying the action in Eq. (4) with respect to the metric:

$$T^{\mu\nu} = \mu \int \left(\dot{X}^\mu \dot{X}^\nu - X'^\mu X'^\nu\right) \delta^{(4)}[x^\lambda - X^\lambda(t,\sigma)]dtd\sigma. \tag{11}$$

The average Fourier transform of the stress-energy tensor over one period of oscillation may be expressed as [31]

$$\tilde{T}^{\mu\nu}(\omega, \mathbf{k}) = \frac{1}{T}\int T^{\mu\nu} e^{-ik_\nu X^\nu} d^4x = \frac{\mu}{2T} I_+^{(\mu} I_-^{\nu)}, \tag{12}$$

where

$$I_\pm^\mu = \frac{T}{\pi}\int_0^{2\pi} X_\pm'^\mu e^{-i\frac{T}{2\pi}k_\nu X_\pm^\nu} d\sigma_\pm, \tag{13}$$

$X_\pm^\mu = [\sigma_\pm, \mathbf{x}_\pm]$, $k^\mu = \omega_j n_{(1)}^\mu$, $\omega_j = 2\pi j/T$ is the angular frequency of the j-th harmonic mode of emission, $n_{(1)}^\mu$ is the unit vector along the direction connecting the source to the observer and $I_+^{(\mu} I_-^{\nu)} = \frac{1}{2}\left(I_+^\mu I_-^\nu + I_-^\mu I_+^\nu\right)$.

The power emitted in the form of gravitational radiation is then given by [32, 33]:

$$P = \Gamma G\mu^2, \quad \text{with} \quad \Gamma \equiv \sum_{j=0}^{+\infty} \Gamma_j = \sum_{j=0}^{+\infty} \int \frac{\omega_j^2}{2^3\pi T^2}\left[\left|I_j^{22} - I_j^{33}\right|^2 + \left|I_j^{23} + I_j^{32}\right|^2\right] d\Omega, \tag{14}$$

where the subscript 'j' is used to indicate that we are considering the contribution of the j-th mode to the corresponding variable, Ω represents the solid angle and we have introduced:

$$I_j^{ik} = I_+^l I_-^m n_{(i)l} n_{(k)m}, \tag{15}$$

where $n_{(i)}^\mu = [1, \mathbf{n}_{(i)}]$ and $\mathbf{n}_{(1)}, \mathbf{n}_{(2)}, \mathbf{n}_{(3)}$ are unit vectors that form the “co-rotating" orthogonal frame (see e.g. [32, 34] for more details). Here, Γ is the total gravitational wave power emitted by the loops in units of $G\mu^2$ and is generally referred to as gravitational wave emission efficiency.

The gravitational wave emission efficiency and the spectrum of emission has been extensively studied in the literature, using a variety of analytical and numerical techniques, for different loop solutions [31, 33, 35–41]. This generally involves computing the integrals in Eq. (13) for a particular loop solution. By looking at the form of these integrals, one may see that, quite generally, they are exponentially suppressed for high harmonic modes and so no significant gravitational wave emission should be expected at high frequencies. However, this is not the case at cusps—as the phase of the integral has a saddle point—or when $X_\pm'^\mu$ is discontinuous, as in kinks [31]. Cusps and kinks should then dominate the gravitational wave emission indeed.

Cusps emit beamed gravitational wave bursts, with an opening angle of $\theta \sim j^{-1/3}$, along the direction of the cusps [31] and their spectrum of emission follows a power law of the form $\Gamma_j \sim j^{-4/3}$ [36, 37]. A kink, on the other hand, emits a 1-dimensional fan-like burst as it travels along the string, also with an opening angle $\theta \sim j^{-1/3}$, but its power spectrum has a different spectral index: $\Gamma_j \sim j^{-5/3}$ [38]. Finally, since loops should generally have kinks propagating in both directions, they may also collide. This gives rise to an isotropic gravitational wave burst with $\Gamma_j \sim j^{-2}$ [40]. We may then express the spectrum of emission of cosmic string loops as:

$$\Gamma_j = \frac{\Gamma}{\zeta(q)} j^{-q}, \tag{16}$$

with $\Gamma = 50$, as suggested by most studies, and $q = 4/3, 5/3, 2$, if the emission is dominated by cusps, kinks and kink-kink collisions respectively, and where ζ represents the Riemann Zeta-function.

2.2 Cosmic string network evolution and loop production

Cosmic string loops are created dynamically in the evolution of a cosmic string network, as a result of the frequent collisions between long strings and of string self-intersections. Understanding defect network dynamics is then essential to determine the number of loops that are created throughout cosmic history and that may contribute to the stochastic gravitational wave background.

When modelling the cosmological evolution of a cosmic string network, the impact of expansion can no longer be neglected and, therefore, one needs to consider Eq. (5) in a Friedmann–Lemaitre–Robertson–Walker (FLRW) background. Note however that, in this case, the conformal and temporal gauge conditions cannot be simultaneously imposed and it is instead convenient to choose temporal-transverse gauge conditions, with

$$\sigma^0 \equiv \eta \quad \text{and} \quad \dot{\mathbf{x}} \cdot \mathbf{x}' = 0, \tag{17}$$

where dots now represent a derivative with respect to conformal time η. With this choice, $\dot{\mathbf{x}}$ remains perpendicular to the string tangent $\mathbf{x}'$ and may then still be interpreted as the velocity of the string. In this background, the equations of motion for the cosmic string assume the form [30]:

$$\ddot{\mathbf{x}} + 2\frac{\dot{a}}{a}\left(1 - \dot{\mathbf{x}}^2\right)\dot{\mathbf{x}} = \epsilon^{-1}\left(\epsilon^{-1}\mathbf{x}'\right)', \tag{18}$$

$$\dot{\epsilon} + 2\frac{\dot{a}}{a}\dot{\mathbf{x}}^2\epsilon = 0, \tag{19}$$

where $\epsilon^2 = \mathbf{x}'^2/(1 - \dot{\mathbf{x}}^2)$.

In a FLRW universe, a cosmic string network may be assumed to be statistically homogeneous and isotropic on sufficiently large scales. Their cosmological evolution may then be described thermodynamically through two macroscopic variables: the Root-Mean-Squared (RMS) velocity, defined as

$$\bar{v}^2 \equiv \langle \dot{\mathbf{x}}^2 \rangle = \frac{\int \dot{\mathbf{x}}^2 \epsilon d\sigma}{\int \epsilon d\sigma}, \tag{20}$$

where $\langle\cdots\rangle$ denotes an average over the whole network, and the characteristic length

$$L \equiv \left(\frac{\mu}{\rho}\right)^{1/2}, \tag{21}$$

which is a measure of the energy density of the network ρ and, within the Nambu–Goto approximation, is also a measure of the average distance between strings.

By averaging Eq. (18), using Eqs. (20) and (21) and assuming that $\langle v^4\rangle = \langle v^2\rangle^2$, we find the following evolution equation for $\bar{v}$:

$$\frac{d\bar{v}}{dt} = (1-\bar{v}^2)\left[\frac{k(\bar{v})}{L} - 2H\bar{v}\right], \tag{22}$$

where

$$k(\bar{v}) \equiv \frac{(1-v^2)v}{\langle(1-\dot{\mathbf{x}}^2)\rangle(\dot{\mathbf{x}}\cdot\hat{\mathbf{u}})} \tag{23}$$

(with $\hat{\mathbf{u}}$ being a unit vector with the direction of the string curvature) is a measure of the average curvature of cosmic strings (here, we will use the *ansatz* proposed in [42]).

The energy density of the cosmic string network, in an expanding universe, should satisfy $\rho \propto E/a^3$, where $E = \mu a(\eta)\int \epsilon d\sigma$ is the total cosmic string energy. By differentiating this expression with respect to time and using Eqs. (19) and (21), one may also obtain an evolution equation for L. It is important to note, however, that the Nambu–Goto action does not describe cosmic string interactions and, therefore, such an equation would not include the impact of loop production. This was shown to result in an energy loss of the form [1]:

$$\dot{\rho}_\ell = -\tilde{c}\bar{v}\frac{\rho}{L}, \quad \text{or equivalently,} \quad \left.\frac{dL}{dt}\right|_{\text{loops}} = \frac{1}{2}\tilde{c}\bar{v}, \tag{24}$$

where $\tilde{c}$ is a phenomenological parameter that described the efficiency of loop chopping. The cosmological evolution of the characteristic length L may then be described by

$$2\frac{dL}{dt} = 2HL(1+\bar{v}^2) + \tilde{c}\bar{v}. \tag{25}$$

Equations (22) and (25) are known as the Velocity-dependent One-Scale (VOS) model [2] and were originally proposed in [42, 43] (see also [44–46] for a generalization to other topological defects). This model, after calibrating its free parameter $\tilde{c}$ with Nambu–Goto numerical simulations (which show that $\tilde{c} = 0.23 \pm 0.04$) provides an accurate description of the cosmological the evolution of cosmic string networks.

[2] This model also includes the impact of friction on cosmic string dynamics. Here, since we are considering only the contributions to the stochastic gravitational wave spectrum generated in the frictionless era, we neglect this effect.

After the friction-dominated era, these networks are known to evolve towards a linear scaling regime characterized by

$$L = \xi t \quad \text{and} \quad \frac{d\bar{v}}{dt} = 0, \tag{26}$$

with

$$\xi^2 = \frac{k(k+\tilde{c})}{4\nu(1-\nu)} \quad \text{and} \quad \bar{v}^2 = \frac{k}{k+\tilde{c}}\frac{1-\nu}{\nu}, \tag{27}$$

for a power-law expansion of the scale factor $a \propto t^\nu$. The existence of such a scaling regime is not only a general prediction of analytical models, but was also observed in Nambu–Goto and field theory simulations. In a realistic ΛCDM background—described by

$$H^2 = H_0^2\left[\Omega_r \mathcal{G}(a)\left(\frac{a_0}{a(t)}\right)^4 + \Omega_m\left(\frac{a_0}{a(t)}\right)^3 + \Omega_\Lambda\right], \tag{28}$$

where $\mathcal{G}(a)$ is a function that accounts for the effect of the decrease of the effective number of relativistic degrees of freedom as the universe expands on the radiation energy density (see e.g. [47] for more details)—we also expect the cosmic string network to evolve in a linear scaling regime deep in the radiation era. However, as the radiation-matter transition is triggered and, later on, dark energy becomes relevant for the dynamics of the universe, this is no longer the case and the network cannot be assumed to evolve according to Eqs. (26) and (27). However, by solving the VOS equations alongside Eq. (28), L and $\bar{v}$ may be characterized throughout the realistic cosmic history (something that is hard to do in numerical simulations). This, as pointed out in [26, 48], allows us to also characterize the energy that is lost as a result of loop production throughout cosmic history—without having to resort to the simplifying assumption that networks are always in a linear scaling regime as was often done in the literature in earlier studies (see e.g. [13, 24, 49])—thus enabling an accurate characterization of the loop number density.

For this to be possible, however, one needs to know the length of the loops that are created in the evolution of a cosmic string network. In this semi-analytical approach [26, 48], the length of the loop at the time of birth is assumed to be determined by the characteristic length of the network:

$$\ell_b = \alpha L, \tag{29}$$

where $\alpha < 1$ is a constant free parameter that may be calibrated with numerical simulations and the subscript 'b' will be used to indicate that we are considering the corresponding quantity at the time of birth of loops. Although the assumption that all loops are created exactly with the same length—or, in other words, that the distribution of the energy density lost to loops in length is a Dirac-δ function—seems rather strong, the results that are obtained under this assumption may be used to construct the loop number density for any distribution of loop length [24, 50] (as, after all, δ functions may be used to construct virtually any function). Moreover, it was shown in [25, 50], that this effect may also be partially accounted for by including an additional parameter, that we introduce in the next paragraph.

Under this assumption, the rate of loop production is given by [26]:

$$\frac{dn_c}{dt} = \frac{\dot{\rho}_\ell}{\ell_b} = \frac{\mathcal{F}}{\sqrt{2}} \frac{\tilde{c}\bar{v}}{\alpha L^4}, \tag{30}$$

where the factor of $1/\sqrt{2}$ was introduced to account for the reduction of the loops' energy caused by the red-shifting of peculiar velocities [13]. Here, $\mathcal{F}$ is the so-called *fuzziness* parameter that accounts for potential uncertainties and/or violations of the assumptions of the model. Besides describing the possibility that not all loops are created with the same length, this parameter may also be used to relax another underlying assumption in this model: that all loops produced by the network decay only by emitting gravitational waves (as suggested in [51]).

After creation, loops start decaying by emitting their energy in the form of gravitational radiation at a roughly constant rate $\Gamma G\mu^2$ (cf. Eq. (14)). As a result, their length decreases as

$$\ell(t) = \alpha L(t_b) - \Gamma G\mu(t - t_b), \tag{31}$$

for $t > t_b$. The problem of computing $n(\ell, t)$ then reduces to finding, using this equation and the evolution of L, the times of birth of all loops that have a length ℓ at the time t, $t_b(\ell, t)$. Taking also the impact of the dilution of loops between t_b and t caused by the expansion of the background into account, we finally have that [26, 50]:

$$n(\ell, t) = \left.\frac{dn_c}{dt}\right|_{t=t_b} \left.\frac{dt}{d\ell}\right|_{t=t_b} \left(\frac{a(t_b)}{a(t)}\right)^3 . \tag{32}$$

2.3 The stochastic gravitational wave background generated by Nambu–Goto strings

We now have all the ingredients needed to compute the stochastic gravitational wave background generated by a network of Nambu-Goto strings throughout cosmic history (by combining Eqs. (2), (16), (31) and (32)). The resulting spectrum is plotted in Fig. 1 for different values of α. Therein, one may see that, although decreasing the size of loops results in a decrease of the amplitude of the spectrum (because smaller loops decay faster and, consequently, evaporate earlier in cosmic history), the shape of the spectrum remains roughly similar. However, there are some changes to the width and relative height of the peak of the spectrum when the length of loops becomes comparable to the gravitational backreaction scale, $\Gamma G\mu$.

This background has contributions from three different loop populations: the loops that are created and decay in the radiation era; the loops that are created in the radiation

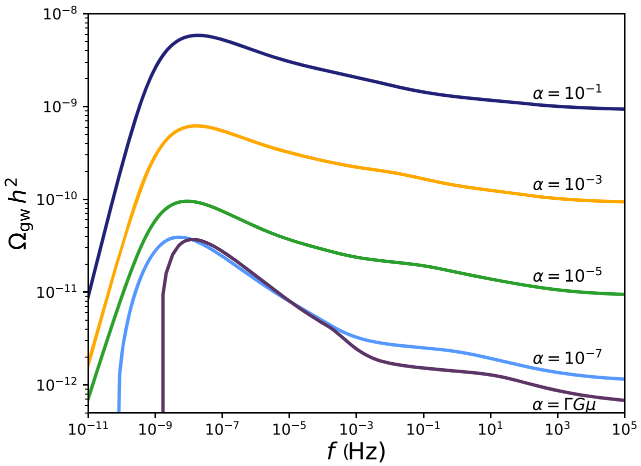

Fig. 1 Stochastic gravitational wave background generated by cosmic string loops for different values of the loop size parameter α. Here, $G\mu = 10^{-10}$, $\mathcal{F} = 1$, $q = 4/3$ and 10^5 harmonic modes of emission are included

era but decay in the matter era; and those that are created in the matter era. The contribution of each of these loops populations to the stochastic gravitational wave background is shown in Fig. 2. The first population of loops (composed of loops that decay in the radiation era) would generate, if the effective number of relativistic degrees of freedom would remain constant, a plateau in the high-frequency region of the spectrum. However, since in a realistic cosmological background this number should decrease as the universe cools and this affects the expansion rate (to which, as is manifest in Eq. (2), the spectrum is highly sensitive), the spectrum generated by this loop population exhibits some step-like features[3] (that may be seen in the spectra in Fig. 1). This plateau should, however, appear at even higher frequencies sourced by loops that have decayed before any changes in the effective number of degrees of freedom (these are outside of the range of the plot). The other two populations generate two distinct peaked contributions in the low frequency range and the shape of the peak of the spectrum is determined by which of these contributions is dominant. The amplitude of the contribution of the radiation-era loops that decay in the matter era scales as $\alpha^{1/2}$, while that of matter era loops remains roughly constant [50]. Therefore, as α decreases, the contribution of matter era loops becomes increasingly important to the stochastic gravitational wave spectrum, which explains the changes to the shape of the peak for small loops.[4]

The amplitude of the plateau of the spectrum is independent of the spectrum of emission of cosmic string loops. The value of q, however, affects the shape of the peak and of the features caused by the decrease of the effective number of relativistic

[3] Interestingly, these features may enable us to use this spectrum to probe the existence of additional degrees of freedom [52].

[4] How small α needs to be for matter era loops to dominate the peak of the spectrum depends on cosmic string tension. As shown in [26], for $G\mu \sim \mathcal{O}(10^{-7})$, matter era loops provide a significant contribution to the peak even for large loops.

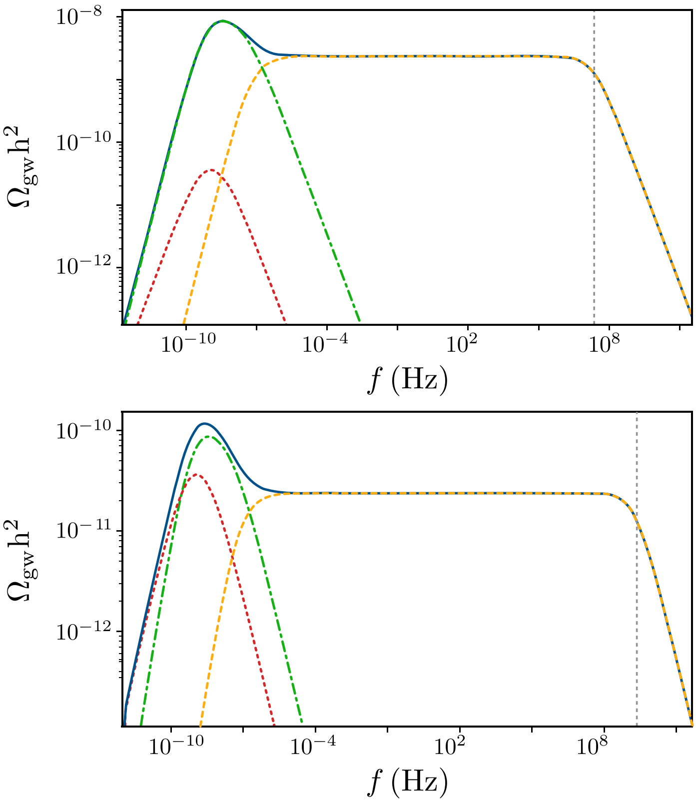

Fig. 2 Contribution of the different loop populations to the stochastic gravitational wave background. Here, $G\mu = 10^{-10}$, $\mathcal{F} = 1$ and $\alpha = 10^{-1}$ (top panel) and $\alpha = 10^{-5}$ (bottom panel). The spectra include only the fundamental mode of emission, so they are independent of q, and the effect of the changes of the number of relativistic degrees of freedom was not included. The solid (blue) lines represent the total SGWB, the dashed (yellow) lines represent the contribution of loops that decay in the radiation era, the dash-dotted (green) lines correspond to the radiation-era loops that decay during the matter era and the dotted (red) lines represent the contribution of loops created in the matter era. This figure was originally published in [50]

degrees of freedom, as is illustrated in Fig. 3. In particular, as q increases, the peak of the spectrum becomes higher and narrower. The peak of the spectrum may then allows to probe the spectrum of emission of cosmic string loops observationally too.

In Fig. 4, we plot the stochastic gravitational wave background generated by cosmic string loops for different values of the cosmic string tension $G\mu$. Therein, one may see that as tension is lowered and loops decay at a slower rate, the peak of the spectrum is shifted towards higher frequencies and the amplitude of the spectrum decreases. A detection of this spectrum should then allows us to determine string tension and to infer the energy of the string-forming phase transition (even though there may be some degeneracy with α).

Nambu–Goto numerical simulations [25, 41, 53] indicate that about 10% of the energy lost by the network goes into the production of large cosmic string loops, with $\alpha \sim 0.34$, while the remainder is in the form of small loops, with lengths comparable

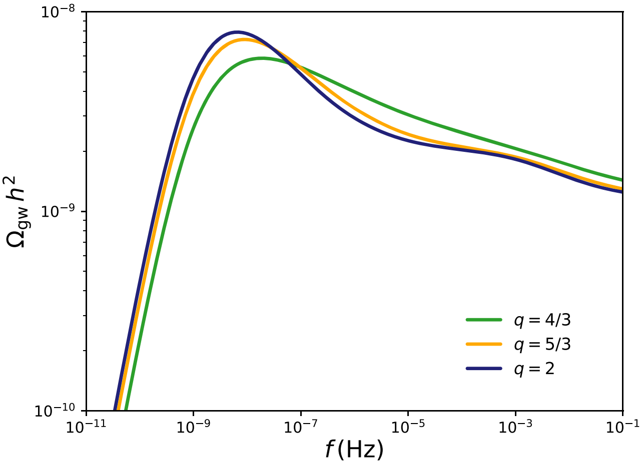

Fig. 3 Stochastic gravitational wave background generated by cosmic string loops for different values of the spectral index q. Here, $G\mu = 10^{-10}$, $\alpha = 10^{-1}$, $\mathcal{F} = 1$ and 10^5 harmonic modes of emission are included

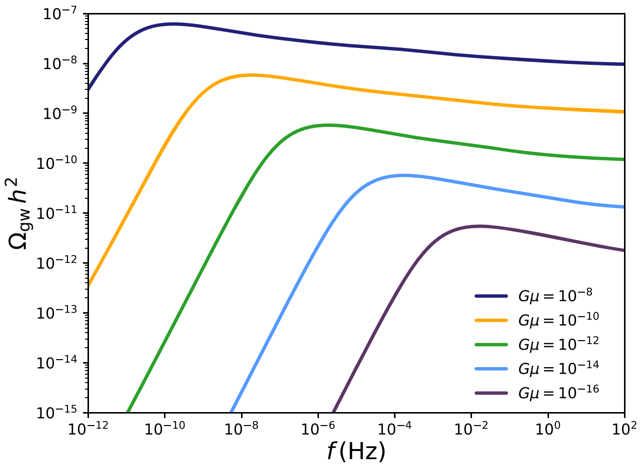

Fig. 4 Stochastic gravitational wave background generated by cosmic string loops for different values of cosmic string tension $G\mu$. Here, we take $\alpha = 10^{-1}$, $\mathcal{F} = 1$, $q = 4/3$ and include the contribution of 10^5 harmonic modes of emission

to the gravitational backreaction scale and high peculiar velocities, that do not make a significant contribution to the spectrum.[5] As shown in [47, 56], this simulation-inferred model is well described by the semi-analytical model described in the previous

[5] Another model inferred from Nambu–Goto simulations [54] predicts the existence of an additional population of very small loops that would provide the dominant contribution to the spectrum. The validity of this model, however, is currently under question [53, 55].

section if one sets $\alpha = 0.34$, $\mathcal{F} = 0.1$ and $q = 4/3$.[6] Both the semi-analytical model introduced in [26, 48] and this simulation-inferred model [25] have been adopted as benchmark models for LISA and have recently been included in LISA's template bank [56]. LISA Cosmology Working Group studies [47, 56] show that this detector will be ideal to probe cosmic strings, since, as tension is lowered, the peak of the spectrum moves into its sensitivity window. LISA should be able to probe Nambu–Goto strings with unprecedented precision—potentially reaching tensions as low as $10^{-16} - 10^{-17}$, corresponding to energy scales of $\mathcal{O}(10^{11})$ GeV—as well as to probe deviations from this simplest string scenario (that would be encoded in parameters α, $\mathcal{F}$ and q) and signatures of beyond-the-standard-model physics (e.g. additional relativistic degrees of freedom and phases of non-standard expansion in the early universe).

3 Going beyond Nambu–Goto: the case of superconducting strings

The Nambu–Goto approximation provides an effective description of thin cosmic strings—or, in other words, of strings whose curvature radius is much larger than their thickness—and that do not have internal degrees of freedom. However, in many situations of interest in cosmology and particle physics one or both of these conditions may be violated. As a matter of fact, there has been an intensive debate in the literature over the past decades about whether or not the Nambu–Goto approximation indeed provides an adequate description of even the simplest local strings, the Abelian-Higgs cosmic strings, as was originally expected. This debate concerns, particularly, how strong a role the emission of scalar and gauge radiation plays in string loop decay. If it were to be significant, this would reduce the amplitude of the stochastic gravitational gravitational wave background significantly and our ability to probe this scenario observationally would be naturally affected [57]. As a matter of fact, in field theory simulations of networks of Abelian-Higgs strings, loops decay quite fast by emitting scalar and gauge radiation [58]. However, this is not necessarily the case in simulations of individual loops [51, 59, 60] and it was observed that radiation is mostly emitted in regions of the string with high curvature and/or when the length of the loop becomes comparable to the thickness of the strings (i.e., when or where the strings cannot be considered to be thin).

A relevant scenario in which this simple Nambu–Goto description definitely does not apply is that of superconducting strings [3]. If electromagnetic gauge invariance is broken in their cores, strings may carry currents and behave as thin superconducting wires. There are several models that may lead to the production of superconducting strings [3, 61–64] and in [65] it is argued that a spontaneous generation of current may occur in the early universe even in currentless strings. In many such scenarios, superconducting strings may also be treated as infinitely-thin, but now there are internal degrees of freedom associated to the currents propagating along the string. An effective action to describe the dynamics of these strings has then to account for the impact of

[6] There are, however, slight differences in the shape of the peak of the spectrum that result from the fact that the emission spectrum considered is that measured in loops removed directly from simulations. Although this spectrum is roughly given by (16), with $q = 4/3$ for large j, there are some differences for small j.

these internal degrees of freedom, as well as for the coupling of the current carriers to electromagnetism (see e.g. [66] for more details).

Superconducting strings, however, may be treated as elastic strings [67]. Elastic strings are characterized by having a distinct tension $\mathcal{T}$ and mass per unit length $\mathcal{U}$ and for having two distinct types of perturbations: transverse perturbations, with a speed $c_E^2 = \mathcal{T}/\mathcal{U}$, and longitudinal perturbations, with a speed $c_L^2 = -d\mathcal{T}/d\mathcal{U}$. When $c_E = c_L < 1$, elastic strings are said to be transonic. Although superconducting strings are not always expected to be transonic [68], transonic strings may be used to describe them in some particular limits [69] and may describe "linear" superconducting string models [34]. In [34], the authors took advantage of the integrability of the equation of motion for transonic strings and used them to study the impact of current on gravitational wave emission. I will review these results in this section.

3.1 Impact of current on the gravitational wave emission of loops

Transonic string loop solutions may also be expressed as in Eq. (10) and the scalar field describing the charge carriers (which is confined to the worldsheet) also admits solutions in terms of left- and right-moving functions $F_{\pm}$:

$$\phi = \frac{T}{2\pi}\left[F_+(\sigma_+) + F_-(\sigma_-)\right]. \tag{33}$$

However now, due to the existence of current, $\mathbf{x}'_{\pm}$ are no longer unit vectors and their magnitude is determined by the value of current instead:

$$\mathbf{x}'^{\,2}_{\pm}(\sigma_{\pm}) = 1 - F'^{\,2}_{\pm}(\sigma_{\pm}). \tag{34}$$

This has important consequences for the formation of cusps, as they cannot form if F'_+ and F'_- are different. Nevertheless, for symmetrical currents, with $F'_+ = F'_- \equiv F'$, a cusp-like point—with the same shape as those that appear in standard strings—may form, but its velocity will be subluminal: $\dot{\mathbf{x}}^2 = 1 - F'^2$ [70]. Indeed, the charge carriers introduce inertia to the strings, which effectively slows them down and should result in a decrease of the GW emission efficiency.

In [34], the impact of current on the gravitational wave emission of cosmic string loops with cusps was studied (see also [71] for a study of chiral superconducting loops). This was achieved by fully characterizing the spectrum of emission of gravitational radiation of a particular class of loops that form cusps at discrete instants of time, known as Burden loops [36], using Eq. (14). They found that current, in fact, leads to a weakening of the gravitational wave emission efficiency that is well described by:

$$\Gamma(F') = \Gamma^{NG}\left(1 - \left|F'\right|\right)^{B^r}, \tag{35}$$

where $\Gamma^{NG} = 50$ is the gravitational wave emission spectrum for currentless (Nambu–Goto) strings, $r = s, c$ is used to label symmetrical currents ($F'_+ = F'_- = F'$) and chiral (null) current (for which either F'_+ or F'_- vanishes and F' labels the non-vanishing current) respectively and $B^s = \sqrt{2}B^c \simeq 2$. Therein, they also found that current causes an exponential suppression of the spectrum of emission with increasing harmonic mode. As a result, it may no longer be described as a power law and is instead of the form:

$$\Gamma_j \sim j^{-q} e^{-j f_r(G)}, \quad \text{with} \quad f_r(G) = a_r (1 - \sqrt{G})^{b_r}, \tag{36}$$

where $G^2 = 1 - F'^2$ and a_r and b_r are fitting constants whose values depend on whether currents are chiral or symmetrical (and may be found in [34]).

The inertia introduced by the charge carriers also leads to a decrease of the speed of propagation of kinks along cosmic strings, which also alters their gravitational wave emission. To study the impact of current on the emission of cusps, in [34], they resorted to the cuspless loops solution introduced in [38] (which were also studied in [71] for the particular case of chiral currents). As for cusps, their results show that currents weaken gravitational wave emission in a way that is also well described by Eq. (35), but now $B^s = \sqrt{2}B^c \simeq 1.5$. They also found that, in this case, the spectrum of emission is unaffected by current and that it may still be described by a power law with spectral index $q = 5/3$. Interestingly, the cuspless loop solution considered in [34] is also a solution of the equation of motion for general superconducting strings—and not only transonic superconducting strings—which seems to support the idea that Eq. (35) should be valid for superconducting strings in general.

3.2 Impact of current on cosmic string loop production

Although the VOS model introduced in Sect. 2.2 was derived directly from the Nambu–Goto action, it may be extended through the addition of phenomenological terms to describe more complex cosmic string scenarios. The semi-analytical approach introduced in [26, 48] is also very flexible and relies on minimal assumptions about the nature of cosmic strings and enables, therefore, the study of the stochastic gravitational wave background for non-standard cosmic string scenarios as well. It was, in fact, successfully used to characterize the spectra generated by cosmic superstrings [50], strings in non-standard cosmological scenarios [52, 56], strings created during inflation [72], strings with friction [27] and several others (see also [73, 74] for a semi-analytical framework to describe the stochastic gravitational wave spectrum generated by domain walls).

The cosmological evolution of networks of current-carrying strings has been recently studied in [75–77]. Therein, the authors extended the VOS model to also describe the evolution of averaged current and to account for its impact on the evolution of cosmic string networks and this model may be used to characterize the number of loops produced for superconducting strings. In [75–77], they found that, during the matter era, current is effectively diluted due to the fast expansion rate and, as a result, strings behave essentially as standard strings (i.e strings without current). During the radiation era, however, provided that there is a current dissipation mechanism, the network experiences a linear scaling regime with non-trivial current characterized by:

$$\xi_r^2 = (1-Y)k(k+\tilde{c}) \quad \text{and} \quad \bar{v}_r^2 = (1-Y)\frac{k}{k+\tilde{c}}. \tag{37}$$

Here, Y is the averaged macroscopic current, related to the microscopic current roughly as $Y \sim \langle F'_{\pm}\rangle^2$. Note that, for superconducting strings, the characteristic length L can no longer be identified as the averaged physical length of strings $L_{\rm ph}$. The latter is actually a measure of the *bare string energy density*, $\rho_0 = \mu/L_{\rm ph}^2$, that does not include the contribution of current, and is given by $L_{\rm ph} = L(1+Y)^{1/2}$. Since the physical length is the relevant quantity to compute the number of loops produced, here $\xi_r \equiv L_{\rm ph}/t$.

The number of loops produced in the evolution of a cosmic string network is, as we have seen in Sect. 2.2, determined by the large scale dynamics of the network. Since current affects the evolution of the network during the radiation era, the number of loops produced during this stage should be affected too. Note that the curvature parameter $k = k(\bar{v})$ depends on the RMS velocity of the network (cf. Eq. (23)), so the impact of current on L and $\bar{v}$ is not as straightforward as Eq. (37) seems to indicate. We display this impact in the top panel of Fig. 5. Therein one may see that the RMS velocity decreases monotonically with increasing current, which by itself contributes to making collisions between strings progressively less likely as current increases. The behaviour of the physical length, however, is somewhat more complex. For small enough values of current, $L_{\rm ph}$ increases as current increases and the network also becomes less dense, which further exacerbates this effect. However, if current is increased even further, for $Y \gtrsim 0.7$, the density of the network quickly increases and string collisions become increasingly frequent. As a result, loop production is somewhat suppressed for small to moderate values of the current, but for very high currents loops will be produced profusely, as is illustrated in the bottom panel of Fig. 5.

3.3 Impact of current on the amplitude of the stochastic gravitational wave background

As we have seen, superconducting strings should behave effectively as currentless strings during the matter era and, as a results, the contribution of matter era loops to the stochastic gravitational wave background should remain unaffected by the inclusion of current. The amplitude of the contribution of the loop populations created in the radiation era, however, will not only be affected by the decrease in Γ (described in Sect. 3.1), but also by the changes in the number of loops produced described in the previous section. Using the analytical approximation to the stochastic gravitational wave background generated by cosmic strings derived in [50], one may see that [34]:

$$\frac{\Omega_{\rm pl}^S}{\Omega_{\rm pl}^{NG}} = \frac{\dot{\rho}_\ell^S}{\dot{\rho}_\ell^{NG}}\left(\frac{\Gamma^{NG}\xi_r^S}{\Gamma^S\xi_r^{NG}}\right)^{1/2}, \tag{38}$$

where $\Omega_{\rm pl}^r$ represents the amplitude of the radiation era plateau and the label $r = S, NG$ is used to indicate that the values of superconducting or Nambu–Goto strings, respectively, are considered. The bottom panel of Fig. 5 displays this ratio, alongside

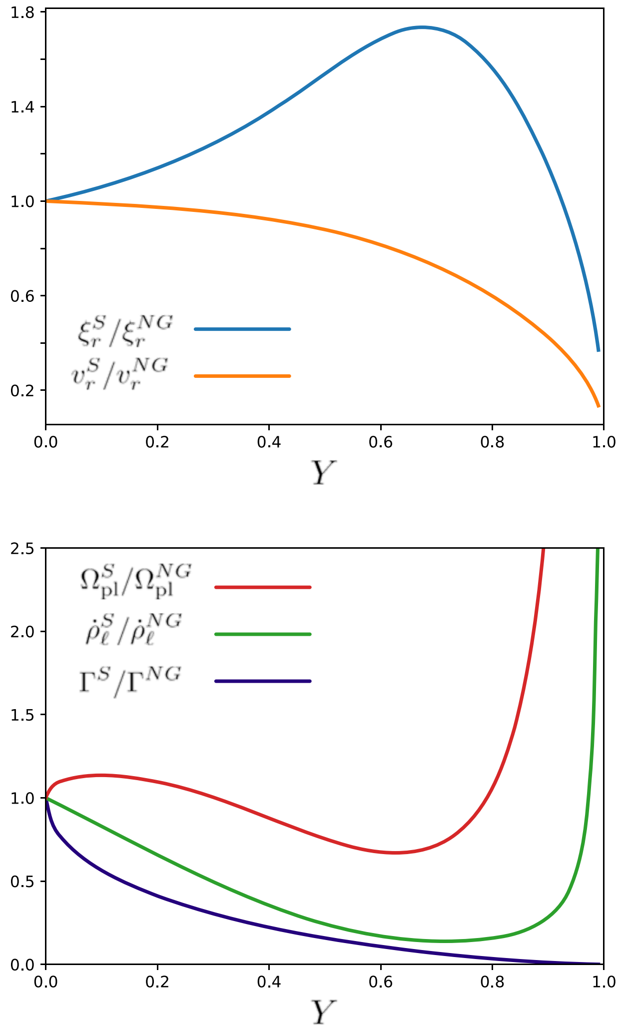

Fig. 5 Impact of current on the amplitude of the stochastic gravitational wave background generated by radiation era loops. The top panel shows the impact of the macroscopic current Y on the averaged physical length ξ^S and the RMS velocity $\bar{v}^S$ (normalized by the corresponding quantities for Nambu–Goto strings). The bottom panel displays the ratio between the amplitude of the radiation plateau of the stochastic gravitational wave background generated by superconducting and cosmic strings $\Omega_{\rm gw}^S/\Omega_{\rm gw}^{NG}$, as well as the ratio between the energy density lost due to the formation of loops $\dot\rho_\ell^S/\dot\rho_\ell^{NG}$ and between the total power emitted in gravitational radiation Γ^S/Γ^{NG}. This figure was adapted from [34]

the energy density lost as a result of loop formation and the gravitational wave emission efficiency, for different values of current Y. Therein one may see that the change in the number of loops produced plays a crucial role. As one increases current and Γ decreases, loops survive longer and radiate their energy in gravitational waves closer to the present time. This, if we ignore the impact on the number of loops produced,

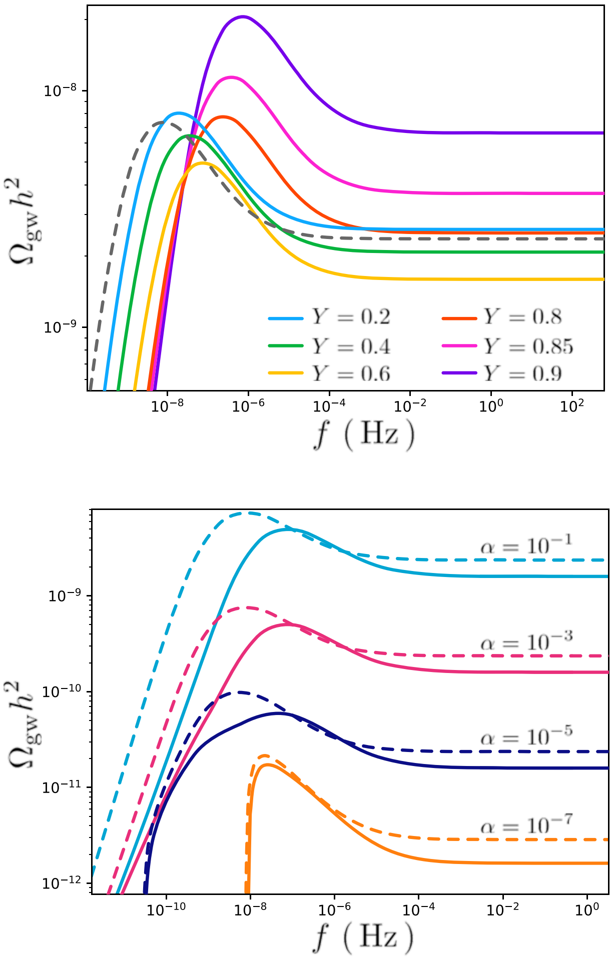

Fig. 6 Stochastic gravitational wave background generated by networks of superconducting strings. The top panel shows the spectrum generated for loops characterized by $\alpha = 10^{-1}$ and different values of macroscopic current Y. The bottom panel displays the stochastic gravitational wave background for networks with a macroscopic current of $Y = 0.6$ and different values of α. Both panels include the spectrum generated by standard strings for comparison (dashed lines). For both standard and superconducting strings, loops with kinks, with a power spectrum characterized by the exponent $q = 5/3$, are considered and 10^5 modes of emission are included. Moreover, $G\mu = 10^{-10}$ and $\Gamma^{NG} = 50$ and currents are assumed to be chiral. This figure was adapted from [34]

would lead to a progressive increase of $\Omega_{\rm gw}$ as one increases current. However, as may be seen in Fig. 5, for $Y \lesssim 0.7$, this increase merely acts to counteract the impact of the suppression of loop production and the impact of current on the amplitude of the spectrum is only moderate (see also the top panel of Fig. 6). For larger currents, however, both effects act to increase the amplitude of $\Omega_{\rm gw}$ and, as a result, the amplitude of the stochastic gravitational wave background increases quickly and sharply in this limit.

The overall impact of current on the spectrum, however, depends not only on the magnitude of the macroscopic current Y, but also on the length of loops.[7] For large enough loops—for which the contribution of matter era loops is generally negligible, as previously discussed—the shape of the spectrum is identical to that of standards strings, but the amplitude and its frequency range depends on the value of current, as may been seen in the top panel of Fig. 6. However, currents affect the ratio between the contribution of radiation-era loops that decay in the matter era and that of loops created in the matter era, and therefore for small enough α it may then affect the shape of the peak of the spectrum, as may be seen in the bottom panel of Fig. 6. In particular, the shift in the peak generated by radiation-era loops that decay in the matter era towards higher frequencies caused by the inclusion of current may lead to a broadening of the peak and the height of the peak (relative to the plateau) may also be affected for smaller values of α. These features may provide distinctive fingerprints that distinguish superconducting strings from standards strings in case of a detection.

Note however that superconducting string loops are also expected to emit bursts of electromagnetic radiation [78, 79], which should result in a weakening of their signal. This additional decay mechanism was not yet included in the computation of the stochastic gravitational wave background.

4 Conclusions

Cosmic string networks generate gravitational waves throughout cosmological history and, as a result, they are expected to generate one of the strongest stochastic gravitational wave backgrounds of cosmological origin [21]. This spectrum not only has a characteristic shape that may help distinguish it from other potential sources of a primordial stochastic gravitational wave background (e.g., first order phase transitions and inflation), but is also very sensitive to the physics of the early universe, potentially allowing us to probe thermal history and cosmological expansion as well [47, 52, 56].

Here, by considering the particular case of superconducting strings, I have shown that the cosmic string stochastic gravitational wave background may have distinctive signatures of the underlying physics. The production of cosmic strings is predicted in several particle physics scenarios and the stochastic gravitational wave background is expected to have specific imprints in many instances [27, 50, 52, 56, 72]. String-forming scenarios may then be validated by a detection of their characteristic spectrum or tightly constrained by the lack thereof. In either case, there is much information to be gained about fundamental physics, as this reduces the vast array of possibilities for the early universe.

Acknowledgements L. S. is supported by FCT—Fundação para a Ciência e a Tecnologia through contract No. DL 57/2016/CP1364/CT0001. Funding for this work has also been provided by FCT through the research Grants UIDB/04434/2020 and UIDP/04434/2020 and through the R&D project 2022.03495.PTDC—Uncovering the nature of cosmic strings.

Author Contributions L.S. wrote and reviewed the manuscript and prepared figures 1, 3 and 4.

[7] Note that, for superconducting strings, it was not yet possible to measure the length of loops in numerical simulations, so the favoured value of α is currently unknown.

Funding Open access funding provided by FCT|FCCN (b-on).

Data Availability No datasets were generated or analysed during the current study.

Declarations

Competing interests The authors declare no competing interests.

Open Access This article is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence, and indicate if changes were made. The images or other third party material in this article are included in the article's Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the article's Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this licence, visit http://creativecommons.org/licenses/by/4.0/.

References

1. Kibble, T.W.B.: Topology of cosmic domains and strings. J. Phys. A **9**, 1387–1398 (1976). https://doi.org/10.1088/0305-4470/9/8/029
2. Preskill, J., Vilenkin, A.: Decay of metastable topological defects. Phys. Rev. D **47**, 2324–2342 (1993). https://doi.org/10.1103/PhysRevD.47.2324. arXiv:hep-ph/9209210
3. Witten, E.: Superconducting strings. Nucl. Phys. B **249**, 557–592 (1985). https://doi.org/10.1016/0550-3213(85)90022-7
4. Davis, R.L.: Cosmic axions from cosmic strings. Phys. Lett. B **180**, 225–230 (1986). https://doi.org/10.1016/0370-2693(86)90300-X
5. Davis, S.C., Davis, A.-C., Trodden, M.: N=1 supersymmetric cosmic strings. Phys. Lett. B **405**, 257–264 (1997). https://doi.org/10.1016/S0370-2693(97)00642-4. arXiv:hep-ph/9702360
6. Sarangi, S., Tye, S.H.H.: Cosmic string production towards the end of brane inflation. Phys. Lett. B **536**, 185–192 (2002). https://doi.org/10.1016/S0370-2693(02)01824-5. arXiv:hep-th/0204074
7. Jones, N.T., Stoica, H., Tye, S.H.H.: The production, spectrum and evolution of cosmic strings in brane inflation. Phys. Lett. B **563**, 6–14 (2003). https://doi.org/10.1016/S0370-2693(03)00592-6. arXiv:hep-th/0303269
8. Lazarides, G., Peddie, I.N.R., Vamvasakis, A.: Semi-shifted hybrid inflation with B-L cosmic strings. Phys. Rev. D **78**, 043518 (2008). https://doi.org/10.1103/PhysRevD.78.043518. arXiv:0804.3661
9. Allys, E.: Bosonic structure of realistic SO(10) supersymmetric cosmic strings. Phys. Rev. D **93**(10), 105021 (2016). https://doi.org/10.1103/PhysRevD.93.105021. arXiv:1512.02029
10. Dror, J.A., Hiramatsu, T., Kohri, K., Murayama, H., White, G.: Testing the seesaw mechanism and leptogenesis with gravitational waves. Phys. Rev. Lett. **124**(4), 041804 (2020). https://doi.org/10.1103/PhysRevLett.124.041804. arXiv:1908.03227
11. Samanta, R., Datta, S.: Gravitational wave complementarity and impact of NANOGrav data on gravitational leptogenesis. JHEP **05**, 211 (2021). https://doi.org/10.1007/JHEP05(2021)211. arXiv:2009.13452
12. Hindmarsh, M.B., Kibble, T.W.B.: Cosmic strings. Rept. Prog. Phys. **58**, 477–562 (1995). https://doi.org/10.1088/0034-4885/58/5/001. arXiv:hep-ph/9411342
13. Vilenkin, A., Shellard, E.P.S.: Cosmic Strings and Other Topological Defects. Cambridge University Press, Cambridge (2000)
14. Agazie, G., et al.: The NANOGrav 15 yr data set: evidence for a gravitational-wave background. Astrophys. J. Lett. **951**(1), 8 (2023). https://doi.org/10.3847/2041-8213/acdac6. arXiv:2306.16213
15. Antoniadis, J., et al.: The second data release from the European Pulsar Timing Array—III. Search for gravitational wave signals. Astron. Astrophys. **678**, 50 (2023). https://doi.org/10.1051/0004-6361/202346844

16. Reardon, D.J., et al.: Search for an isotropic gravitational-wave background with the Parkes pulsar timing array. Astrophys. J. Lett. **951**(1), 6 (2023). https://doi.org/10.3847/2041-8213/acdd02. arXiv:2306.16215
17. Xu, H., et al.: Searching for the nano-hertz stochastic gravitational wave background with the Chinese pulsar timing array data release i. Res. Astron. Astrophys. **23**(7), 075024 (2023). https://doi.org/10.1088/1674-4527/acdfa5. arXiv:2306.16216
18. Afzal, A., et al.: The NANOGrav 15 yr data set: search for signals from new physics. Astrophys. J. Lett. **951**(1), 11 (2023). https://doi.org/10.3847/2041-8213/acdc91. arXiv:2306.16219
19. Antoniadis, J., et al.: The second data release from the European Pulsar Timing Array - IV. Implications for massive black holes, dark matter, and the early Universe. Astron. Astrophys. **685**, 94 (2024). https://doi.org/10.1051/0004-6361/202347433
20. Janssen, G., et al.: Gravitational wave astronomy with the SKA. PoS AASKA14, 037 (2015) https://doi.org/10.22323/1.215.0037 arXiv:1501.00127
21. Auclair, P., et al.: Cosmology with the laser interferometer space antenna. Living Rev. Rel. **26**(1), 5 (2023). https://doi.org/10.1007/s41114-023-00045-2. arXiv:2204.05434
22. Vilenkin, A.: Gravitational radiation from cosmic strings. Phys. Lett. B **107**, 47–50 (1981). https://doi.org/10.1016/0370-2693(81)91144-8
23. Accetta, F.S., Krauss, L.M.: The stochastic gravitational wave spectrum resulting from cosmic string evolution. Nucl. Phys. B **319**, 747–764 (1989). https://doi.org/10.1016/0550-3213(89)90628-7
24. Sanidas, S.A., Battye, R.A., Stappers, B.W.: Constraints on cosmic string tension imposed by the limit on the stochastic gravitational wave background from the European Pulsar Timing Array. Phys. Rev. D **85**, 122003 (2012). https://doi.org/10.1103/PhysRevD.85.122003. arXiv:1201.2419
25. Blanco-Pillado, J.J., Olum, K.D., Shlaer, B.: The number of cosmic string loops. Phys. Rev. D **89**(2), 023512 (2014). https://doi.org/10.1103/PhysRevD.89.023512. arXiv:1309.6637
26. Sousa, L., Avelino, P.P.: Stochastic gravitational wave background generated by cosmic string networks: velocity-dependent one-scale model versus scale-invariant evolution. Phys. Rev. D **88**(2), 023516 (2013). https://doi.org/10.1103/PhysRevD.88.023516. arXiv:1304.2445
27. Mukovnikov, S., Sousa, L.: Ultra-high frequency gravitational waves from cosmic strings with friction (2024) arXiv:2404.13213
28. Aghanim, N., et al.: Planck 2018 results. VI. Cosmological parameters. Astron. Astrophys. **641**, 6 (2020). https://doi.org/10.1051/0004-6361/201833910. arXiv:1807.06209
29. Olum, K.D., Blanco-Pillado, J.J.: Radiation from cosmic string standing waves. Phys. Rev. Lett. **84**, 4288–4291 (2000). https://doi.org/10.1103/PhysRevLett.84.4288. arXiv:astro-ph/9910354
30. Turok, N.: Grand unified strings and galaxy formation. Nucl. Phys. B **242**, 520–541 (1984). https://doi.org/10.1016/0550-3213(84)90407-3
31. Damour, T., Vilenkin, A.: Gravitational wave bursts from cusps and kinks on cosmic strings. Phys. Rev. D **64**, 064008 (2001). https://doi.org/10.1103/PhysRevD.64.064008. arXiv:gr-qc/0104026
32. Durrer, R.: Gravitational angular momentum radiation of cosmic strings. Nucl. Phys. B **328**, 238–271 (1989). https://doi.org/10.1016/0550-3213(89)90103-X
33. Allen, B., Shellard, E.P.S.: Gravitational radiation from cosmic strings. Phys. Rev. D **45**, 1898–1912 (1992). https://doi.org/10.1103/PhysRevD.45.1898
34. Rybak, I.Y., Sousa, L.: Emission of gravitational waves by superconducting cosmic strings. JCAP **11**, 024 (2022). https://doi.org/10.1088/1475-7516/2022/11/024. arXiv:2209.01068
35. Vachaspati, T., Vilenkin, A.: Gravitational radiation from cosmic strings. Phys. Rev. D **31**, 3052 (1985). https://doi.org/10.1103/PhysRevD.31.3052
36. Burden, C.J.: Gravitational radiation from a particular class of cosmic strings. Phys. Lett. B **164**, 277–281 (1985). https://doi.org/10.1016/0370-2693(85)90326-0
37. Vachaspati, T.: Gravity of cosmic loops. Phys. Rev. D **35**, 1767–1775 (1987). https://doi.org/10.1103/PhysRevD.35.1767
38. Garfinkle, D., Vachaspati, T.: Radiation from kinky, cuspless cosmic loops. Phys. Rev. D **36**, 2229 (1987). https://doi.org/10.1103/PhysRevD.36.2229
39. Scherrer, R.J., Quashnock, J.M., Spergel, D.N., Press, W.H.: Properties of realistic cosmic string loops. Phys. Rev. D **42**, 1908–1914 (1990). https://doi.org/10.1103/PhysRevD.42.1908
40. Binetruy, P., Bohe, A., Hertog, T., Steer, D.A.: Gravitational wave bursts from cosmic superstrings with y-junctions. Phys. Rev. D **80**, 123510 (2009). https://doi.org/10.1103/PhysRevD.80.123510. arXiv:0907.4522

41. Blanco-Pillado, J.J., Olum, K.D.: Stochastic gravitational wave background from smoothed cosmic string loops. Phys. Rev. D **96**(10), 104046 (2017). https://doi.org/10.1103/PhysRevD.96.104046. arXiv:1709.02693
42. Martins, C.J.A.P., Shellard, E.P.S.: Extending the velocity dependent one scale string evolution model. Phys. Rev. D **65**, 043514 (2002). https://doi.org/10.1103/PhysRevD.65.043514. arXiv:hep-ph/0003298
43. Martins, C.J.A.P., Shellard, E.P.S.: Quantitative string evolution. Phys. Rev. D **54**, 2535–2556 (1996). https://doi.org/10.1103/PhysRevD.54.2535. arXiv:hep-ph/9602271
44. Avelino, P.P., Sousa, L.: Domain wall network evolution in (N+1)-dimensional FRW universes. Phys. Rev. D **83**, 043530 (2011). https://doi.org/10.1103/PhysRevD.83.043530. arXiv:1101.3360
45. Sousa, L., Avelino, P.P.: p-brane dynamics in (N+1)-dimensional FRW universes: a unified framework. Phys. Rev. D **83**, 103507 (2011). https://doi.org/10.1103/PhysRevD.83.103507. arXiv:1103.1381
46. Sousa, L., Avelino, P.P.: The cosmological evolution of p-brane networks. Phys. Rev. D **84**, 063502 (2011). https://doi.org/10.1103/PhysRevD.84.063502. arXiv:1107.4582
47. Auclair, P., et al.: Probing the gravitational wave background from cosmic strings with LISA. JCAP **04**, 034 (2020). https://doi.org/10.1088/1475-7516/2020/04/034. arXiv:1909.00819
48. Sousa, L., Avelino, P.P.: Stochastic gravitational wave background generated by cosmic string networks: the small-loop regime. Phys. Rev. D **89**(8), 083503 (2014). https://doi.org/10.1103/PhysRevD.89.083503. arXiv:1403.2621
49. Caldwell, R.R., Allen, B.: Cosmological constraints on cosmic string gravitational radiation. Phys. Rev. D **45**, 3447–3468 (1992). https://doi.org/10.1103/PhysRevD.45.3447
50. Sousa, L., Avelino, P.P., Guedes, G.S.F.: Full analytical approximation to the stochastic gravitational wave background generated by cosmic string networks. Phys. Rev. D **101**(10), 103508 (2020). https://doi.org/10.1103/PhysRevD.101.103508. arXiv:2002.01079
51. Hindmarsh, M., Lizarraga, J., Urio, A., Urrestilla, J.: Loop decay in Abelian-Higgs string networks. Phys. Rev. D **104**(4), 043519 (2021). https://doi.org/10.1103/PhysRevD.104.043519. arXiv:2103.16248
52. Cui, Y., Lewicki, M., Morrissey, D.E., Wells, J.D.: Cosmic archaeology with gravitational waves from cosmic strings. Phys. Rev. D **97**(12), 123505 (2018). https://doi.org/10.1103/PhysRevD.97.123505. arXiv:1711.03104
53. Blanco-Pillado, J.J., Olum, K.D.: Direct determination of cosmic string loop density from simulations. Phys. Rev. D **101**(10), 103018 (2020). https://doi.org/10.1103/PhysRevD.101.103018. arXiv:1912.10017
54. Lorenz, L., Ringeval, C., Sakellariadou, M.: Cosmic string loop distribution on all length scales and at any redshift. JCAP **10**, 003 (2010). https://doi.org/10.1088/1475-7516/2010/10/003. arXiv:1006.0931
55. Blanco-Pillado, J.J., Olum, K.D., Wachter, J.M.: Energy-conservation constraints on cosmic string loop production and distribution functions. Phys. Rev. D **100**(12), 123526 (2019). https://doi.org/10.1103/PhysRevD.100.123526. arXiv:1907.09373
56. Blanco-Pillado, J.J., Cui, Y., Kuroyanagi, S., Lewicki, M., Nardini, G., Pieroni, M., Rybak, I.Y., Sousa, L., Wachter, J.M.: Gravitational waves from cosmic strings in LISA: reconstruction pipeline and physics interpretation (2024) arXiv:2405.03740
57. Hindmarsh, M., Kume, J.: Multi-messenger constraints on Abelian-Higgs cosmic string networks. JCAP **04**, 045 (2023). https://doi.org/10.1088/1475-7516/2023/04/045. arXiv:2210.06178
58. Hindmarsh, M., Lizarraga, J., Urrestilla, J., Daverio, D., Kunz, M.: Scaling from gauge and scalar radiation in Abelian Higgs string networks. Phys. Rev. D **96**(2), 023525 (2017). https://doi.org/10.1103/PhysRevD.96.023525. arXiv:1703.06696
59. Matsunami, D., Pogosian, L., Saurabh, A., Vachaspati, T.: Decay of cosmic string loops due to particle radiation. Phys. Rev. Lett. **122**(20), 201301 (2019). https://doi.org/10.1103/PhysRevLett.122.201301. arXiv:1903.05102
60. Blanco-Pillado, J.J., Jiménez-Aguilar, D., Lizarraga, J., Lopez-Eiguren, A., Olum, K.D., Urio, A., Urrestilla, J.: Nambu–Goto dynamics of field theory cosmic string loops. JCAP **05**, 035 (2023). https://doi.org/10.1088/1475-7516/2023/05/035. arXiv:2302.03717
61. Everett, A.E.: New mechanism for superconductivity in cosmic strings. Phys. Rev. Lett. **61**, 1807–1810 (1988). https://doi.org/10.1103/PhysRevLett.61.1807
62. Davis, A.-C., Perkins, W.B.: Generic current carrying strings. Phys. Lett. B **390**, 107–114 (1997). https://doi.org/10.1016/S0370-2693(96)01358-5. arXiv:hep-ph/9610292

63. Garaud, J., Volkov, M.S.: Superconducting non-Abelian vortices in Weinberg–Salam theory—electroweak thunderbolts. Nucl. Phys. B **826**, 174–216 (2010). https://doi.org/10.1016/j.nuclphysb.2009.10.003. arXiv:0906.2996
64. Lilley, M., Di Marco, F., Martin, J., Peter, P.: Nonabelian bosonic currents in cosmic strings. Phys. Rev. D **82**, 023510 (2010). https://doi.org/10.1103/PhysRevD.82.023510. arXiv:1003.4601
65. Davis, A.-C., Peter, P.: Cosmic strings are current carrying. Phys. Lett. B **358**, 197–202 (1995). https://doi.org/10.1016/0370-2693(95)00966-O. arXiv:hep-ph/9506433
66. Spergel, D.N., Piran, T., Goodman, J.: Dynamics of superconducting cosmic strings. Nucl. Phys. B **291**, 847–875 (1987). https://doi.org/10.1016/0550-3213(87)90499-8
67. Carter, B.: Duality relation between charged elastic strings and superconducting cosmic strings. Phys. Lett. B **224**, 61–66 (1989). https://doi.org/10.1016/0370-2693(89)91051-4
68. Carter, B.: Stability and characteristic propagation speeds in superconducting cosmic and other string models. Phys. Lett. B **228**, 466–470 (1989). https://doi.org/10.1016/0370-2693(89)90976-3
69. Carter, B.: Brane dynamics for treatment of cosmic strings and vortons. In: 2nd Mexican School on Gravitation and Mathematical Physics (1997)
70. Rybak, I.Y.: Revisiting Y junctions for strings with currents: transonic elastic case. Phys. Rev. D **102**(8), 083516 (2020). https://doi.org/10.1103/PhysRevD.102.083516. arXiv:2001.07262
71. Babichev, E., Dokuchaev, V.: Oscillation damping of chiral string loops. Phys. Rev. D **66**, 025007 (2002). https://doi.org/10.1103/PhysRevD.66.025007. arXiv:hep-ph/0204304
72. Guedes, G.S.F., Avelino, P.P., Sousa, L.: Signature of inflation in the stochastic gravitational wave background generated by cosmic string networks. Phys. Rev. D **98**(12), 123505 (2018). https://doi.org/10.1103/PhysRevD.98.123505. arXiv:1809.10802
73. Grüber, D., Sousa, L., Avelino, P.P.: Stochastic gravitational wave background generated by domain wall networks. Phys. Rev. D **110**(2), 023505 (2024). https://doi.org/10.1103/PhysRevD.110.023505. arXiv:2403.09816
74. Grüber, D., Avelino, P.P., Sousa, L.: Domain walls in light of Cosmic Microwave Background and Pulsar Timing Array data (2024) arXiv:2406.02288
75. Martins, C.J.A.P., Peter, P., Rybak, I.Y., Shellard, E.P.S.: Generalized velocity-dependent one-scale model for current-carrying strings. Phys. Rev. D **103**(4), 043538 (2021). https://doi.org/10.1103/PhysRevD.103.043538. arXiv:2011.09700
76. Martins, C.J.A.P., Peter, P., Rybak, I.Y., Shellard, E.P.S.: Charge-velocity-dependent one-scale linear model. Phys. Rev. D **104**(10), 103506 (2021). https://doi.org/10.1103/PhysRevD.104.103506. arXiv:2108.03147
77. Rybak, I.Y., Martins, C.J.A.P., Peter, P., Shellard, E.P.S.: Cosmological evolution of Witten superconducting string networks. Phys. Rev. D **107**(12), 123514 (2023). https://doi.org/10.1103/PhysRevD.107.123514. arXiv:2304.00053
78. Blanco-Pillado, J.J., Olum, K.D.: Electromagnetic radiation from superconducting string cusps. Nucl. Phys. B **599**, 435–445 (2001). https://doi.org/10.1016/S0550-3213(00)00771-9. arXiv:astro-ph/0008297
79. Wachter, J.M., Olum, K.D.: Electromagnetic backreaction from currents on a straight string. Phys. Rev. D **90**(2), 023510 (2014). https://doi.org/10.1103/PhysRevD.90.023510. arXiv:1405.2097

Publisher's Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.