Effect of Pasta Phase on Oscillations in Neutron Stars

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Abstract

We show that the shear modes in the neutron star crust are quite sensitive to the existence of nonuniform nuclear structures, the so-called "pasta". Due to the existence of pasta phase, the frequencies of shear modes are reduced, where the dependence of fundamental frequency is different from that of overtones. Since the torsional shear frequencies depend strongly on the structure of pasta phase, through the observations of stellar oscillations, one can probe the pasta structure in the crust, although that is quite difficult via the other observations. Additionally, considering the effect of pasta phase, we show the possibility to explain the all observed frequencies in the SGR 1806-20 with using only crust torsional shear modes.

1 Introduction

The soft gamma repeaters (SGRs) are considered as one of the most promising candidate of magnetars, which are neutron stars with strong magnetic fields. The sporadic X- and gamma-ray bursts are radiated from SGRs, while SGRs rarely emit much stronger gamma-rays called "giant flares". Up to now, at least three giant flares have been detected, which are the SGR 0526-66, the SGR 1900+14, and the SGR 1806-20. Through the timing analysis of these decaying tail, the quasi-periodic oscillations (QPOs) have discovered, which are in the range from tens Hz up to a few kHz [1]. Since the QPOs are believed as the outcomes of the neutron star oscillations, the observations of QPOs in SGRs could be first evidences to detect the neutron star oscillations directly.

It is considered that the neutron star crust exists from the bottom of the ocean of melted iron inward to the boundary with the inner fluid core at a density of order the saturation density of nuclear matter. Although nuclei in the crust form a bcc lattice due to Coulomb interactions, according to the recent studies, the nuclear structure in the bottom of crust could be nonuniform, i.e., with increasing the density, the shape of nuclear matter region is changing from sphere (bcc lattice) into cylinder, slab, cylindrical hole, and uniform matter (inner fluid core). This variation of nuclear structure is known as the so-called "pasta structure". The density that the cylinder structure appears, ρ_p, depends on the nuclear symmetry energy expressed with the density symmetry coefficient L, which is suggested to be order $\rho_p \sim 10^{13}$ g/cm^3 via the calculations of the ground state of matter in the crust. However, it might be quite difficult to verify the existence of pasta structure by using the observation of neutron star properties such as mass and radius, because the width of pasta phase is around 10% of the crust, which is less than a few hundred meters. In contrast, in this article, we will calculate the torsional oscillations of neutron star with pasta phase and show that the frequencies of shear modes depend strongly on the presence of pasta phase and on ρ_p. The more details of this study can be seen in [2].

2 Shear Modulus in Crust

Previously, there are many calculations about the oscillations of magnetars (e.g., [3–6]), where they assume that nuclei form bcc lattice in the crust when the crust torsional oscillations are considered. With this assumption, the shear modulus of the crust is suggested as

$$\mu = 0.1194 \times n_i (Ze)^2 / a, \tag{2.1}$$

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where n_i is the ion number density, $a = (3/4\pi n_i)^{1/3}$ is the average ion spacing, and $+Ze$ is the ion charge. On the other hand, it is pointed out that the elastic properties in the pasta phase could be liquid crystals rather than a crystalline solid [7]. That is, unlike the case of bcc lattice, the shear modulus should be decreasing in the pasta phase as increasing the density. This picture is thought to be natural, because the structure of nuclear matter changes gradually as mentioned the above and at last the shear modulus becomes zero in the fluid core. In order to realize such a relation about the shear modulus, we adopt Eq. (2.1) in the crust except for the pasta phase, while in the pasta phase it is assumed that the shear modulus can be expressed as the cubic function with respect to the density, which satisfies that μ should connect to Eq. (2.1) smoothly at $\rho = \rho_p$ and become zero smoothly at the boundary with the core, i.e., $\mu = c_1(\rho - \rho_c)^2(\rho - c_2)$, where c_1 and c_2 are some constants determined by the boundary conditions (see Fig. 1). Of course, this simple relation of μ in the pasta phase might be a kind of toy model, although expressed the rough behavior, because that should depend on the microscopic structure including the matter composition and/or the nuclear symmetry energy. Still, this relation is thought to be enough to examine the dependence of shear oscillations on the existence of pasta phase as a first step. Anyway, as a result of the fall-off of shear modulus, one can expect that the shear velocity will become smaller, and that the frequencies of shear modes will also decrease.

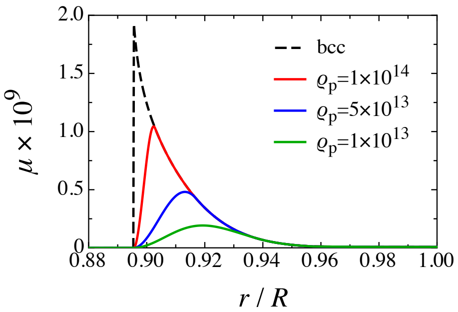

Figure 1: Adopted shear modulus μ as a function of relative radius r/R.

3 Torsional Shear Modes

The torsional shear modes can be described with using one perturbation variable, which is the angular displacement of the stellar matter, $\mathcal{Y}(t,r)$. The non-zero component of perturbed matter quantities is ϕ-component of the perturbed 4-velocity of fluid, δu^ϕ, which is expressed with $\mathcal{Y}$ as

$$\delta u^\phi = e^{-\Phi}\partial_t \mathcal{Y}(t,r)\sin^{-1}\theta\partial_\theta P_\ell(\cos\theta). \tag{3.1}$$

Assuming that the perturbed variable has a harmonic time dependence, such that $\mathcal{Y}(t,r) = e^{i\omega t}\mathcal{Y}(r)$, the perturbation equation reduces to

$$\mathcal{Y}'' + \left[\left(\frac{4}{r} + \Phi' - \Lambda'\right) + \frac{\mu'}{\mu}\right]\mathcal{Y}' + \left[\frac{\epsilon + p}{\mu}\omega^2 e^{-2\Phi} - \frac{(\ell+2)(\ell-1)}{r^2}\right]e^{2\Lambda}\mathcal{Y} = 0, \tag{3.2}$$

where ϵ and p correspond to the energy density and pressure, respectively, and the prime denotes the derivative with respect to r. With appropriate boundary conditions, the problem to solve becomes the eigenvalue problem. We impose a zero traction condition at the boundary between the inner core and the crust, and the zero-torque condition at the stellar surface.

4 Numerical Results

We examine the frequencies of torsional shear modes, as varying the value of ρ_p in the range of $\rho_p = 10^{13} - 10^{14}$ g/cm^3, because that value is not certain but suggested to be order 10^{13} g/cm^3 as mentioned before. The left panel of Fig. 2 shows the fundamental frequencies of torsional shear modes with $\ell = 2$ as a function of the stellar mass, where the broken line corresponds to the frequencies for the stellar model without pasta phase and the solid lines are corresponding to those with pasta phase with different values of ρ_p. Obviously, one can observe that the frequencies of shear modes depend strongly on the existence of pasta phase. In fact, compared with the frequency for the stellar model without pasta phase, those with pasta phase are reduced to 12.0%, 34.9%, and 49.3% for $\rho_p = 1 \times 10^{14}$, 4×10^{13}, and 1×10^{13} g/cm^3, respectively. While, in the right panel of Fig. 2, the frequencies of 1st overtones of shear modes with $\ell = 2$ are plotted as a function of the stellar mass. Unlike the fundamental modes, the frequencies with higher ρ_p are almost same as that without pasta phase. Still, one can see the dependence of the frequency on the existence of pasta phase with lower ρ_p. In fact, the frequencies with pasta phase are different from that without pasta phase in 0.4%, 46.0%, and 67.0% for $\rho_p = 1 \times 10^{14}$, 4×10^{13}, and 1×10^{13} g/cm^3, respectively. This dependence might be difficult to explain in analogy with the Newtonian limit, but one could be possible to obtain the additional information of the crust property via the observation of frequencies of overtones.

Since the both frequencies of fundamental and overtone shear modes depend strongly on the presence of pasta phase, whose effect has been neglected so far, one needs to consider this effect on the shear oscillations. Additionally, owing to this strong dependence, it could be possible to probe the properties of pasta phase via the observations of stellar oscillations and stellar mass, although the constraint on the pasta phase is quite difficult via the other observations of neutron stars. In contrast to simple relation of μ adopted in this article, we will make an examination with more realistic shear model in the pasta phase in the future.

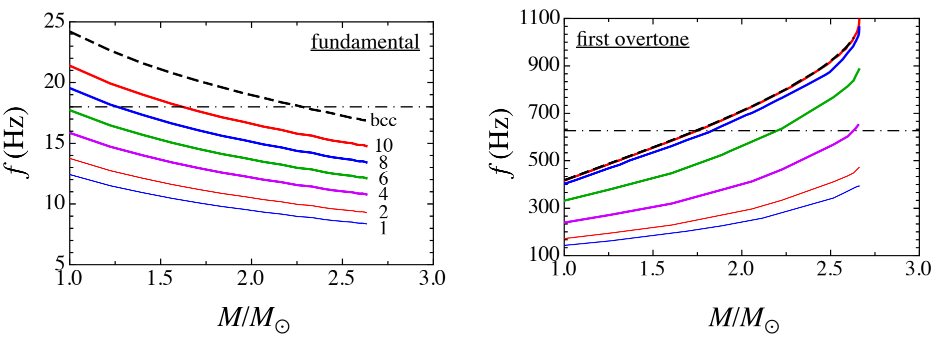

Figure 2: Frequencies of fundamental torsional shear modes (left) and first overtones (rihgt) with $\ell = 2$ as a function of neutron star mass.

At last, we will compare the calculated frequencies of shear modes with the observed QPO frequencies in giant flare. Especially, in this article we focus on the QPO frequencies in the SGR 1806-20, i.e., 18, 26, 30, and 92.5 Hz in less than 100 Hz, because this phenomenon has the most observed frequencies among the detected giant flares in the past [1]. The previous calculations are shown the difficulty to explain the all observed frequencies with using only crust shear modes [3]. That is, the fundamental $\ell = 2$ mode, which is the possible lowest frequency, is considered to correspond to the observed frequency of 18 Hz and the fundamental $\ell = 3$ mode is corresponding to 26 or 30 Hz. But, the spacing of shear frequencies with different ℓ is larger than the spacing between the observed frequencies of 26 and 30 Hz. In practice, according to such a traditional identification, we can explain the observed frequencies with the crust shear modes with pasta phase as shown in the left panel of Fig. 3, i.e., 18, 30, 92.5 Hz can be identified as $\ell = 2$, 3, and 10 fundamental modes within a few percent accuracy, where the expected stellar mass is $M = 1.5M_\odot$. However, we find the possibility to explain the all observed frequencies with using only crust shear modes, if ρ_p would be small. As shown in the right panel of Fig. 3, the observed frequencies of 18, 26, 30, and 92.5 Hz can be identified as $\ell = 3$, 4, 5, and 16 fundamental shear modes within a few percent accuracy again, where the expected stellar mass is $M = 1.5M_\odot$. This is important suggestion to explain the observed QPO frequencies in giant flares, which could become a directing post in the asteroseismology with neutron stars.

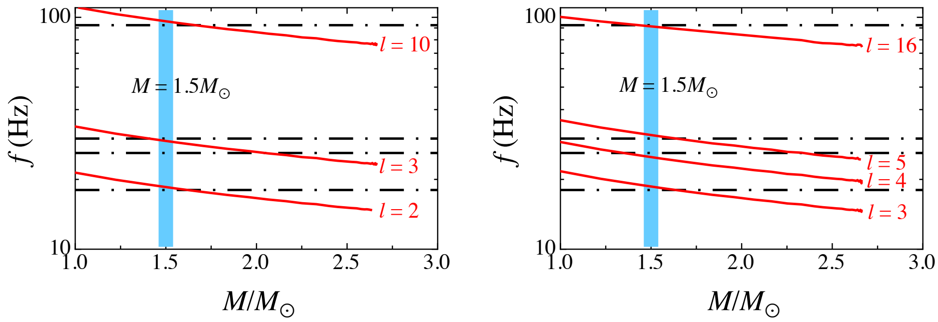

Figure 3: Comparison for the frequencies with the observed QPO frequencies in SGR 1806-20 with $\rho_{\rm p} = 10^{14}$ g/cm^{-3} (left) and with $\rho_{\rm p} = 2 \times 10^{13}$ g/cm^{-3} (right).

5 Conclusion

We consider the effect of nonuniform nuclear structure, the so-called "pasta", in the neutron star crust on the torsional shear modes. Based on the suggestion that the elastic properties in the pasta phase could be liquid crystals, the frequencies of shear modes are calculated with simple relation of shear modulus. As a result, due to the existence of pasta phase, one can observe the smaller frequencies than those expected without pasta phase. This result indicates not only the importance to take into account the pasta phase, but also the possibility to probe the pasta structure via the observations of the stellar oscillations, such as the QPO frequencies in giant flares. Furthermore, we show the possibility to explain the observed QPO frequencies with using only crust shear modes with pasta phase, which is quite difficult with the traditional identification without pasta phase. In order to find the most suitable stellar model with the observations, we need to adopt more realistic pasta structure and to examine systematically with different EOSs, where the additional effects, such as superfluidity inside the star as well as the stellar magnetic field, should be also taken into account. Still, we believe that the pasta structure in the neutron star curst could play an important role in the stellar oscillations.

Acknowledgements

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