Neutrinos in the Cosmos: Propagation of Ultrahigh Energy Neutrinos

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Efforts are under way to detect ultra high energy (UHE) neutrinos, $\nu(UHE)$. However, the reaction $\nu(UHE)+\bar{\nu}(CB)\rightarrow e^- + e^+$ has the threshold at $10^{13}\,eV$ for a neutrino of mass $0.058\,eV$ (suggested by the recent experiments on neutrino oscillations). This serves the major hindrance for the ultra high energy neutrinos, as neutrino halos, arising from the clustering of cosmic background neutrinos (CBν's), are taken into account. Owing to the 25% dark matter as compared to 5% visual ordinary matter, it is of utmost importance to "verify experimentally" whether neutrino halos account for this 25% dark matter. According to the Standard Model realized at the beginning of the 21st Century, there are no other "natural" dark-matter candidates other than neutrino halos. Thus, the behaviors of the associated neutrino halos near visual heavy objects, such as stars, planets, etc., need to be synthesized in detail. The "presumed" formation of a black hole of visual ordinary matter is stopped by the invisible "incompressible" dark-matter neutrino halo - "incompressible" due to Pauli's exclusion principle. Thus, the mass effect, due to the tiny neutrino mass of $0.058\,eV$, writes the last chapter of the story on the Standard Model.

1 Prelude

Could the neutrinos or antineutrinos travel freely in our Universe? Could the interactions with the "believed" cosmic background 1.9° neutrinos be completely negligible? We know that the cosmic background neutrinos would be of 1.9° if uniformly distributed [1], but neutrinos are now known to have masses so that they should cluster and hence deviate for the uniform 1.9° distribution.

Newton's gravitational law states that the force between two objects of mass m_1 and m_2 at the distance r is

$$force = G_N \frac{m_1 m_2}{r^2}, \qquad (1)$$

with the gravitational constant G_N. In Einstein's general relativity, the law becomes an equation in differential geometry.

Our first point is as follows: With such a tiny gravitational constant G_N, the objects in question must be macroscopic, that is, that each object would be (many) moles of the smallest units of matter. The masses m_1 and m_2 of the objects would be at least 10^{24} molecules. For planets or stars, it would be $10^{24}\cdot 10^{36}$ molecules (e.g., a star of five solar mass) - so, it goes through many complexities if enumerating from the very simple. Thus, Newton's gravitational law is a macroscopic law, *not* a microscopic law. As a macroscopic law, it has to build up from all the involved complexities and the simplicity must be spelled out in terms of symmetries.

The second important point has to do with the presence of the 25% dark matter. Our view is that these dark-matter particles are cosmic background (CB) ν's (i.e., neutrinos and antineutrinos, to be abbreviated simply as "neutrinos"). In the ideal case, each visual ordinary-matter macroscopic object, such as the Earth or the Venus, should carry five times in weight the neutrino halo. The reason for that is from the simplicity of the Newton's gravitational law (from Einstein's general relativistic law). *To sum up, a re-scaling of G_N by a factor of six (five plus self) is needed, if the dark matter of five times can be consistently taken into account.*

We are so much accustomed to the unified view that we always put the gravitational force at the same level of the strong and electroweak

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forces. But it is difficult to ignore the obstacles posed by all the complexities of the macroscopic law - Newton's gravitational law should be one of them.

Nowadays it is firmly established that there are three generations of quarks and three generations of leptons, at the level of the so-called "point-like Dirac particles". According to another newly established belief in Cosmology, the content of the current Universe would be 25% in the dark matter while only 5% in the ordinary matter, the latter described by the "minimal Standard Model" (mSM). In this language, the dark-matter particles are supposed to be described by a general Standard Model. Thus, there is certain urgent need to look for a general Standard Model.

Specifically, we suggest that we live in the quantum 4-dimensional Minkowski space-time with the force-fields gauge-group structure $SU_c(3) \times SU_L(2) \times U(1) \times SU_f(3)$ built-in from the outset. In this background, the quark world is accepted because of the (123) symmetry (i.e., under $SU_c(3) \times SU_L(2) \times U(1)$), while the lepton world is accepted in view of the other (123) symmetry (i.e., under $SU_L(2) \times U(1) \times SU_f(3)$). This gives us "our world" [2].

Here $SU_f(3)$ stands for the $SU_{family}(3)$ family gauge theory, rather than $SU_{flavor}(3)$ which derives from the isospin symmetry - the latter *not* a gauge theory.

Soon after the hot Big Bang, the particles of different kinds were manufactured with the neutrinos (abbreviated for "neutrinos and antineutrinos") as one of the central products. Even though they are oscillating from one kind into another, the neutrinos are basically "stable". In fact, neutrinos are only long-lived dark-matter particles [3], in the Standard Model. In view of the weak interactions which they participated, these neutrinos were decoupled slightly earlier than the $3^\circ K$ cosmic microwave background (CMB), that is why it was cooled longer, expected at $1.9^\circ K$ [1]. But these cosmic background neutrinos (CBν's), owing to the tiny mass, should cluster, into the neutrino halos, around the visual ordinary matter objects such as planets, stars, etc. The non-zero tiny masses help to re-write the story.

These CBν's are anticipated to form neutrino halos, forming the so-called dark-matter halos, around the visual ordinary-matter objects, such as planets, stars, etc. - all because of the Newton's gravitational force due to the fact that neutrinos have masses [3]. According to Newton's gravitational law, the object being exerted would be independent of the object's mass - so the neutrino's mass of $0.058\,eV$, though tiny, is irrelevant so long as the neutrino has mass. This is one mystery associated with the Newton's gravitational law.

The other important point is that the candidate for the 25% dark matter is neutrinos and antineutrinos, if we look at the early history of our Universe [4]. There is no other long-lived dark-matter particle, beside CBν's.

We may treat the eight major planets and the Sun having the same origin - having similar dark-matter (neutrino) halos. For a system that was formed very early in our Universe history, we believe that the possibility of the neutrino halos getting rip off would be rather small.

We would assume that the eight major planets and the Sun, and other large systems, all have their own neutrino halos. The reason is due to both that CBν's were there before the creation of the first stars and that neutrino halos need their own heavy centers. Basically, a neutrino halo is a feebly-interacting fermi gas, which was "attached" to the ordinary-matter chunk, when the latter was formed. It is the magic of the Newton's gravitational law.

The Earth also has the invisible neutrino halo, according to our view. But we have to go to the Venus or the Mercury to detect the feeble effect arising from their neutrino halos - basically, the effect to be detected is smaller than the feeble backgrounds which we could think of. We are not so sure if the Venus or the Mercury would fulfill our criterions. But we have to try.

Therefore, for the Venus or the Mercury, we assume that the neutrino halos would be (2 - 3) times the radius of the "mother" planet and, according to the 25% dark matter versus the 5% visual ordinary matter, its "weight" is five times the weight of the mother planet.

In what follows, we do estimates only for the Venus. There are different reasons why we keep the Mercury in the picture - since there may be no day-night difference (no self-rotation, or such effects small), no atmosphere presence, etc. Of course, the proximity to the Earth will make the "experiments" more feasible.

For the Venus, we have the following basic information:

$$R = 6,051\,Km,$$
$$M = 4.87 \times 10^{24} Kg,$$
$$T = 224.7\,days. \quad (2)$$

These numbers are quite close to those for the Earth, the twin brother ($R = 6378\,Km$, $M = 5.794 \times 10^{24} Kg$).

Assuming, for CBν's, that the five times of the mass distributed uniformly over 2.5 times over the radius, we obtain

$$Density = 1.68\,g/cm^3 = 5.61 \times 10^{32}\,(eV/c^2)/cm^3. \quad (3)$$

There would be a total of 10^{34} neutrinos of the largest mass $0.058\,eV$; divided by six (3 flavors, plus antiparticles), etc.

We'll use this number to get our final estimate.

For us the earthlings, the Mercury, the Venus, the Earth, and the Mars, why only on Earth are there living things? Soon or later, we hope that we could understand the origins of these planets. Why this difference? In fact, we might be able to understand these questions through the interactions of solar neutrinos shining for over billions of years - the messengers that could go very deep into these planets and yet change the species at the nuclear level. For these very reasons, we should spend time trying to investigate these questions - let's call it "Extraterrestrial Solar Neutrino Physics" [5].

As pointed out earlier [3], there is a huge neutrino halo associated with a galaxy. Locally, it would follow the distribution of the visual world, according to Newton's gravitational law. Thus, the Sun would be a local center of the CBν's, and, to be sensible, eight planets the eight centers - since the neutrino halos shouldn't be so huge because of the gravitational force (even for the neutrino halos as the relativistic Fermi gas).

If we could observe $\nu_e(Solar) + \bar{\nu}_e(CB) \to e^- + e^+$ using solar neutrinos, it would be easier on the Venus (or, the Mercury) than on the Earth, in view of the tininess of the cross section. It seems that, on the Venus and on the Mercury, the background for the proposed experiment might be minimal, especially away from the side of the sunshine.

Thus, the existence of the neutrino mass implies that the CBν's cluster according to the local environments, according to Newton's gravitational law. Therefore, the detection of the CBν's might be feasible on the Venus or on the Mercury. The existence of the CBν's is one of the fundamental issues that we, all physicists, are facing these days.

2 The Standard Model of All Centuries

In light of the Standard Model of all centuries [6, 2], the interest in the global behaviors of the CBν's is tremendous. The Standard Model [6] is basically the description of point-like particles, such as electrons, neutrinos, quarks, etc., basing on the Einstein's relativity principle and the quantum principle. We name it as "of all centuries" since it could be there a thousand years from now. We imagine that this Standard Model would replace the Newton's classic era, sooner or later.

In the beginning of the 21st Century, we could declare that we are in fact living in the quantum 4-dimensional Minkowski space-time with the force-fields gauge-group structure $SU_c(3) \times SU_L(2) \times U(1) \times SU_f(3)$ built-in at the very beginning - this defines the overall "background". The "background" can see the lepton world, of atomic sizes, which has the symmetry under $SU_L(2) \times U(1) \times SU_f(3)$. It can also see the quark world, of nuclear sizes, which has the well-known (123) symmetry. Altogether, it is called "the Standard Model" [6].

The observed persistent existence of the $3^\circ\,K$ cosmic microwave background (CMB) in our Universe is the *evidence* of that the force-fields gauge-group structure is "built-in from the very beginning". The CMB is rather uniform, to the level of one part in 10^5, due to the massless of the photons.

Theoretically, we need the family $SU_f(3)$ symmetry for the lepton world to make sure that the lepton world is asymptotically free and is free of Landau ghosts. Experimentally, it explains the three-generation problem and it also explains neutrino oscillations [6].

The "peculiar" characteristics are as follows: All the couplings in the lepton world are dimensionless in the 4-dimensional Minkowski space-time. All the couplings in the quark world are also dimensionless. Apart from the "ignition" term, all the couplings are dimensionless in the gauge and Higgs sector (forming the overall "background").

"Dimensionless-ness" means that it is the property of the 4-dimensional Minkowski space-time. It is truly "critical" in the quantum 4-dimensional Minkowski space-time as all these couplings would *not* be there if the dimension would be displaced by an infinitesimal amount from the integer four. So, we believe that these dimensionless couplings are determined *globally* by the 4-dimensional Minkowski space-time.

In this Standard Model [6], the neutrinos are the *only long-lived* dark-matter particles [4]. Yet, the theory excludes any additional particles from entering the theory - thus, it is complete as a theory.

Thus, we should base on this Standard Model, in analyzing the various basic particle-physics problems.

3 $\nu(Solar) + \bar\nu(CB) \to e^- + e^+$ on the Venus

First of all, we may make some crucial estimates to show why we could try to make the measurement on $\nu(Solar) + \bar\nu(CB) \to e^- + e^+$ on the Venus, provided that the surface of the Venus is rather "clean", without anything for the experimental background.

The major bunch of solar neutrinos are in the range of slightly above $1\,MeV$ while the CBν's (in an invisible gas) are at rest by comparison. The cross section σ is a couple of $10^{-45} E_\nu (in\ MeV) cm^2$ [7]. We know that the largest mass of ν's is about $0.058\,eV$.

So, on the surface of the Venus, the unit volume of the presumably CBν's (cf. Eq. (4) in the above) would be bombarded by the flux of solar neutrinos, yielding

$$\begin{aligned} &\phi_\nu \cdot N_{CB} \cdot \sigma \\ &\sim ((6,378/6051)^2 \cdot 6.0 \times 10^{10}/cm^2/sec) \cdot \\ &(5.61 \times 10^{32}/0.058/cm^3) \cdot (10^{-45} cm^2) \\ &\sim O(1)/sec/cm^3, \end{aligned} \tag{4}$$

which is close to unity, amazingly.

Thus, the neutrino halo of the Venus should be detectible unless the background is formidable (like in the Earth).

Coming back to think about the reactions induced by the cosmic background (CB) ν's assuming all the normal conditions in our nearby environments, we realize that the reaction $\nu(Solar) + \bar\nu(CB) \to e^- + e^+$, with some tiny cross sections, turns out to be the only choice.

We proceed to investigate $\nu(Solar) + \bar\nu(CB) \to e^- + e^+$ to see if it would be relevant on the Venus (or, on the Mercury). Here the $\nu(Solar)$ is the beam from the Sun and the $\bar\nu(CB)$ is the dark-matter cosmic background (CB) neutrino inherent on the Venus.

We have, for the reaction $\nu(Solar) + \bar\nu(CB) \to e^- + e^+$,

$$\nu(p) + \bar\nu(p') \to e^-(p_e) + e^+(p'_e), \tag{5}$$

the initial $\nu(Solar)$ would be ν_e but very soon would oscillate away (to ν_μ or to ν_τ). These neutrino states are the flavor states (*not* eigen-states) which are linear combinations of the mass Dirac eigen-states. Note that the solutions to the Dirac equation define the mass eigen-states, but *not* the flavor "eigen-states".

We could use the final $(e^- e^+)$ pair to "tag" the transition amplitudes: $T(\nu_e\bar\nu_e \to e^- e^+)$, $T(\nu_\mu\bar\nu_\mu \to e^- e^+)$, and $T(\nu_\tau\bar\nu_\tau \to e^- e^+)$ for producing the $(e^- e^+)$ from the visual nothing (neutrinos).

For example, we write the transition amplitude $T(\nu_e\bar\nu_e \to e^- e^+)$ [7],

$$\begin{aligned} &T(\nu_e\bar\nu_e \to e^- e^+) \\ =\ &\frac{G}{\sqrt{2}} i\bar u(p_e)\gamma_\lambda(g_V + g_A\gamma_5)v(p'_e) \\ &\cdot \sum_j \bar v^j(p'_\nu)U^\dagger_{ej}\gamma_\lambda(1+\gamma_5)\sum_i U_{ei}u^i(p_\nu) \end{aligned} \tag{6}$$

And $T(\nu_\mu\bar\nu_\mu \to e^- e^+)$ may be given below:

$$\begin{aligned} &T(\nu_\mu\bar\nu_\mu \to e^- e^+) \\ =\ &\frac{G}{\sqrt{2}} i\bar u(p_e)\gamma_\lambda(g'_V + g'_A\gamma_5)v(p'_e) \\ &\cdot \sum_j \bar v^j(p'_\nu)U^\dagger_{\mu j}\gamma_\lambda(1+\gamma_5)\sum_i U_{\mu i}u^i(p_\nu) \\ &+\frac{G'}{\sqrt{2}} i\bar u(p_e)(1-\gamma_5) \times \sum_i U_{\mu i}u^i(p_\nu) \\ &\cdot \sum_j \bar v^j(p'_\nu)U^\dagger_{\mu j}(1-\gamma_5) \times v(p'_e). \end{aligned} \tag{7}$$

Similar expression could be written for $T(\nu_\tau \bar{\nu}_\tau \to e^- e^+)$.

Here, in $T(\nu_\mu \bar{\nu}_\mu \to e^- e^+)$, the second term arises from the charged family Higgs exchange, presumably much smaller than exchange of $W^\pm$ or Z^0 (cf. [6]) at these energies.

Since we need to use the Dirac spinors for the neutrinos and since the Dirac spinors are written in terms of mass eigen-states, we are forced to use linear combinations to get the flavor states [8]. This sounds like a trivial point but may bring in a lot of subtleties on the basic concepts.

Thus, we have, for the $\nu_e \bar{\nu}_e \to e^- e^+$ channel,

$$\begin{aligned} &\sum_{av} \mid T \mid^2 \\ \approx\; &\frac{1}{4}(\frac{G_F}{\sqrt{2}})^2 \frac{1}{EE'k_0k_0'} \\ &\{(g_V^2 + g_A^2)2(p\cdot k'p'\cdot k + p\cdot kp'\cdot k') \\ &+2g_Vg_A2(p\cdot k'p'\cdot k - p\cdot kp'\cdot k')\}. \end{aligned} \quad (8)$$

Using the kinematics,

$$k_\mu = (\vec{k}, ik_0), \qquad k'_\mu = (0, im_\nu);$$
$$p_\mu = (\frac{\vec{k}}{2} + \vec{\delta}, iE), \qquad p'_\mu = (\tfrac{\vec{k}}{2} - \vec{\delta}, iE), \quad (9)$$

we find

$$\sum_{av} \mid T \mid^2 \approx (\frac{G_F}{\sqrt{2}})^2 (g_V^2 + g_A^2)(1 - \frac{1}{2}\frac{k^2}{k_0 E}). \quad (10)$$

Using the formula on the differential cross section,

$$\frac{d\sigma}{d\Omega} = \frac{\frac{k^2}{4} + \delta^2}{(2\pi)^2} \sum_{av} \mid T \mid^2, \quad (11)$$

it completes the part on the formulae.

Numerically, it is a peculiar kinematics since $m_\nu = 0.058eV$, $m_e = 0.511MeV$, and the energy of the solar neutrino somewhat larger than $1\,MeV$. Only three-body mode, or B^8, N^{13}, and O^{15} solar neutrinos have enough energy to go, according to the above "table" [1]. As an estimate, we have [7]

$$\sigma \sim 10^{-45} cm^2, \quad (12)$$

which is really a tiny cross section. In view of its importance in verifying the cosmic background (CB) neutrinos, we should try as one of the first experiments in the 21st Century.

4 $\nu(UHE) + \bar{\nu}(CB) \to e^- + e^+$

The dark matter, 25% in our Universe as compared to 5% of ordinary matter, should be cosmic background neutrinos and antineutrinos (CBν's), according to the Standard Model [6] in which CBν's are the *only long-lived invisible particles* [4].

What happens in the Cosmos to the ultra high energy (UHE) neutrinos? UHE neutrinos may encounter the antineutrinos in the CBν's environment, so annihilating to give the $(e^- e^+)$ pairs, or the $e^- \mu^+$ pairs (at higher energies), or the $\mu^- \mu^+$ pairs (at even higher energies), etc. UHE neutrinos may encounter the light nuclei, e.g., $\bar{\nu}_e(UHE) + p \to e^+ + n$, $\bar{\nu}_e(UHE) + ^4He \to e^+ + ^3H + n$, etc. Or, UHE neutrinos may encounter the electrons, such that $\bar{\nu}_e(UHE) + e^- \to W^* \to$ *many things*. It is clear that the whole particle physics would be at play.

In terms of the thresholds (for the reactions), the $e^- e^+$ pairs would be on the lowest threshold, the encounter with light nuclei would be the next, etc. But the elastic scattering such as $\nu(UHE) + e^- \to e^- + \nu$, $\nu(UHE) + p \to p + \nu$, etc. is free of the threshold. The location that the human being sits can not exempt from electrons, from protons, and so on.

Let's set aside the problem due to that our local environments are so complicated in extracting the information on UHE neutrinos. We shall focus on $\nu(UHE) + \bar{\nu} \to e^- + e^+$.

Again, we may consider the kinematics

$$k_\mu = (\vec{k}, ik_0), \qquad k'_\mu = (0, im_\nu);$$
$$p_\mu = (\frac{\vec{k}}{2} + \vec{\delta}, iE), \qquad p'_\mu = (\tfrac{\vec{k}}{2} - \vec{\delta}, iE), \quad (13)$$

but making k_0 in the energy range that is greater than $10^{13}\,eV$, the energy of the ultra high energy (UHE) neutrino. In this case, the gauge-boson masses may not be the only large energy in the problem. Thus, the transition amplitude, in the

U-gauge, should be replaced by

$$\begin{aligned} & T(\nu_e(UHE)+\bar{\nu}_e(CB)\to e^-+e^+) \\ = & \frac{G}{\sqrt{2}}\{m_W^2\frac{\delta_{\lambda\lambda'}+(q')_\lambda(q')_{\lambda'}/m_W^2}{(q')^2+m_W^2-i\epsilon} \\ & i\bar{u}(p_e)\gamma_\lambda(1+\gamma_5)v(p_e')+ \\ & m_Z^2\frac{\delta_{\lambda\lambda'}+q_\lambda q_{\lambda'}/m_Z^2}{q^2+m_Z^2-i\epsilon} \\ & i\bar{u}(p_e)\gamma_\lambda(g_V'+g_A'\lambda_5)v(p_e')\} \\ & \cdot\sum_j \bar{v}^j(p_\nu')U_{ej}^\dagger\gamma_{\lambda'}(1+\gamma_5) \\ & \sum_i U_{ei}u^i(p_\nu). \end{aligned} \quad (14)$$

Here $(q')_\mu = k_\mu - p_\mu$ is the four-momentum transfer going through the $W^\pm$ boson (in the t-channel) while $q_\mu = k_\mu + k'_\mu$ is going through the Z^0 boson in the s-channel. In the $t-$channel, $(q')^2$ and q'_μ could be very large for the ultrahigh energy neutrino:

$$\begin{aligned} (q')^2 &= (\frac{\vec{k}}{2}-\vec{\delta})^2-(k_0-E)^2; \\ q'_\mu &= (\frac{\vec{k}}{2}-\vec{\delta}, i(k_0-E)). \end{aligned} \quad (15)$$

In fact, we have, for $k_0 >> m_e >> m_\nu$,

$$\begin{aligned} & (q')^2 \\ = & \frac{k^2}{4}+\delta^2 \\ & -\{\sqrt{k^2+m_\nu^2}-\sqrt{(k^2/4)+\delta^2+m_e^2}\}^2 \\ = & +(2\delta^2+m_e^2-\frac{m_\nu^2}{2}+...), \end{aligned} \quad (16)$$

such that, by treating δ as the same order as m_e, the $(q')^2$ has to be of the same order as m_e^2.

Likewise, the $s-$channel is controlled by $q^2 = \vec{k}^2-(k_0^2+m_\nu)^2 = -(2m_\nu k+m_\nu^2+...)$, a competing number in the range of $\pm m_e^2$ (at $k_0 = 10^{13}\,eV$).

The term in $q'_\lambda q'_{\lambda'}/m_W^2$ or in $q_\lambda q_{\lambda'}/m_Z^2$, when squared in $\mid T\mid^2$, would appear as functions of $(q')^2$, or q^2, or, perhaps in interference terms, $q'\cdot q$. As a result, they are tiny because of the extra $(q')^2/m_W^2$ or q^2/m_Z^2 factor, and so on.

Similar discussions can be applied to the other transition amplitude:

$$\begin{aligned} & T(\nu_\mu(UHE)+\bar{\nu}_\mu(CB)\to e^-+e^+) \\ = & \frac{G}{\sqrt{2}}m_Z^2\frac{\delta_{\lambda\lambda'}+q_\lambda q_{\lambda'}/m_Z^2}{q^2+m_Z^2-i\epsilon} \\ & i\bar{u}(p_e)\gamma_\lambda(g_V'+g_A'\gamma_5)v(p_e') \\ & \cdot\sum_j \bar{v}^j(p_\nu')U_{\mu j}^\dagger\gamma_{\lambda'}(1+\gamma_5)\sum_i U_{\mu i}u^i(p_\nu) \\ & +\frac{G'}{\sqrt{2}}m_H^2\frac{1}{(q')^2+m_H^2-i\epsilon}i\bar{u}(p_e)(1-\gamma_5) \\ & \times\sum_i U_{\mu i}u^i(p_\nu)\cdot\sum_j \bar{v}^j(p_\nu')U_{\mu j}^\dagger(1-\gamma_5) \\ & \times v(p_e'). \end{aligned} \quad (17)$$

And we have a similar expression for $\nu_\tau(UHE)+\bar{\nu}_\tau(CB)\to e^-+e^+$.

Our discussions lead to the following simplified transition amplitudes:

$$\begin{aligned} & T(\nu_e(UHE)+\bar{\nu}_e(CB)\to e^-+e^+) \\ = & \frac{G}{\sqrt{2}}\{i\bar{u}(p_e)\gamma_\lambda(1+\gamma_5)v(p_e')+ \\ & i\bar{u}(p_e)\gamma_\lambda(g_V'+g_A'\lambda_5)v(p_e')\} \\ & \cdot\sum_j \bar{v}^j(p_\nu')U_{ej}^\dagger\gamma_{\lambda'}(1+\gamma_5) \\ & \sum_i U_{ei}u^i(p_\nu). \end{aligned} \quad (18)$$

$$\begin{aligned} & T(\nu_\mu(UHE)+\bar{\nu}_\mu(CB)\to e^-+e^+) \\ = & \frac{G}{\sqrt{2}}i\bar{u}(p_e)\gamma_\lambda(g_V'+g_A'\gamma_5)v(p_e') \\ & \cdot\sum_j \bar{v}^j(p_\nu')U_{\mu j}^\dagger\gamma_{\lambda'}(1+\gamma_5)\sum_i U_{\mu i}u^i(p_\nu) \\ & +\frac{G'}{\sqrt{2}}i\bar{u}(p_e)(1-\gamma_5)\times\sum_i U_{\mu i}u^i(p_\nu) \\ & \cdot\sum_j \bar{v}^j(p_\nu')U_{\mu j}^\dagger(1-\gamma_5)\times v(p_e'). \end{aligned} \quad (19)$$

Again, we use the formula for the differential cross section for the ultrahigh energy neutrinos, say, at $10^{13}\,eV$,

$$\frac{d\sigma}{d\Omega} = \frac{\frac{k^2}{4}+\delta^2}{(2\pi)^2}\sum_{av}\mid T\mid^2, \quad (20)$$

We would obtain

$$\sigma \sim 10^{-31}cm^2, \quad (21)$$

a much larger cross section but, for the UHE neutrinos, this is not subject to observations. Maybe the LHC could make $10^{12}\,eV$ or TeV neutrinos subject to experimentations.

For $\nu_\mu + \bar{\nu}_\mu \to e^- + e^+$, the first term, in fact from the Z^0 boson, is from the $s-$channel. The second term, arising the family Higgs boson, involves the cross-dot products and so have the $t-$channel behavior.

So far, we have used the U-gauge in our analysis; in the tree approximations, we note that the results are complete to this order.

5 Invisible Neutrino Halos and Black Holes

Do neutrino halos stop the formation of black holes? That is, there is no black hole formed when the physics of neutrino halos is taken into account.

Neutrinos are fermions; they would be stacking up like fermi liquids since one state can accommodate only one neutrino.

In the limit of the very low temperature and very high densities, we have [10]

$$U = \tfrac{3}{5} N\epsilon_F [1 + \tfrac{5}{12}\pi^2 (\tfrac{kT}{\epsilon_F})^2 + ...];$$
$$P = \tfrac{2}{3}\tfrac{U}{V} = \tfrac{2}{5}\tfrac{\epsilon_F}{v}[1 + \tfrac{5\pi^2}{12}(\tfrac{kT}{\epsilon_F})^2 + ...];$$
$$\epsilon_F = \tfrac{\hbar^2}{2m}(\tfrac{6\pi^2}{gv})^{2/3}; \qquad v \equiv \tfrac{V}{N}. \tag{22}$$

In this limit, it is characterized by a large Fermi energy ϵ_F, of very small kinetic energy. That is, the very small mass is accompanied by a very large Fermi energy ϵ_F.

So, we need to apply these formulae to the neutrino gas in the case of the neutrino halo for the "visual" collapsed star - to see if the neutrino halo would stop the collapse of the star.

First of all, we note from the factor $\hbar^2/(2m)$ that it is an effect arising from the quantum principle and that it is also due to the non-zero mass in this limit. Secondly, it may be difficult to determine v, the volume per particle, in the same limit. But the white dwarf gives an example. It gives, for a dwarf star of a solar mass ($10^{33}\,gm$) [10],

$$\epsilon_F \approx \frac{\hbar^2}{2m_e}\frac{1}{v^{2/3}} \approx 20\,MeV. \tag{23}$$

Using this formula for the neutrino halo with $m_\nu = 0.058\,eV$,

$$\epsilon_F^\nu \approx \frac{\hbar^2}{2m_\nu}\frac{1}{v^{2/3}} \approx 200\,TeV. \tag{24}$$

Maybe we could generalize the uncertainty relation as follows:

$$(\epsilon_F v^{2/3}) \cdot m \geq \hbar^2. \tag{25}$$

In light of the tiny neutrino mass, there is some quantity "conjugate" to the mass quantity for the uncertainty relation. The "conjugate" quantity to the mass is "the Fermi energy times the occupied area per particle". According to our presentation, this uncertainty relation might make some sense though we don't know the unit volume v which may be the same as the electron case.

The Schwarzschild radius, or the horizon, is so tiny (by many orders) in this game while the neutrino halo has the minimum volume that is much bigger (by many orders) than the horizon (of the presumed black hole) - so that the pulling-back gravitational force exerted by the neutrino halo becomes overwhelming at the very last stage. The visual story is dictated by the invisible partner, the neutrino halo. The conclusion is that a black hole consisting of visual ordinary matter cannot be formed, because of the incompressible dark-matter neutrino halo.

Thus, the destiny of a visual ordinary-matter star plus its five times in weight invisible neutrino halo will be determined by the neutrino halo, as a degenerate (invisible) Fermi gas, well ahead of the presumed "black hole" of the visual ordinary-matter lump. A reasonable theory usually gives us a conclusion that is a little boring. We believe that the Standard Model would prevail.

6 Concluding Remarks

Neutrinos have been with us for nearly a whole century, since the early 1930's. Because the feeble nature of its interactions with the environments, we do not understand the neutrinos yet. We thought that we understand neutrinos in terms of the Standard Model [6] owing to the beauty, the consistency, and the completeness of the theory. Basically, the Standard Model is the description of the point-like particles on the basis of the Einstein's relativity principle and the quantum principle. The Standard Model already covers *all* point-like particles since it is redundant on adding to it some new particle(s). It really is our world, and nothing more.

Basing on this Standard Model [6], cosmic background neutrinos and antineutrinos (CBν's) are the 25% dark matter, as contrast with the 5% ordinary matter in our Universe [4].

The neutrino halos (i.e. 25% dark matter) always accompany the 5% visual ordinary-matter objects, such as stars, planets, etc., such that the world behaves normally and that the collapse of a visual star into a black hole never happens.

Since the 5 % ordinary matter are accompanied by the 25 % invisible dark matter (in "incompressible" neutrino halos), the story of formation of black holes of visual ordinary matter is just a fiction.

In this paper, we just try to "convert" those unknowns to those which we could understand. That is, we could base on the Standard Model in trying to understand our Universe.

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