




Inflation models with Peccei–Quinn symmetry and axion kinetic misalignment

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Abstract We propose a consistent framework with the $U(1)$ Peccei–Quinn (PQ) symmetry for obtaining the initial condition for axion kinetic misalignment from inflation. We introduce a PQ complex scalar field and an extra Higgs doublet, which are conformally coupled to gravity, and three right-handed neutrinos for the seesaw mechanism. In the DFSZ type scenarios for the axion, we obtain the PQ anomalies from the Standard Model quarks carrying nonzero PQ charges in some of two Higgs doublet models, solving the strong CP problem by the QCD potential for the axion. Assuming that the PQ symmetry is explicitly violated in the scalar potential by quantum gravity effects, we show that a sufficiently large initial axion velocity can be obtained before the QCD phase transition while avoiding the axion quality problem. As inflation is driven by the radial distance from the origin in the space of scalar fields close to the pole of the kinetic terms in the Einstein frame, we obtain successful inflationary predictions and set the initial axion velocity at the end of inflation. Focusing on the pure PQ inflation with a small running quartic coupling for the PQ field, we discuss the post-inflationary dynamics for the inflaton and the axion. As a result, we show that a sufficiently high reheating temperature can be obtained dominantly from the Higgs-portal couplings to the PQ field, while being consistent with axion kinetic misalignment, the stability for the Higgs fields during inflation and the non-restoration of the PQ symmetry after reheating.

1 Introduction

The QCD axion is a pseudo-Goldstone boson of the $U(1)$ Peccei–Quinn (PQ) symmetry and it solves the strong CP problem in the Standard Model (SM) by the dynamical relaxation mechanism during the cosmological evolution after the QCD phase transition [1]. It is also a candidate for decaying cold dark matter [2–4], with feeble interactions to gluons and photons and potentially other SM particles [5–8]. The relic abundance for the axion is determined by the misalignment mechanism [2–4] where the axion is assumed to be displaced from the origin of the axion potential during or after inflation and becomes a coherently oscillating cold dark matter after the QCD phase transition.

Recently, the possibility that the axion has a nonzero initial velocity before the QCD phase transition has drawn much attention [9]. In this case, even after the QCD phase transition, the axion is not trapped into one of the minima of the periodic potential until its kinetic energy is comparable to the axion potential energy. Thus, the axion oscillation is delayed until a late time after the standard oscillation time of the axion without a initial velocity, so the dilution of the axion energy density due to the Hubble expansion is milder, leading to the correct relic density even for a smaller initial energy density of the axion or a smaller axion decay constant [9]. In order to achieve a nonzero initial velocity for the axion, however, an explicit violation of the PQ symmetry is required, while the axion quality is maintained to solve the strong CP problem. Thus, it is necessary to make the overall coefficient of the PQ violating potential dynamical, so there must be a period in the early universe where the radial mode of the PQ complex scalar field is much larger than the one in the vacuum and it decays away [9–11].

In this article, we aim to achieve a nonzero initial axion velocity for the axion kinetic misalignment in a consistent

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model with the $U(1)$ PQ symmetry for inflation, reheating and all the way to the axion oscillation. We extend our previous results of the PQ inflation in KSVZ type axion models [10] to the case in DFSZ type axion models where one more Higgs doublet and the PQ complex scalar field are added beyond the SM [7, 8], instead of an extra heavy vector-like colored quark. We also introduce the interactions for three right-handed (RH) neutrinos being consistent with the PQ symmetry and obtain the masses for the RH neutrinos due to the spontaneous breaking of the PQ symmetry. Thus, neutrino masses can be generated in the seesaw mechanism with a small mixing between the active neutrinos and the heavy RH neutrinos.

We assume that the PQ field and two Higgs doublets are coupled conformally to gravity and the scalar potential is composed of PQ invariant renormalizable terms and PQ violating higher dimensional terms. In this setup, we derive the effective Lagrangians for the inflaton in the cases for both the Higgs-PQ mixed and pure PQ inflations. The inflaton is identified as the radial distance from the origin in the field space of scalar fields, driving a slow-roll inflation close to the pole of the kinetic terms [12]. We also consider the stabilization of all the non-inflaton scalar fields during inflation and identify the initial axion velocity at the end of inflation.

Focusing on the pure PQ inflation where the correct inflation scale is achieved from a small running quartic coupling of the PQ field, we analyze the inflationary predictions, the post-inflationary dynamics of the inflaton and the perturbative reheating, and determine the relic density from the axion kinetic misalignment. We show how the stabilization conditions for the Higgs fields and the non-restoration of the PQ symmetry after reheating can constrain the parameter space for the axion kinetic misalignment.

The paper is organized as follows. We begin with the setup for DFSZ type axion models and present two Higgs doublet models based on Z_2 parities and anomaly coefficients for the PQ symmetry in each model. Then, we discuss the vacuum structure of the model from the scalar potential for charge-neutral scalar fields and discuss the axion quality problem with general PQ violating potentials. Next, the detailed discussion on the effective theory for the inflaton with the Higgs-PQ mixed field or the pure PQ field is presented and the stabilization of the non-inflaton scalar fields is shown. Afterwards, we discuss the slow-roll inflation and the initial axion velocity in the case of the pure PQ inflation. We also consider the reheating procedure and determine the amount of dark radiation from the axions produced during reheating. From the post-inflationary evolution of the axion velocity, we also get the relic density from the axion kinetic misalignment after the QCD phase transition. Finally, conclusions are drawn.

2 The setup

We present the model setup for the PQ inflation in the DFSZ axion model where there are a complex PQ scalar field and an extra Higgs doublet beyond the SM. We identify four types of two Higgs doublet models depending on Z_2 parities and include three right-handed neutrinos for getting neutrino masses by the seesaw mechanism, being consistent with the PQ symmetry. We also show the axion couplings to gluons and photon in each of two Higgs doublet models.

We first consider the bosonic part of the Lagrangian in the Jordan frame in the DFSZ axion model as

$$\frac{\mathcal{L}_J}{\sqrt{-g_J}} = -\frac{1}{2}M_P^2 \Omega R(g_J) + \mathcal{L}_{\text{kin},J} - \Omega^2 V_E \quad (1)$$

where Ω is the frame function of H_1 , H_2 and Φ , the kinetic terms are given by

$$\mathcal{L}_{\text{kin},J} = |D_\mu H_1|^2 + |D_\mu H_2|^2 + |\partial_\mu \Phi|^2 \quad (2)$$

and V_E is the Einstein frame potential, composed of PQ invariant and PQ violating terms as $V_E = V_{\text{PQ}} + V_{\text{PQV}}$.

Taking the conformal couplings of the scalar fields to gravity by

$$\Omega = 1 - \frac{1}{3M_P^2}|H_1|^2 - \frac{1}{3M_P^2}|H_2|^2 - \frac{1}{3M_P^2}|\Phi|^2, \quad (3)$$

we obtain the Einstein frame Lagrangian as

$$\begin{aligned} \frac{\mathcal{L}_E}{\sqrt{-g_E}} = & -\frac{1}{2}M_P^2 R + \frac{1}{\Omega} \left(|D_\mu H_1|^2 + |D_\mu H_2|^2 + |\partial_\mu \Phi|^2 \right) \\ & + \frac{3}{4}M_P^2 \frac{(\partial_\mu \Omega)^2}{\Omega^2} - V_E. \end{aligned} \quad (4)$$

2.1 Scalar potential

We assume that the scalar fields transform under the PQ symmetry as

$$\begin{aligned} \Phi & \rightarrow e^{iq\Phi\alpha} \Phi, \\ H_1 & \rightarrow e^{iq_1\alpha} H_1, \\ H_2 & \rightarrow e^{iq_2\alpha} H_2. \end{aligned}$$

Then, the PQ-invariant Einstein frame potential is given by

$$\begin{aligned} V_{\text{PQ}} = & \lambda_\Phi |\Phi|^4 + \lambda_1 |H_1|^4 + \lambda_2 |H_2|^4 + \lambda_3 |H_1|^2 |H_2|^2 \\ & + \lambda_4 \left(H_1^\dagger H_2 \right) \left(H_2^\dagger H_1 \right) \\ & + \lambda_{1\Phi} |H_1|^2 |\Phi|^2 + \lambda_{2\Phi} |H_2|^2 |\Phi|^2 \\ & + \left(2^{p/2-1} \kappa_p H_1^\dagger H_2 \Phi^p + \text{h.c.} \right) \\ & + \mu_\Phi^2 |\Phi|^2 + m_1^2 |H_1|^2 + m_2^2 |H_2|^2 + V_0, \end{aligned} \quad (5)$$

with V_0 being a cosmological constant. Then, if the PQ charges satisfy $pq_\Phi - q_1 + q_2 = 0$, the $H_1^\dagger H_2 \Phi^p$ term is PQ

Table 1 Z_2 parities and PQ charges for two Higgs doublet models

Z_2	Φ	H_1	H_2	q_L	u_R	d_R	l_L	e_R
Type I	+	-	+	+	+	+	+	+
Type II	+	-	+	+	+	-	+	-
Type X	+	-	+	+	+	+	+	-
Type Y	+	-	+	+	+	-	+	+
PQ	Φ	H_1	H_2	q_L	u_R	d_R	l_L	e_R
Type I	q_Φ	q_1	$q_1 - pq_\Phi$	0	$q_1 - pq_\Phi$	$-q_1 + pq_\Phi$	0	0
Type II	q_Φ	q_1	$q_1 - pq_\Phi$	0	$q_1 - pq_\Phi$	$-q_1$	0	$-q_1$
Type X	q_Φ	q_1	$q_1 - pq_\Phi$	0	$q_1 - pq_\Phi$	$-q_1 + pq_\Phi$	0	$-q_1$
Type Y	q_Φ	q_1	$q_1 - pq_\Phi$	0	$q_1 - pq_\Phi$	$-q_1$	0	$-q_1 + pq_\Phi$

invariant.¹ Since the PQ symmetry is broken dominantly by the VEV of the Φ field for the invisible axion, we need to choose $q_\Phi \neq 0$. We introduce a cosmological constant V_0 in the Einstein frame to set the vacuum energy to zero.

The PQ symmetry can be broken explicitly due to quantum gravity, with the PQ violating terms in the potential, as follows,

$$V_{\text{PQV}} = \sum_{n,l} \sum_{k=0}^{[l/2]} \frac{c_{n,l,k}}{2M^{2n+l-4}} (H_1^\dagger H_2)^n |\Phi|^{2k} \Phi^{l-2k} + \text{h.c.}, \tag{6}$$

for $n(q_2 - q_1) + q_\Phi(l - 2k) \neq 0$, which are constrained by the quality of the axion for solving the strong CP problem. Henceforth, we set $M_P = 1$ for convenience, but we recover M_P whenever necessary.

2.2 Two Higgs doublet models

The most general Yukawa interactions will be of the form

$$\mathcal{L}_Y = y_{ij} \bar{f}_L H_1 f_R + y'_{ij} \bar{f}_L H_2 f_R, \tag{7}$$

up to the replacement of $H_{1,2}$ with $\tilde{H}_{1,2}$ if f_R is an up-type quark or an RH neutrino. However, the general Yukawa couplings would lead to dangerous FCNCs. Thus, there are several phenomenologically viable possibilities for the flavor-independent assignments for Z_2 parities and PQ charges, summarized in Table 1.

Looking at Table 1, we see that for the Type I model, all quarks couple to just one of the Higgs fields, which we take to be H_2 . In contrast, in the Type II model, u_R couples to H_2 but d_R and e_R couple to H_1 . In the Type X, both u_R and d_R couple to H_2 but e_R couples to H_1 , and finally, in the Type Y (flipped), u_R and e_R couple to H_2 and d_R couples to H_1 .

For instance, in the case of the Type II model in Table 1, we obtain the PQ and Z_2 invariant Yukawa couplings,

$$\mathcal{L}_Y = -Y_u \bar{q}_L \tilde{H}_2 u_R - Y_d \bar{q}_L H_1 d_R - Y_e \bar{l}_L H_1 e_R. \tag{8}$$

We comment on the neutrino Yukawa couplings and the mass terms for the right-handed (RH) neutrinos, N_R . If the RH neutrinos are Z_2 -even, we can introduce the neutrino Yukawa couplings for Dirac and Majorana neutrino masses, as follows,

$$\mathcal{L}_v^{(1)} = -Y_\nu \bar{l}_L \tilde{H}_2 N_R - \frac{1}{2} y_N \overline{N_R^c} \Phi N_R. \tag{9}$$

Then, we need to impose $q_{N_R} = q_1 - pq_\Phi = -\frac{1}{2}q_\Phi$, so $q_\Phi = \frac{2}{2p-1}q_1$ and $q_{N_R} = -\frac{q_1}{2p-1}$. On the other hand, if the RH neutrinos carry Z_2 -odd parities, we need to take alternative mass terms for neutrinos, as follows,

$$\mathcal{L}_v^{(2)} = -Y_\nu \bar{l}_L \tilde{H}_1 N_R - \frac{1}{2} y_N \overline{N_R^c} \Phi N_R \tag{10}$$

In this case, we need to take the PQ charge for N_R as $q_{N_R} = q_1 = -\frac{1}{2}q_\Phi$ instead of $q_{N_R} = q_1 - pq_\Phi$ in Table 1, so $q_\Phi = -2q_1$ and $q_{N_R} = q_1$.

2.3 Axion couplings to gluons and photon

The color and electromagnetic anomalies of the axial current associated with the axion are given in the following Lagrangian,

$$S_{\text{anom}} = \frac{k_G}{8\pi^2} \int \theta \text{tr}(G \wedge G) + \frac{k_F}{8\pi^2} \int \theta F \wedge F, \tag{11}$$

with $\theta = a/f_a$. Then, we obtain the effective axion-photon coupling below the PQ symmetry breaking scale, as follows,

$$\mathcal{L}_{\text{photon}} = -\frac{1}{4} g_{a\gamma\gamma} a F_{\mu\nu} \tilde{F}^{\mu\nu} \tag{12}$$

with

$$g_{a\gamma\gamma} = \frac{\alpha}{2\pi f_a \xi} \left(\frac{E}{N} - 1.92 \right). \tag{13}$$

Here, $\xi = k_G$ and $E/N = 2k_F/k_G$ from $N = \frac{1}{2}k_G$ and $E = k_F$, and the domain wall number is given by $N_{\text{DW}} =$

¹ For renormalizable interactions, we can take $p = 1$ or $p = 2$.

Table 2 PQ anomalies and axion-photon couplings in two Higgs doublet models

	k_G	k_F	E/N
Type I	0	$3(pq_\Phi - q_1)$	—
Type II	$3pq_\Phi$	$4pq_\Phi$	$\frac{8}{3}$
Type X	0	$3pq_\Phi$	—
Type Y	$3pq_\Phi$	pq_Φ	$\frac{2}{3}$

$\xi = k_G$ [1]. The values for k_G, k_F in DFSZ-like models [7, 8] are shown in Table 2. Here, we note that $E/N = \frac{8}{3}$ for Type II, which is consistent with unification. For comparison, in KSVZ model with a neutral vector-like colored triplet [5, 6], the anomaly coefficients are given by $k_G = 1$ and $k_F = 0$, so $E/N = 0$.

and discuss the axion quality problem in the presence of higher dimensional PQ violating terms in the potential.

3.1 Einstein frame Lagrangian for neutral scalars

We parametrize the charge-neutral components of Higgs and the PQ fields in the following,

$$H_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ h_1 e^{i\eta_1} \end{pmatrix}, \quad H_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ h_2 e^{i\eta_2} \end{pmatrix},$$

$$\Phi = \frac{1}{\sqrt{2}} \rho e^{i\theta}. \tag{14}$$

Then, the kinetic terms of the Einstein-frame Lagrangian in Eq. (4) become

$$\frac{\mathcal{L}_{\text{kin},E}}{\sqrt{-g_E}} = -\frac{1}{2}R + \frac{(\partial_\mu h_1)^2 + (\partial_\mu h_2)^2 + h_1^2 (\partial_\mu \eta_1)^2 + h_2^2 (\partial_\mu \eta_2)^2 + (\partial_\mu \rho)^2 + \rho^2 (\partial_\mu \theta)^2}{2(1 - \frac{1}{6}h_1^2 - \frac{1}{6}h_2^2 - \frac{1}{6}\rho^2)}$$

$$+ \frac{3 \left(\partial_\mu \left(\frac{1}{6}h_1^2 + \frac{1}{6}h_2^2 + \frac{1}{6}\rho^2 \right) \right)^2}{4 \left(1 - \frac{1}{6}h_1^2 - \frac{1}{6}h_2^2 - \frac{1}{6}\rho^2 \right)^2}. \tag{15}$$

It was recently pointed out that if there is no fractionally charged color-singlet particle, the SM gauge groups are globally identified by $[SU(3)_C \times SU(2)_L \times U(1)_Y]/Z_6$, which corresponds to $[SU(3)_C \times U(1)_{\text{em}}]/Z_3$ after electroweak symmetry breaking. In this case, the anomaly coefficients, k_G, k_F , are quantized by $k_G \in \mathbb{Z}$ and $\frac{2}{3}k_G + k_F \in \mathbb{Z}$ [13–16]. The first quantization condition means $3pq_\Phi \in \mathbb{Z}$ for both Type II and Y. On the other hand, since $\frac{2}{3}k_G + k_F = 6pq_\Phi, 3pq_\Phi$ for Type II and Y, respectively, we find that an integer, $k_G = 3pq_\Phi$, is sufficient for the second quantization condition. If q_Φ is an integer, which is a sufficient condition for the first quantization condition, the domain wall number is a multiple of three or six from $N_{\text{DW}} = k_G = 3pq_\Phi$ for $p = 1$ or $p = 2$. So, if the PQ symmetry is spontaneously broken after reheating, there is a notorious domain wall problem in this case. However, if we tolerate the quantization condition to an integer k_G , we can take q_Φ as a multiple of $1/(3p)$, avoiding the domain wall problem in the standard scenarios.² We note that E/N is the same, independent of p in the PQ-invariant potential, $H_1^\dagger H_2 \Phi^p$.

3 PQ symmetry breaking and axion quality problem

We consider the Lagrangian for charge-neutral scalars in the Einstein frame in order to discuss the spontaneous breaking of electroweak symmetry and PQ symmetry. We show the conditions for the stable electroweak vacuum in the model

Moreover, we can rewrite the PQ-invariant Einstein-frame potential for neutral scalars from Eq. (5) as

$$V_{\text{PQ}} = V_0 + \frac{1}{2}m_1^2 h_1^2 + \frac{1}{2}m_2^2 h_2^2$$

$$+ \frac{1}{4}\lambda_1 h_1^4 + \frac{1}{4}\lambda_2 h_2^4 + \frac{1}{4}(\lambda_3 + \lambda_4)h_1^2 h_2^2$$

$$+ \frac{1}{2}\mu_\Phi^2 \rho^2 + \frac{1}{4}\lambda_\Phi \rho^4$$

$$+ \frac{1}{4}\lambda_{1\Phi} h_1^2 \rho^2 + \frac{1}{4}\lambda_{2\Phi} h_2^2 \rho^2$$

$$+ \frac{1}{2}\kappa_p h_1 h_2 \rho^p \cos(\eta_2 - \eta_1 + p\theta), \tag{16}$$

and the PQ violating Einstein-frame potential in Eq. (6) as

$$V_{\text{PQV}} = \sum_{n,l} \sum_{k=0}^{\lfloor l/2 \rfloor} \frac{|c_{n,l,k}|}{2^{n+l/2}} h_1^n h_2^n \rho^l$$

$$\cos \left[n(\eta_2 - \eta_1 + p\theta) + (l - 2n - 2k)\theta + A_{n,l,k} \right], \tag{17}$$

with $A_{n,l,k}$ being constant phase shifts.

3.2 Vacuum structure

We determine the vacuum structure of the model for breaking the PQ and electroweak symmetries simultaneously.

As the VEVs of the PQ and Higgs fields are much smaller than the Planck scale, we can approximate the kinetic terms in the Einstein-frame in Eq. (15) to be almost of canonical

² Nonetheless, the domain wall problem reappears due to non-invertible domain walls [16].

form. In order to determine the vacuum, we consider the minimization conditions for the PQ-invariant potential assuming that the PQ violating potential is small enough for axion quality problem. Minimizing one of the angular modes by $\eta_2 - \eta_1 + p\theta$ from the cosine potential in Eq. (16), we get the PQ-invariant potential in the following form,

$$\begin{aligned}
 V_{\text{PQ}} = & V_0 + \frac{1}{2}m_1^2 h_1^2 + \frac{1}{2}m_2^2 h_2^2 + \frac{1}{4}\lambda_1 h_1^4 \\
 & + \frac{1}{4}\lambda_2 h_2^4 + \frac{1}{4}(\lambda_3 + \lambda_4)h_1^2 h_2^2 \\
 & + \frac{1}{2}\mu_\Phi^2 \rho^2 + \frac{1}{4}\lambda_\Phi \rho^4 \\
 & + \frac{1}{4}\lambda_{1\Phi} h_1^2 \rho^2 + \frac{1}{4}\lambda_{2\Phi} h_2^2 \rho^2 \\
 & - \frac{1}{2}|\kappa_p| h_1 h_2 \rho^p. \tag{18}
 \end{aligned}$$

Then, the minimization conditions for the PQ-invariant potential are given by

$$\begin{aligned}
 \mu_\Phi^2 \rho + \lambda_\Phi \rho^3 + \frac{1}{2}\lambda_{1\Phi} h_1^2 \rho \\
 + \frac{1}{2}\lambda_{2\Phi} h_2^2 \rho - \frac{1}{2}p|\kappa_p| h_1 h_2 \rho^{p-1} = 0, \tag{19}
 \end{aligned}$$

$$\begin{aligned}
 m_1^2 h_1 + \lambda_1 h_1^3 + \frac{1}{2}(\lambda_3 + \lambda_4)h_1 h_2^2 \\
 + \frac{1}{2}\lambda_{1\Phi} h_1 \rho^2 - \frac{1}{2}|\kappa_p| h_2 \rho^p = 0, \tag{20}
 \end{aligned}$$

$$\begin{aligned}
 m_2^2 h_2 + \lambda_2 h_2^3 + \frac{1}{2}(\lambda_3 + \lambda_4)h_1^2 h_2 \\
 + \frac{1}{2}\lambda_{2\Phi} h_2 \rho^2 - \frac{1}{2}|\kappa_p| h_1 \rho^p = 0. \tag{21}
 \end{aligned}$$

We denote the VEVs by $\langle h_1 \rangle = v_1 = v \cos \beta$ and $\langle h_2 \rangle = v_2 = v \sin \beta$, with $v^2 = v_1^2 + v_2^2$, and $\langle \rho \rangle \equiv v_\Phi$. Then, we get a nonzero VEV for the PQ field from Eq. (19) as

$$v_\Phi = \sqrt{\frac{1}{\lambda_\Phi} \left(-\mu_\Phi^2 - \frac{1}{2}\lambda_{1\Phi} v^2 \cos^2 \beta - \frac{1}{2}\lambda_{2\Phi} v^2 \sin^2 \beta + \frac{1}{2}|\kappa_2| v^2 \sin 2\beta \right)}, \tag{22}$$

for $p = 2$, and v_Φ is a solution to the cubic equation in Eq. (19) for $p = 1$. We can also rewrite Eqs. (20) and (21) into the equations determining the electroweak scale v and $\sin 2\beta$ in terms of the effective Higgs mass parameters and quartic couplings, as follows,

$$\frac{v^2}{2} = \frac{(1 - \cos 2\beta)m_{2,\text{eff}}^2 - (1 + \cos 2\beta)m_{1,\text{eff}}^2}{(1 + \cos 2\beta)^2 \lambda_1 - (1 - \cos 2\beta)^2 \lambda_2}, \tag{23}$$

and

$$\begin{aligned}
 \sin 2\beta \left(m_{1,\text{eff}}^2 + m_{2,\text{eff}}^2 \right. \\
 \left. + \frac{1}{2}(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4)v^2 + \frac{1}{2}(\lambda_1 - \lambda_2)v^2 \cos 2\beta \right)
 \end{aligned}$$

$$= |\kappa_p|(v_\Phi)^p. \tag{24}$$

with $m_{1,\text{eff}}^2 \equiv m_1^2 + \frac{1}{2}\lambda_{1\Phi} v_\Phi^2$ and $m_{2,\text{eff}}^2 \equiv m_2^2 + \frac{1}{2}\lambda_{2\Phi} v_\Phi^2$ being the effective Higgs mass parameters. We note that the electroweak scale is controlled by $m_{i,\text{eff}}^2$ and λ_i ($i = 1, 2$) for a given $\sin 2\beta$. For $\kappa_p = 0$, we get $\sin 2\beta = 0$, so only the Type I 2HDM would be a viable option for getting all the SM fermion masses. But, in this case, there is no QCD anomaly for the PQ symmetry, so there is no axion solution to the strong CP problem. Thus, we need a nonzero κ_p for realizing the consistent electroweak symmetry breaking and the QCD axion, in particular, in Type II and Y 2HDMs where the QCD anomalies for the PQ symmetry are nonzero.

We remark that the linear combination of h_1 and h_2 has a tachyonic mass near the origin for electroweak scale. Namely, for \mathcal{M}^2 being the squared mass matrix for h_1 and h_2 , we need $\det \mathcal{M}^2 < 0$, resulting in the following necessary condition,

$$m_{1,\text{eff}}^2 m_{2,\text{eff}}^2 < \frac{1}{4}|\kappa_p|^2 (v_\Phi)^{2p}. \tag{25}$$

Otherwise, $h_1 = h_2 = 0$ would be a stable minimum of the potential and electroweak symmetry breaking would not occur. The above condition is automatically satisfied if the signs of $m_{1,\text{eff}}^2$ or $m_{2,\text{eff}}^2$ are opposite. Otherwise, the effective mass parameters are bounded.

We also note that Eq. (24) with $|\sin 2\beta| < 1$ leads to the upper bound on $|\kappa_p|$, as follows,

$$\begin{aligned}
 |\kappa_p|(v_\Phi)^p < m_{1,\text{eff}}^2 + m_{2,\text{eff}}^2 \\
 + \frac{1}{2}(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4)v^2 + \frac{1}{2}(\lambda_1 - \lambda_2)v^2 \cos 2\beta. \tag{26}
 \end{aligned}$$

Thus, the effective mass parameters for the Higgs doublets, $m_{1,\text{eff}}^2$, $m_{2,\text{eff}}^2$ and $|\kappa_p|(v_\Phi)^p$, are constrained by Eqs. (25) and (26), apart from the condition for the electroweak scale

in Eq. (23).

We comment on the upper bounds on the couplings between the Higgs doublets and the PQ field from electroweak symmetry breaking. First, the effective mass parameters, $m_{1,\text{eff}}^2$, $m_{2,\text{eff}}^2$, should not be far from of the weak scale, to get the electroweak symmetry scale in Eq. (23) without fine-tuning parameters. Then, the mixing quartic couplings, $\lambda_{1\Phi}$, $\lambda_{2\Phi}$, contributing to the the effective mass parameters, should be sufficiently small for $v \ll v_\Phi$. For instance, for $v_\Phi \sim 10^8$ GeV and $|m_{1,\text{eff}}^2|, |m_{2,\text{eff}}^2| \lesssim (1 \text{ TeV})^2$, we need $|\lambda_{1\Phi}|, |\lambda_{2\Phi}| \lesssim 10^{-10}$. Moreover, Eq. (24) is satisfied if $|\kappa_p|(v_\Phi)^p \lesssim |m_{1,\text{eff}}^2|, |m_{2,\text{eff}}^2| \lesssim (1 \text{ TeV})^2$, so we need $|\kappa_2| \lesssim 10^{-10}$ for $p = 2$ or $|\kappa_1| \lesssim 10^{-2}$ GeV for $p = 1$. If

there is fine-tuning between larger values of $m_{1,\text{eff}}^2$ and $m_{2,\text{eff}}^2$ in Eq. (23), we can allow for larger values of $|\lambda_{1\Phi}|$, $|\lambda_{2\Phi}|$ and $|\kappa_p|(v_\Phi)^\rho$, as far as Eq. (24) is satisfied. In this work, we take $|\lambda_{1\Phi}|$, $|\lambda_{2\Phi}|$, κ_p as being the free parameters as far as the running quartic coupling λ_Φ is small enough for inflation. So, we need to take $|\lambda_{1\Phi}|$, $|\lambda_{2\Phi}|$, $|\kappa_2| \lesssim 10^{-5}$ and Eq. (26) leads to $|\kappa_1| \lesssim (m_{1,\text{eff}}^2 + m_{2,\text{eff}}^2)/v_\Phi \lesssim \frac{1}{2}(\lambda_{1\Phi} + \lambda_{2\Phi})v_\Phi \lesssim 10^3 \text{ GeV}$ for $v_\Phi \sim 10^8 \text{ GeV}$.

3.3 Axion quality problem

In order to discuss the axion quality problem in the presence of PQ violating terms in the potential, we expand the Higgs and PQ fields around their VEVs, as follows,

$$\begin{aligned} H_1 &= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ (v_1 + \rho_1)e^{ia_1/v_1} \end{pmatrix}, \\ H_2 &= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ (v_2 + \rho_2)e^{ia_2/v_2} \end{pmatrix}, \\ \Phi &= \frac{1}{\sqrt{2}}(v_\Phi + s)e^{ia_\Phi/v_\Phi}. \end{aligned} \tag{27}$$

Then, among the neutral Goldstone bosons, a_1 , a_2 , a_Φ , we can identify the would-be Goldstone boson G , eaten by the Z boson, and the heavy pseudo-scalar A , and the remaining orthogonal pseudo-scalar \bar{a} , as follows,

$$G = \frac{1}{v}(v_1 a_1 + v_2 a_2), \tag{28}$$

$$A = N \left(\frac{a_2}{v_2} - \frac{a_1}{v_1} + \frac{pa_\Phi}{v_\Phi} \right), \tag{29}$$

$$\bar{a} = K \left(v_2 a_1 - v_1 a_2 + \frac{v_\Phi v^2}{pv_1 v_2} a_\Phi \right), \tag{30}$$

with $v = \sqrt{v_1^2 + v_2^2}$, $N = 1/\sqrt{v_1^{-2} + v_2^{-2} + p^2 v_\Phi^{-2}}$ and $K = 1/\sqrt{v_1^2 + v_2^2 + v_\Phi^2 v^4/(p^2 v_1^2 v_2^2)}$.

On the other hand, the QCD axion is identified from the PQ Noether current, as follows,

$$a = \frac{1}{f_a} (q_1 v_1 a_1 + q_2 v_2 a_2 + q_\Phi v_\Phi a_\Phi) \tag{31}$$

where $f_a = \sqrt{(q_1 v_1)^2 + (q_2 v_2)^2 + (q_\Phi v_\Phi)^2}$ is the axion decay constant. Here, we note that the PQ charges are constrained to $p q_\Phi - q_1 + q_2 = 0$ for the PQ invariance. We also can rewrite the QCD axion in terms of the orthogonal set of fields as

$$a = \frac{1}{v f_a} \left[(q_1 v_1^2 + q_2 v_2^2) G + \frac{p q_\Phi v_1 v_2}{v K} \bar{a} \right]. \tag{32}$$

Therefore, in unitary gauge with $G = 0$, the QCD axion is identical to the orthogonal scalar \bar{a} . In this case, for $v_\Phi \gg v_1, v_2$, we can approximate $K \simeq \frac{pv_1 v_2}{v^2 v_\Phi}$ and $f_a \simeq q_\Phi v_\Phi$, so $a \simeq \bar{a}$. In the decoupling limit of the heavy pseudo-scalar

with $A = 0$, the QCD axion is dominated by the angular mode of the PQ field as $a \simeq a_\Phi$ from Eq. (31).

For $v_\Phi \gg v_1, v_2$, the PQ violating terms only with the PQ fields affect the axion quality problem most. So, we can take $n = 0$ terms and replace ρ by its VEV in the first line in the PQ violating potential in Eq. (17). Then, for $a \simeq a_\Phi$ and $f_a \simeq q_\Phi v_\Phi$, the PQ violating terms at the order of f_a^l are approximated to

$$V_{\text{PQV}} \simeq \sum_{k=0}^{[l/2]} \frac{|c_{0,l,k}| f_a^l}{2^{l/2} M_P^{l-4} q_\Phi^l} \cos \left[(l-2k) \frac{q_\Phi a}{f_a} + A_{0,l,k} \right]. \tag{33}$$

In the presence of the PQ anomalies, we get the effective gluon couplings, as follows,

$$\mathcal{L}_{\text{gluons}} = \frac{g_s^2}{32\pi^2} \left(\bar{\theta} + \xi \frac{a}{f_a} \right) G_{\mu\nu}^a \tilde{G}^{a\mu\nu} \tag{34}$$

where ξ is the PQ anomaly coefficient, which is set to $\xi = 1$ in KSVZ models and $\xi = k_G = 6q_\Phi$ in DFSZ models. Here, we can check the PQ anomaly coefficient ξ explicitly for Type II and Y models in Table 2. After PQ and electroweak symmetries are broken, the effective Yukawa interactions for the pseudo-scalars, $a_1 \equiv \theta_1 v_1$, $a_2 \equiv \theta_2 v_2$, appearing in the Higgs fields, are given by

$$\begin{aligned} \mathcal{L}_{\text{eff},\theta_{1,2}} &= -\frac{1}{\sqrt{2}} v_1 Y_d e^{i\theta_1} \bar{d}_L d_R \\ &\quad - \frac{1}{\sqrt{2}} v_2 Y_u e^{-i\theta_2} \bar{u}_L u_R + \text{h.c.} \end{aligned} \tag{35}$$

Then, after making chiral rotations of quarks for three generations by $u_L \rightarrow e^{-i\theta_2/2} u_L$, $u_R \rightarrow e^{i\theta_2/2} u_R$, $d_L \rightarrow e^{i\theta_1/2} d_L$ and $d_R \rightarrow e^{-i\theta_1/2} d_R$ [1], we obtain the anomalous shift of the Lagrangian as

$$\Delta\mathcal{L} = 3 \times \frac{\theta_1 - \theta_2}{32\pi^2} G_{\mu\nu}^a \tilde{G}^{a\mu\nu}. \tag{36}$$

Here, from Eq. (29), we can rewrite $\theta_1 - \theta_2 = \frac{a_1}{v_1} - \frac{a_2}{v_2} = \frac{p a_\Phi}{v_\Phi} - \frac{A}{N}$. Thus, setting $A = 0$ and using $a_\Phi \simeq a$ and $f_a \simeq q_\Phi v_\Phi$, we obtain $\theta_1 - \theta_2 \simeq \frac{p q_\Phi a}{f_a}$, so the anomaly coefficient ξ becomes $\xi = 3p q_\Phi$.

Then, after the QCD phase transition, there appears an extra contribution to the axion potential, in the following form,

$$\Delta V_E = -\Lambda_{\text{QCD}}^4 \cos \left(\bar{\theta} + \xi \frac{a}{f_a} \right). \tag{37}$$

After the radial mode settles down to the minimum of the potential, i.e. $\langle \rho \rangle \simeq v_\Phi$, from Eqs. (33) and (37), the effective

potential for the axion after the QCD phase transition is given by

$$\begin{aligned}
 V_{\text{eff}}(a) = & -\Lambda_{\text{QCD}}^4 \cos\left(\bar{\theta} + \xi \frac{a}{f_a}\right) \\
 & + M_P^4 \left(\frac{f_a}{\sqrt{2}q_\Phi M_P}\right)^l \\
 & \times \sum_{k=0}^{\lfloor l/2 \rfloor} |c_{0,l,k}| \cos\left((l-2k)\frac{q_\Phi a}{f_a} + A_{0,l,k}\right).
 \end{aligned} \tag{38}$$

In order to solve the strong CP problem by the axion, the axion potential needs to relax the effective θ term dynamically to satisfy the EDM bound,

$$|\theta_{\text{eff}}| = \left| \bar{\theta} + \xi \frac{\langle a \rangle}{f_a} \right| < 10^{-10}. \tag{39}$$

Then, from the minimization of the effective potential for the axion in Eq. (38), namely, $\frac{dV_{\text{eff}}}{da} = 0$, we obtain

$$\begin{aligned}
 a_{\text{phys}} \equiv a + \frac{f_a \bar{\theta}}{\xi} \simeq & \frac{f_a^{l-1}}{2^{l/2} M_P^{l-4} q_\Phi^{l-1} m_a^2} \\
 & \times \sum_{k=0}^{\lfloor l/2 \rfloor} |c_{0,l,k}| (l-2k) \sin\left(A_{0,l,k} - \frac{l-2k}{\xi} q_\Phi \bar{\theta}\right)
 \end{aligned} \tag{40}$$

where $m_a^2 = \frac{\xi^2}{f_a^2} \Lambda_{\text{QCD}}^4$ is the squared mass for the axion due to QCD only, and we assumed $(l-2k)q_\Phi \frac{a_{\text{phys}}}{f_a} \ll 1$ and $(l-2k)^2 |c_{0,l,k}| f_a^{l-2} / (2^{l/2} M_P^{l-4} q_\Phi^{l-2}) \cos\left(A_{0,l,k} - \frac{l-2k}{\xi} q_\Phi \bar{\theta}\right) \lesssim m_a^2$ for all k .

As a result, from Eq. (40) with Eq. (39), we can solve the strong CP problem if

$$\begin{aligned}
 & \frac{\xi f_a^{l-2}}{2^{l/2} M_P^{l-4} q_\Phi^{l-1} m_a^2} \sum_{k=0}^{\lfloor l/2 \rfloor} |c_{0,l,k}| (l-2k) \\
 & \times \sin\left(A_{0,l,k} - \frac{l-2k}{\xi} q_\Phi \bar{\theta}\right) < 10^{-10}.
 \end{aligned} \tag{41}$$

Unless there is a cancellation between various contributions at the same order, each term in the PQ violating potential at the order of f_a^l is constrained by

$$\left(\frac{f_a}{M_P}\right)^l \lesssim \frac{2^{l/2} \xi q_\Phi^{l-1}}{(l-2k)|c_{0,l,k}|} \left(\frac{\Lambda_{\text{QCD}}}{M_P}\right)^4 \times 10^{-10}. \tag{42}$$

As compared to the KSVZ models [10], there is a mild dependence on the charge of the PQ field q_Φ , but the order of magnitude estimation of the axion quality problem remains the same. For a given axion decay constant, we can set a bound on the order of the PQ violating potential. For instance, choosing $f_a = 10^{12}(10^8)$ GeV, $k = 0$, $|c_{0,l,0}| = \mathcal{O}(1)$ and $\xi = k_G = 3pq_\Phi = 6$ in Type II and Y models in Table 2, we need $l \gtrsim 13(8)$ for the axion quality. As will be discussed later, the bound from the CMB normalization leads to $3^{l/2}|c_{0,l,k}| \lesssim 10^{-10}$ for all k , so it is sufficient to take $l \gtrsim 11(7)$ for $f_a = 10^{12}(10^8)$ GeV, $\xi = 6$ and $|c_{0,l,k}| \lesssim 10^{-12}$.

4 PQ inflation and predictions

We derive the effective theory for a single-field slow-roll inflation. The first case is that the mixture of the radial components of PQ and neutral Higgs fields is responsible for the inflaton. The second case is that the pure radial component of the PQ field is the inflaton. We check the stability of the non-inflaton directions during inflation. Then, focusing on the pure PQ inflation, we present the inflationary predictions and the initial condition for the axion velocity at the end of inflation due to the PQ violating potential.

4.1 Effective theory for PQ-Higgs mixed inflation

For general inflation dynamics in DFSZ models, the Higgs fields can participate to the inflationary dynamics, so their background field values are nonzero, i.e. $h_1 \neq 0, h_2 \neq 0$. In this case, slow-roll inflaton takes place near the pole of the kinetic term, that is, $\chi^2 \equiv \rho^2 + h_1^2 + h_2^2 \rightarrow 6$. On the other hand, the orthogonal directions to the inflaton, denoted as $\tau_1 = h_1/\rho$ and $\tau_2 = h_2/\rho$, must be stabilized [17, 18]. Then, noting $\rho^2 = \chi^2 / (1 + \tau_1^2 + \tau_2^2)$, we can rewrite the kinetic terms in Eq. (15) for χ, τ_1, τ_2 and the angular modes, as follows,

$$\begin{aligned} \frac{\mathcal{L}_{\text{kin},E}}{\sqrt{-g_E}} &= -\frac{1}{2}R + \frac{(\partial_\mu(\tau_1\rho))^2 + (\partial_\mu(\tau_2\rho))^2 + (\tau_1\rho)^2(\partial_\mu\eta_1)^2 + (\tau_2\rho)^2(\partial_\mu\eta_2)^2 + (\partial_\mu\rho)^2 + \rho^2(\partial_\mu\theta)^2}{2(1 - \frac{1}{6}\chi^2)} \\ &+ \frac{3}{4} \frac{(\partial_\mu(\frac{1}{6}\chi^2))^2}{(1 - \frac{1}{6}\chi^2)^2} = -\frac{1}{2}R + \frac{1}{2} \frac{(\partial_\mu\chi)^2}{(1 - \frac{1}{6}\chi^2)^2} + \frac{\chi^2}{2(1 - \frac{1}{6}\chi^2)} \frac{1}{(1 + \tau_1^2 + \tau_2^2)} \left[(\partial_\mu\theta)^2 + \tau_1^2(\partial_\mu\eta_1)^2 + \tau_2^2(\partial_\mu\eta_2)^2 \right] \\ &+ \frac{\chi^2}{2(1 - \frac{1}{6}\chi^2)} \frac{1}{(1 + \tau_1^2 + \tau_2^2)^2} \left[(\partial_\mu\tau_1)^2 + (\partial_\mu\tau_2)^2 + (\tau_2\partial_\mu\tau_1 - \tau_1\partial_\mu\tau_2)^2 \right]. \end{aligned} \tag{43}$$

Introducing the canonical inflaton by

$$\chi = \sqrt{6} \tanh\left(\frac{\phi}{\sqrt{6}}\right), \tag{44}$$

we can simplify the above kinetic terms as

$$\begin{aligned} \frac{\mathcal{L}_{\text{kin},E}}{\sqrt{-g_E}} &= -\frac{1}{2}R + \frac{1}{2}(\partial_\mu\phi)^2 + \frac{3 \sinh^2\left(\frac{\phi}{\sqrt{6}}\right)}{(1 + \tau_1^2 + \tau_2^2)} \\ &\times \left[(\partial_\mu\theta)^2 + \tau_1^2(\partial_\mu\eta_1)^2 + \tau_2^2(\partial_\mu\eta_2)^2 \right] \\ &+ \frac{3 \sinh^2\left(\frac{\phi}{\sqrt{6}}\right)}{(1 + \tau_1^2 + \tau_2^2)^2} \\ &\times \left[(\partial_\mu\tau_1)^2 + (\partial_\mu\tau_2)^2 + (\tau_2\partial_\mu\tau_1 - \tau_1\partial_\mu\tau_2)^2 \right]. \end{aligned} \tag{45}$$

For $\langle\tau_1\rangle \neq 0$ and $\langle\tau_2\rangle \neq 0$, there is a nonzero PQ invariant potential for one of the angular modes, $\tilde{A} \equiv \eta_2 - \eta_1 + p\theta$, which is stabilized at either 0 or π . Moreover, $\langle\tau_1^2\rangle\eta_1 + \langle\tau_2^2\rangle\eta_2$ is eaten by the Z boson as in the vacuum in Section 3.3. In unitary gauge, we can choose $\langle\tau_2^2\rangle\eta_2 = -\langle\tau_1^2\rangle\eta_1$, so the heavy angular direction becomes $\tilde{A} \equiv -(1 + \langle\tau_1^2\rangle/\langle\tau_2^2\rangle)\eta_1 + p\theta$. Thus, the Lagrangian for the angular modes become

$$\begin{aligned} \frac{\mathcal{L}_{\text{angles},E}}{\sqrt{-g_E}} &= \frac{3 \sinh^2\left(\frac{\phi}{\sqrt{6}}\right)}{(1 + \tau_1^2 + \tau_2^2)} \\ &\times \left[(\partial_\mu\theta)^2 + \frac{\tau_1^2\langle\tau_2^4\rangle + \tau_2^2\langle\tau_1^4\rangle}{(\langle\tau_1^2\rangle + \langle\tau_2^2\rangle)^2} (\partial_\mu\tilde{A} - p\partial_\mu\theta)^2 \right] \\ &- V_{\text{angles}}(\tilde{A}), \end{aligned} \tag{46}$$

with

$$V_{\text{angles}}(\tilde{A}) = \frac{\kappa_p \tau_1 \tau_2 \chi^{p+2}}{2(1 + \tau_1^2 + \tau_2^2)^{(p+2)/2}} \cos \tilde{A}. \tag{47}$$

Then, we can stabilize $\tilde{A} = 0$ or π , depending on $\lambda_{12\phi} < 0$

or $\lambda_{12\phi} > 0$. For instance, for $\langle\tau_1\rangle = \langle\tau_2\rangle$ and $p = 2$, the effective mass for \tilde{A} becomes

$$m_{\tilde{A},\text{eff}}^2 = \frac{6|\kappa_2| \sinh^2\left(\frac{\phi}{\sqrt{6}}\right)}{(1 + 2\langle\tau_1^2\rangle) \cosh^4\left(\frac{\phi}{\sqrt{6}}\right)}, \tag{48}$$

which is approximated to $m_{\tilde{A},\text{eff}}^2 \simeq \frac{6|\kappa_2|}{1+2\langle\tau_1^2\rangle} \sqrt{\frac{3}{4}}\epsilon$ for $\phi \gg \sqrt{6}$ during inflation. So, the angular mode \tilde{A} is decoupled during inflation, if $m_{\tilde{A},\text{eff}}^2 \gg H_I^2 \simeq 3\lambda\phi$.

Furthermore, after stabilizing \tilde{A} , we can recast the scalar potential in terms of χ, τ_1, τ_2 into

$$\begin{aligned} V_{\text{PQ}} &= \frac{\chi^4}{4(1 + \tau_1^2 + \tau_2^2)^2} \left(\lambda_1\tau_1^4 + \lambda_2\tau_2^4 + (\lambda_3 + \lambda_4)\tau_1^2\tau_2^2 \right. \\ &\quad \left. + \lambda_\phi + \lambda_{1\phi}\tau_1^2 + \lambda_{2\phi}\tau_2^2 \right. \\ &\quad \left. - \frac{2|\kappa_p|\chi^{p-2}\tau_1\tau_2}{(1 + \tau_1^2 + \tau_2^2)^{(p-2)/2}} \right), \end{aligned} \tag{49}$$

$$\begin{aligned} V_{\text{PQV}} &= \sum_{n,l} \sum_{k=0}^{[l/2]} \frac{|c_{n,l,k}|(\tau_1\tau_2)^n \chi^{2n+l}}{2^{n+l/2} M_P^{2n+l-4} (1 + \tau_1^2 + \tau_2^2)^{(2n+l)/2}} \\ &\times \cos \left[(l - 2n - 2k)\theta + A_{n,l,k} + n\alpha \right], \end{aligned} \tag{50}$$

with $\alpha = 0$ or π , depending on whether \tilde{A} is stabilized at 0 or π .

As a result, after stabilizing \tilde{A} , the effective Lagrangian for the general inflation is given by

$$\begin{aligned} \frac{\mathcal{L}_{\text{eff},E}}{\sqrt{-g_E}} &= -\frac{1}{2}R + \frac{1}{2}(\partial_\mu\phi)^2 \\ &+ \frac{3 \sinh^2\left(\frac{\phi}{\sqrt{6}}\right)}{(1 + \tau_1^2 + \tau_2^2)} \left(1 + \frac{p^2\tau_1^2\langle\tau_2^4\rangle + p^2\tau_2^2\langle\tau_1^4\rangle}{(\langle\tau_1^2\rangle + \langle\tau_2^2\rangle)^2} \right) \\ &\times (\partial_\mu\theta)^2 + \frac{3 \sinh^2\left(\frac{\phi}{\sqrt{6}}\right)}{(1 + \tau_1^2 + \tau_2^2)^2} \\ &\times \left[(\partial_\mu\tau_1)^2 + (\partial_\mu\tau_2)^2 + (\tau_2\partial_\mu\tau_1 - \tau_1\partial_\mu\tau_2)^2 \right] \\ &- V_E(\phi, \theta, \tau_1, \tau_2). \end{aligned} \tag{51}$$

Here, $V_E = V_{PQ} + V_{PQV}$, and the effective scalar potentials are written as factorizable forms for $p = 2$,

$$V_{PQ} = U(\phi) \cdot W(\tau_1, \tau_2), \tag{52}$$

with

$$U(\phi) = 9\lambda_\phi \tanh^4\left(\frac{\phi}{\sqrt{6}}\right) \tag{53}$$

and

$$W = \frac{1}{\lambda_\phi(1 + \tau_1^2 + \tau_2^2)^2} \left(\lambda_1\tau_1^4 + \lambda_2\tau_2^4 + (\lambda_3 + \lambda_4)\tau_1^2\tau_2^2 + \lambda_\phi + \lambda_{1\phi}\tau_1^2 + \lambda_{2\phi}\tau_2^2 - 2|\kappa_2|\tau_1\tau_2 \right), \tag{54}$$

and

$$V_{PQV} = \sum_{n,l} \sum_{k=0}^{\lfloor l/2 \rfloor} \frac{3^{n+l/2} |c_{n,l,k}|}{M_P^{2n+l-4}} \frac{(\tau_1\tau_2)^n \tanh^{2n+l}\left(\frac{\phi}{\sqrt{6}}\right)}{(1 + \tau_1^2 + \tau_2^2)^{(2n+l)/2}} \times \cos \left[(l - 2n - 2k)\theta + A_{n,l,k} + n\alpha \right]. \tag{55}$$

However, for $p = 1$, the PQ-invariant effective scalar potential is not of factorizable form, but instead it has a correction to the potential for ϕ , proportional to the Higgs VEVs, as follows,

$$V_{PQ} = U(\phi) \left(A(\tau_1, \tau_2) + P(\phi)B(\tau_1, \tau_2) \right), \tag{56}$$

with

$$A(\tau_1, \tau_2) = \frac{1}{\lambda_\phi(1 + \tau_1^2 + \tau_2^2)^2} \left(\lambda_1\tau_1^4 + \lambda_2\tau_2^4 + (\lambda_3 + \lambda_4)\tau_1^2\tau_2^2 + \lambda_\phi + \lambda_{1\phi}\tau_1^2 + \lambda_{2\phi}\tau_2^2 \right), \tag{57}$$

$$B(\tau_1, \tau_2) = \frac{\tau_1\tau_2}{(1 + \tau_1^2 + \tau_2^2)^{(p+2)/2}}, \tag{58}$$

$$P(\phi) = -2 \cdot 6^{(p-2)/2} \frac{|\kappa_P|}{\lambda_\phi} \tanh^{p-2}\left(\frac{\phi}{\sqrt{6}}\right). \tag{59}$$

Now we check the stability for τ_1 and τ_2 . For $\lambda_1 = \lambda_2$ and $\lambda_{1\phi} = \lambda_{2\phi}$, the Lagrangian is symmetric under the exchange of τ_1 and τ_2 , so are the VEVs, that is, $\langle \tau_1 \rangle = \langle \tau_2 \rangle$. Otherwise, $\langle \tau_1 \rangle \neq \langle \tau_2 \rangle$. The minimization conditions, $\frac{\partial V_{PQ}}{\partial \tau_1} = 0$ and $\frac{\partial V_{PQ}}{\partial \tau_2} = 0$, are satisfied for $\langle \tau_1 \rangle = \langle \tau_2 \rangle = 0$, which is the case for pure PQ inflation in the next subsection. But, $\langle \tau_1 \rangle \neq 0$, $\langle \tau_2 \rangle = 0$ or $\langle \tau_1 \rangle = 0$, $\langle \tau_2 \rangle \neq 0$ are not the minimum of the potential for $\kappa_p \neq 0$, due to the effective tadpole term for either directions [19]. Thus, if $\langle \tau_1 \rangle \neq 0$, we need $\langle \tau_2 \rangle \neq 0$, and vice versa. The values of $\langle \tau_1 \rangle$ and $\langle \tau_2 \rangle$ are bounded because

$$\chi = \rho(1 + \tau_1^2 + \tau_2^2)^{1/2} = \sqrt{6} \tanh\left(\frac{\phi}{\sqrt{6}}\right) \leq \sqrt{6}. \tag{60}$$

The general solutions to $\frac{\partial V_{PQ}}{\partial \tau_1} = 0$ and $\frac{\partial V_{PQ}}{\partial \tau_2} = 0$ are involved, but the stable solutions for inflation exist, as far

as the Hessian of the squared mass matrix for τ_1, τ_2 in the vacuum is positive and the inflation vacuum energy is positive for $W(\langle \tau_1 \rangle, \langle \tau_2 \rangle) > 0$ for $p = 2$ and $A(\langle \tau_1 \rangle, \langle \tau_2 \rangle) + P(\phi)B(\langle \tau_1 \rangle, \langle \tau_2 \rangle) > 0$ for $p = 1$ [17].

After τ_1 and τ_2 are stabilized, the effective kinetic term for θ in Eq. (51) becomes

$$\frac{3 \sinh^2\left(\frac{\phi}{\sqrt{6}}\right)}{(1 + \langle \tau_1^2 \rangle + \langle \tau_2^2 \rangle)} \left(1 + \frac{p^2 \langle \tau_1^2 \rangle \langle \tau_2^2 \rangle}{\langle \tau_1^2 \rangle + \langle \tau_2^2 \rangle} \right) (\partial_\mu \theta)^2. \tag{61}$$

In particular, for $\langle \tau_1 \rangle = \langle \tau_2 \rangle$, the above kinetic term for θ becomes simplified to

$$\left(\frac{2 + p^2 \langle \tau_1^2 \rangle}{2(1 + 2\langle \tau_1^2 \rangle)} \right) 3 \sinh^2\left(\frac{\phi}{\sqrt{6}}\right) (\partial_\mu \theta)^2, \tag{62}$$

which is of the same form as in KSVZ models for $p = 2$ [10]. Similarly, the PQ invariant potential Eq. (52) become functions of ϕ only and the PQ violating potential in Eq. (55) are functions of ϕ and θ only.

We remark that the effective vacuum energy during inflation is controlled by $W(\tau_1, \tau_2)$ in Eq. (54) or $A(\tau_1, \tau_2)$ in Eq. (58) with the VEVs of τ_1 and τ_2 . Then, for $\langle \tau_1 \rangle \neq 0$ or $\langle \tau_2 \rangle \neq 0$, the quartic couplings for two Higgs doublets contribute to the effective vacuum energy. So, the CMB normalization requires a suppressed vacuum energy during inflation as will be discussed later, we need very small $\langle \tau_1 \rangle$ and $\langle \tau_2 \rangle$ or the quartic couplings for two Higgs doublets must run to very small values during inflation [12]. In both cases, we would need nontrivial relations between the running couplings for small Higgs VEVs or vanishing beta functions during inflation. Thus, as will be discussed in the next subsection, we focus on the case for pure PQ inflation where the Higgs VEVs are stabilized at the origin during inflation.

4.2 Effective theory for pure PQ inflation

We take the inflation direction along the radial mode of the PQ field, so the background field values for two Higgs doublets are set to $\langle h_1 \rangle = \langle h_2 \rangle = 0$ or $\langle \tau_1 \rangle = \langle \tau_2 \rangle = 0$. In this case, the angular modes also vanish in the polar representations in Eq. (14). So, instead we need to include the imaginary partners of the neutral Higgs scalars in the Cartesian representations, instead of η_1 and η_2 in Eq. (14), that is, $\frac{1}{\sqrt{2}} h_i e^{i\theta_i} \rightarrow \frac{1}{\sqrt{2}} (h_i + i\tilde{\eta}_i)$ with $i = 1, 2$. Then, it is obvious that the imaginary parts of the neutral Higgs scalars receive the same masses as for $\bar{\tau}_1, \bar{\tau}_2$, due to the symmetry of the potential respecting the SM gauge symmetry during inflation. We can still use the polar representation for the PQ field as in Eq. (14), because the radial mode of the PQ field is nonzero during inflation so the angular mode of the PQ field is kept.

For the pure PQ inflation, we get the effective Lagrangian for describing the evolution of the background, that is, the

inflaton and the angular mode associated with the PQ field, as

$$\frac{\mathcal{L}_{\text{bkg},E}}{\sqrt{-g_E}} = -\frac{1}{2}R + \frac{1}{2}(\partial_\mu\phi)^2 + 3\sinh^2\left(\frac{\phi}{\sqrt{6}}\right)(\partial_\mu\theta)^2 - V_E(\phi, \theta) \tag{63}$$

where the inflaton potential in Einstein frame is given by $V_E = V_{\text{PQ}}(\phi) + V_{\text{PQV}}(\phi, \theta)$, with

$$V_{\text{PQ}}(\phi) = 9\lambda_\phi \tanh^4\left(\frac{\phi}{\sqrt{6}}\right) = U(\phi), \tag{64}$$

$$V_{\text{PQV}}(\phi, \theta) = \sum_{k=0}^{\lfloor l/2 \rfloor} \frac{3^{l/2} |c_{0,l,k}|}{M_P^{l-4}} \tanh^l\left(\frac{\phi}{\sqrt{6}}\right) \times \cos((l-2k)\theta + A_{0,l,k}). \tag{65}$$

Here, the background field values for the Higgs-dependent terms in the Lagrangian were set to zero and we kept all the terms contributing to ρ^l in the PQ violating potential. The inflaton dynamics is governed dominantly by the quartic interaction in V_{PQ} whereas the angular mode θ associated with the PQ field receives a nonzero velocity due to V_{PQV} . The resulting effective Lagrangian for the PQ inflation in DFSZ models takes the same form as in KSVZ models [10].

The effective Lagrangian for the real parts of the Higgs perturbations, \bar{h}_1, \bar{h}_2 , or $\bar{\tau}_1, \bar{\tau}_2$, up to quadratic terms, are given by

$$\frac{\mathcal{L}_{\text{pert},E}}{\sqrt{-g_E}} = 3\sinh^2\left(\frac{\phi}{\sqrt{6}}\right)\left((\partial_\mu\bar{\tau}_1)^2 + (\partial_\mu\bar{\tau}_2)^2\right) - \frac{1}{2}m_{11}^2\bar{\tau}_1^2 - \frac{1}{2}m_{22}^2\bar{\tau}_2^2 - m_{12}^2\bar{\tau}_1\bar{\tau}_2 \tag{66}$$

with

$$m_{ii}^2 = \left(\frac{1}{2}\lambda_{i\phi} - \lambda_\phi\right)\rho^4 = 18(\lambda_{i\phi} - 2\lambda_\phi)\tanh^4\left(\frac{\phi}{\sqrt{6}}\right), \tag{67}$$

$i = 1, 2,$

$$m_{12}^2 = -\frac{1}{2}\kappa_p\rho^{p+2} = -\frac{1}{2}6^{p/2+1}\kappa_p\tanh^{p+2}\left(\frac{\phi}{\sqrt{6}}\right). \tag{68}$$

Then, after diagonalizing the mass matrix for $\bar{\tau}_1, \bar{\tau}_2$, we obtain the mass eigenvalues as

$$m_\pm^2 = \frac{1}{4}\left[\lambda_{1\phi} + \lambda_{2\phi} - 4\lambda_\phi \pm \sqrt{(\lambda_{1\phi} - \lambda_{2\phi})^2 + 4\rho^{2p-4}\kappa_p^2}\right]\rho^4, \tag{69}$$

which are positive definite as far as

$$(\lambda_{1\phi} - 2\lambda_\phi)(\lambda_{2\phi} - 2\lambda_\phi) > \rho^{2p-4}\kappa_p^2 \simeq 6^{p-2}\kappa_p^2. \tag{70}$$

For instance, for $p = 2$, the effective masses for the canonical Higgs perturbations are

$$m_{\pm,\text{eff}}^2 = \frac{m_\pm^2}{6\sinh^2\left(\frac{\phi}{\sqrt{6}}\right)} \simeq \frac{3}{2}\left[\lambda_{1\phi} + \lambda_{2\phi} - 4\lambda_\phi \pm \sqrt{(\lambda_{1\phi} - \lambda_{2\phi})^2 + 4\kappa_p^2}\right] \cdot \frac{\sinh^2\left(\frac{\phi}{\sqrt{6}}\right)}{\cosh^4\left(\frac{\phi}{\sqrt{6}}\right)}, \tag{71}$$

where $\sinh^2(\frac{\phi}{\sqrt{6}})/\cosh^4(\frac{\phi}{\sqrt{6}}) \simeq 4e^{-2\phi/\sqrt{6}} \simeq \sqrt{\frac{3}{4}}\epsilon$ during inflation. So, the Higgs directions are decoupled during inflation, as far as $m_{\pm,\text{eff}}^2 \gg H_I^2 \simeq 3\lambda_\phi$.

4.3 Slow-roll inflation and axion rotation

After stabilizing the field directions other than ϕ and θ , we consider the slow-roll inflation and the initial condition for the axion velocity at the end of inflation. As discussed in the previous subsection, we can stabilize the Higgs fields at the origin due to their large positive effective masses for which the pure PQ inflation is realized by ϕ . In this case, the axion field θ receives a nonzero initial velocity due to the PQ violating potential.

From the effective Lagrangian in Eq. (63), we obtain the equation of motion for the radial and angular modes of the PQ field in the following,

$$\ddot{\phi} + 3H\dot{\phi} - \sqrt{6}\sinh\left(\frac{\phi}{\sqrt{6}}\right)\cosh\left(\frac{\phi}{\sqrt{6}}\right)\dot{\theta}^2 = -\frac{\partial V_E}{\partial\phi}, \tag{72}$$

$$6\sinh^2\left(\frac{\phi}{\sqrt{6}}\right)\left[\ddot{\theta} + 3H\dot{\theta} + \frac{2}{\sqrt{6}}\coth\left(\frac{\phi}{\sqrt{6}}\right)\dot{\phi}\dot{\theta}\right] = -\frac{\partial V_E}{\partial\theta}. \tag{73}$$

Here, the Hubble parameter is determined by

$$H^2 = \frac{1}{3}\left(\frac{1}{2}(\partial_\mu\phi)^2 + 3\sinh^2\left(\frac{\phi}{\sqrt{6}}\right)(\partial_\mu\theta)^2 + V_E\right). \tag{74}$$

which is approximate to $H^2 \simeq \frac{V_E}{3} \simeq 3\lambda_\phi$ during inflation.

For a slow-roll inflation with $\dot{\phi} \ll H\phi$ and $\dot{\phi} \ll H$ as well as $\ddot{\theta} \ll H\dot{\theta}$ and $\dot{\theta} \ll H$, we simply approximate Eqs. (72) and (73) as

$$\dot{\phi} \simeq -\frac{1}{3H}\frac{\partial V_E}{\partial\phi} = -\sqrt{2\epsilon_\phi}M_P H, \tag{75}$$

$$\dot{\theta} \simeq -\frac{1}{3H}\frac{\frac{\partial V_E}{\partial\theta}}{6M_P^2\sinh^2\left(\frac{\phi}{\sqrt{6}M_P}\right)} = -\frac{\sqrt{2\epsilon_\theta}H}{6\sinh^2\left(\frac{\phi}{\sqrt{6}M_P}\right)} \tag{76}$$

where $\epsilon_\phi, \epsilon_\theta$ are the slow-roll parameters for the radial and angular modes, respectively. Then, the radial mode derives a slow-roll inflation while the dynamics of the angular mode is subdominant during inflation. As a result, we can set the

initial kinetic misalignment of the axion by using Eq. (76) at the end of inflation [10], giving rise to the PQ Noether charge at the end of inflation as

$$n_{\theta, \text{end}} = 6 \sinh^2 \left(\frac{\phi_{\text{end}}}{\sqrt{6}} \right) |\dot{\theta}_{\text{end}}| \simeq \sqrt{2\epsilon_{\theta, \text{end}}} H_{\text{end}}. \tag{77}$$

Here, $\epsilon_{\theta, \text{end}}$, H_{end} , ϕ_{end} are the quantities evaluated at the end of inflation.

We assume the PQ invariant potential to be dominant for the pure PQ inflation, so the inflaton potential is approximately given by

$$V_E(\phi) \simeq V_I \left[\tanh^4 \left(\frac{\phi}{\sqrt{6}} \right) \right], \tag{78}$$

with $V_I \equiv 9\lambda_\phi$. Then, the slow-roll parameters during inflation are given by

$$\epsilon = \frac{16}{3} \left[\sinh \left(\frac{2\phi}{\sqrt{6}} \right) \right]^{-2}, \tag{79}$$

$$\eta = -\frac{8}{3} \left[\cosh \left(\frac{2\phi}{\sqrt{6}} \right) - 4 \right] \left[\sinh \left(\frac{2\phi}{\sqrt{6}} \right) \right]^{-2}. \tag{80}$$

The number of efoldings is

$$\begin{aligned} N &= \frac{1}{M_P} \int_{\phi_e}^{\phi_*} \frac{\text{sgn}(V'_E) d\phi}{\sqrt{2\epsilon}} \\ &= \frac{3}{8} \left[\cosh \left(\frac{2\phi_*}{\sqrt{6}} \right) - \cosh \left(\frac{2\phi_e}{\sqrt{6}} \right) \right] \end{aligned} \tag{81}$$

where ϕ_* , ϕ_e are the values of the radial mode at horizon exit and the end of inflation, respectively. As a result, using Eqs. (79), (80) and (81) and $N \simeq \frac{3}{8} \cosh \left(\frac{2\phi_*}{\sqrt{6}M_P} \right)$ for $\phi_* \gg \sqrt{6}$ during inflation, we obtain the slow-roll parameters at horizon exit in terms of the number of efoldings as

$$\epsilon_* \simeq \frac{3}{4(N^2 - \frac{9}{64})}, \quad \eta_* \simeq \frac{3 - 2N}{2(N^2 - \frac{9}{64})}. \tag{82}$$

Thus, we get the spectral index in terms of the number of efoldings and the tensor-to-scalar ratio, as follows,

$$n_s = 1 - \frac{4N + 3}{2(N^2 - \frac{9}{64})}, \tag{83}$$

$$r = 16\epsilon_* = \frac{12}{N^2 - \frac{9}{64}}. \tag{84}$$

We note that if a higher order PQ invariant potential such as $\beta_m |\Phi|^{2m}$ with $m > 2$ is introduced instead of the quartic potential $\lambda_\phi |\Phi|^4$, the inflaton potential is changed to the form, $V_E(\phi) \propto \tanh^{2m}(\phi/\sqrt{6})$, but the inflationary predictions are insensitive to the value of m [10].

The inflationary predictions in the PQ pole inflation are $n_s = 0.966$ and $r = 0.0033$ for $N = 60$, which are consistent with the observations, namely, $n_s = 0.967 \pm 0.0037$ [20] and $r < 0.036$ [21]. On the other hand, from the CMB normalization, $A_s = \frac{1}{24\pi^2} \frac{V_I}{\epsilon_*} = 2.1 \times 10^{-9}$ [20], we need to

set the quartic coupling for the PQ field to $\lambda_\phi = 1.1 \times 10^{-11}$ during inflation. Moreover, the PQ invariant mass term is bounded by $|\mu_\phi| < 1.4 \times 10^{13}$ GeV, and there are similar bounds on the PQ violating terms as

$$\begin{aligned} V_{\text{PQV}} &= 3^{l/2} \sum_{k=0}^{[l/2]} |c_{0,l,k}| \cos \left((l - 2k)\theta_i + A_{0,lk} \right) \\ &< 1.0 \times 10^{-10}. \end{aligned} \tag{85}$$

We note that the slow-roll parameter ϵ_θ appearing in the PQ Noether charge in Eq. (77) depends on the PQ violating potential in Eq. (64), so it is smaller than unity as far as the coefficients of the PQ violating potential at the order l of the PQ field satisfy $3^{l/2}|c_{0,l,k}| < 1.0 \times 10^{-10}$ for each k from Eq. (85). In this case, as discussed in the end of Section 3.3, there is no axion quality problem as far as the order of the PQ violating potential is given by $l \gtrsim 11(7)$ for $f_a = 10^{12}(10^8)$ GeV, $\xi = 6$ and $|c_{0,l,k}| \lesssim 10^{-12}$.

We remark on the multiple dynamics other than the radial component of the PQ field. In both the PQ-Higgs mixed inflation and the pure PQ inflation, it was shown that the field directions orthogonal to the inflaton field can receive masses much greater than the Hubble scale during inflation. But, if the masses of the non-inflaton fields are comparable to or smaller than the Hubble scale in a certain parameter space, we need to consider the dynamics of multiple fields during inflation, for instance, for the non-Gaussianity of CMB [22]. However, the detailed discussion on the dynamics of multiple fields and the non-Gaussianity is beyond the scope of our work, so we postpone it in a future work.

5 Reheating

In this section, we discuss the basics of the inflaton condensate after inflation and provide the results for the rates for inflaton decays and scatterings. Using the results, we determine the reheating temperature and axion dark radiation and comment on the condition for the PQ symmetry restoration after reheating.

5.1 Inflaton condensate

When inflation is driven by the radial mode of the PQ field, the inflaton field value at the end of inflation is given by $\phi_{\text{end}} = \sqrt{\frac{3}{8}} \ln \left(\frac{1}{6}(35 + 8\sqrt{19}) \right) \simeq 1.50$ from $\ddot{a} = 0$, for which the inflaton energy density at the end of inflation is given by $\rho_{\text{end}} = \frac{3}{2} V_E(\phi_{\text{end}})$. The post-inflationary potential of the inflaton for $|\Phi| \ll \sqrt{3}$ takes $V_E(\phi) \simeq \frac{1}{4} \lambda_\phi \phi^4$, so the equation of state for the inflaton during reheating is radiative-like, i.e. $w_\phi = p_\phi/\rho_\phi = \frac{1}{3}$.

The inflaton condensate during reheating takes $\phi = \phi_0(t)\mathcal{P}(t)$ where $\phi_0(t)$ is the amplitude of the inflaton, which is constant over one oscillation.³ Here, $\mathcal{P}(t)$ is the periodic function satisfying $\dot{\mathcal{P}}^2 = \frac{2\rho_\phi}{\phi_0^2}(1-\mathcal{P}^4)$, which is given [18, 23] by

$$\mathcal{P}(t) = \text{cn}\left(\bar{\omega}t, \frac{1}{2}\right) \tag{86}$$

where $\bar{\omega} = \sqrt{\lambda_\phi}\phi_0$ and $\text{cn}(u, m) = \cos\varphi$ is the Jacobi cosine for $u = \int_0^\varphi d\theta/\sqrt{1-m^2\sin^2\theta}$. We note that $\bar{\omega}$ is different from the angular frequency of oscillation ω , which is given [10, 12] by

$$\omega = m_\phi\sqrt{\frac{2\pi}{3}\frac{\Gamma[\frac{3}{4}]}{\Gamma[\frac{1}{4}]}} = \sqrt{2\pi\lambda_\phi}\frac{\Gamma[\frac{3}{4}]}{\Gamma[\frac{1}{4}]}\phi_0 \tag{87}$$

where we used $m_\phi^2 = V_E''(\phi_0) = 3\lambda_\phi\phi_0^2$, that is, $\omega = 0.847\bar{\omega}$. As a result, we can make a Fourier expansion of the periodic function \mathcal{P} by $\mathcal{P}(t) = \sum_{n=-\infty}^\infty \mathcal{P}_n e^{-in\omega t}$. Then, the first few nonzero coefficients are given by $2\mathcal{P}_1 = 0.9550$, $2\mathcal{P}_3 = 0.04305$, $2\mathcal{P}_5 = 0.001859$, etc. Similarly, for the Fourier expansion of \mathcal{P}^2 as $\mathcal{P}^2(t) = \sum_{n=-\infty}^\infty (\mathcal{P}^2)_n e^{-in\omega t}$, the first few nonzero coefficients are given by $2(\mathcal{P}^2)_2 = 0.4972$, $2(\mathcal{P}^2)_4 = 0.04289$, $2(\mathcal{P}^2)_6 = 0.002778$, etc.

5.2 Reheating

For the pure PQ inflation, reheating can take place due to the PQ invariant interactions to the Higgs fields and the Yukawa couplings to the RH neutrinos,

$$\begin{aligned} \mathcal{L}_{\text{int}} \supset & -\sum_{i=1,2} \lambda_{i\phi}|H_i|^2|\Phi|^2 - 2^{p/2-1}\kappa_p H_1^\dagger H_2 \Phi^p \\ & -\frac{1}{2}y_N \overline{N}_R^c \Phi N_R + \text{h.c.}, \end{aligned} \tag{88}$$

or Planck-scale suppressed derivative interactions to the Higgs fields of type $\phi^2(\partial_\mu h_i)^2$, $\phi h_i(\partial_\mu \phi)(\partial^\mu h_i)$, as seen from the non-canonical kinetic terms of Eq. (15). In our work, we remind that it is necessary to introduce $\kappa_p H_1^\dagger H_2 \Phi^p$ for realizing the consistent electroweak symmetry breaking and the QCD axion.

The PQ invariant interactions are responsible for reheating by the inflaton scattering with the Higgs-portal couplings including the $p = 2$ term, namely, $\phi\phi \rightarrow H_1^\dagger H_1$, $H_2^\dagger H_2$, $H_1^\dagger H_2$, $H_2^\dagger H_1$ or the inflaton decay with the Higgs-portal coupling with $p = 1$, such as $\phi \rightarrow H_1^\dagger H_2$, $H_2^\dagger H_1$, and the inflaton decay/scattering with the Yukawa couplings to the RH neutrinos, such as $\phi \rightarrow N_R \overline{N}_R^c$, $\phi\phi \rightarrow N_R N_R$,

and their hermitian conjugates. Moreover, due to the self-interactions of the PQ field, we also need to consider the inflaton scattering into a pair of axions, i.e. $\phi\phi \rightarrow aa$.

During reheating, the Boltzmann equations for the averaged energy density for the inflaton and the radiation energy density ρ_R are given by

$$\dot{\rho}_\phi + 3(1+w_\phi)H\rho_\phi = -\Gamma_\phi(1+w_\phi)\rho_\phi, \tag{89}$$

$$\dot{\rho}_R + 4H\rho_R = \Gamma_\phi(1+w_\phi)\rho_\phi \tag{90}$$

where Γ_ϕ is the sum of inflaton decay or scattering rates, given by

$$\begin{aligned} \Gamma_\phi = & \Gamma_{\phi \rightarrow N_R N_R, \overline{N}_R \overline{N}_R} + \Gamma_{\phi\phi \rightarrow N_R N_R, \overline{N}_R \overline{N}_R} \\ & + \Gamma_{\phi\phi \rightarrow H_i^\dagger H_i} + \Gamma_{\phi \rightarrow H_1^\dagger H_2, H_2^\dagger H_1} \delta_{p1} \\ & + \Gamma_{\phi\phi \rightarrow H_1^\dagger H_2, H_2^\dagger H_1} \delta_{p2} + \Gamma_{\phi\phi \rightarrow aa} \end{aligned} \tag{91}$$

with δ_{p1} , δ_{p2} being the Kronecker delta.

First, we get the scattering rate of the inflaton condensate for $\phi\phi \rightarrow aa$, as follows,

$$\Gamma_{\phi\phi \rightarrow aa} = \frac{1}{8\pi(1+w_\phi)\rho_\phi} \sum_{n=1}^\infty |M_n^a|^2 (E_n \beta_n^a), \tag{92}$$

with

$$|M_n^a|^2 = 4\lambda_\phi^2 \phi_0^4 |(\mathcal{P}^2)_n|^2, \tag{93}$$

and $\beta_n^a = \sqrt{1 - \frac{m_a^2}{E_n^2}}$ and $E_n = n\omega$. For $\phi \gg f_a$ during reheating, the effective mass of the axion is given by $m_a = \sqrt{\lambda_\phi}\phi$. Since $\omega = 0.847\sqrt{\lambda_\phi}\phi_0$, so $2\omega > m_a$, $\phi\phi \rightarrow aa$ is open kinematically. After the Fourier expansion of the inflaton condensate, the averaged scattering rate for $\phi\phi \rightarrow aa$ is given by

$$\langle \Gamma_{\phi\phi \rightarrow aa} \rangle = \frac{9\lambda_\phi^2 \phi_0^2 \omega}{2\pi m_\phi^2} \Sigma_a \left\langle \left(1 - \frac{m_a^2}{\omega^2 n^2}\right)^{1/2} \right\rangle \tag{94}$$

with $\Sigma_a = \sum_{n=1}^\infty n |(\mathcal{P}^2)_n|^2$.

Similarly, from the Higgs-portal interactions of the PQ inflaton, $\mathcal{L}_{\text{int}} = -\lambda_{i\phi}|H_i|^2|\Phi|^2$, we obtain the scattering rate of the inflaton condensate, as follows,

$$\begin{aligned} \Gamma_{\phi\phi \rightarrow H_i^\dagger H_i} = & \frac{1}{8\pi(1+w_\phi)\rho_\phi} \sum_{n=1}^\infty |M_n^{H_i}|^2 (E_n \beta_n^{H_i}), \\ & i = 1, 2, \end{aligned} \tag{95}$$

with

$$|M_n^{H_i}|^2 = 4\lambda_{H_i\phi}^2 \phi_0^4 |(\mathcal{P}^2)_n|^2 \tag{96}$$

and $\beta_n^{H_i} = \sqrt{1 - \frac{m_{H_i}^2}{E_n^2}}$. Here, the effective masses for the Higgs fields are given by $m_{H_i}^2 = m_i^2 + \frac{1}{2}\lambda_{H_i\phi}\phi^2(t)$, $i = 1, 2$, where m_i^2 are the bare Higgs mass parameters. Thus,

³ The amplitude of the inflaton undergoes damping during the Hubble expansion as $\phi_0(t) = \phi_{\text{end}}\sqrt{t_{\text{end}}/t}$ in the case of $V_E(\phi) = \frac{1}{4}\lambda_\phi\phi^4$ during reheating.

after the Fourier expansion, the averaged scattering rate for $\phi\phi \rightarrow H_i^\dagger H_i$ is given by

$$\langle \Gamma_{\phi\phi \rightarrow H_i^\dagger H_i} \rangle = \frac{9\lambda_{H_i\Phi}^2 \phi_0^2 \omega}{2\pi m_\phi^2} \hat{\Sigma}_H \left\langle \left(1 - \frac{m_{H_i}^2}{\omega^2 n^2} \right)^{1/2} \right\rangle, \quad i = 1, 2, \tag{97}$$

with $\hat{\Sigma}_H = \sum_{n=1}^\infty n |\mathcal{P}_n|^2$. We note that we need to take the Higgs-portal coupling to be small enough, namely, $|\lambda_{H_i\Phi}| \lesssim 10^{-5}$, in order to keep the running quartic coupling λ_Φ at the order of 10^{-11} . As $m_{H_i}^2/\omega^2 \sim 0.70\lambda_{H_i\Phi}^2/\lambda_\Phi$, $\phi\phi \rightarrow H_i^\dagger H_i$ for the lowest inflaton modes is kinematically open if $|\lambda_{H_i\Phi}| \lesssim 1.2\sqrt{\lambda_\Phi} \simeq 4 \times 10^{-6}$ for $\lambda_\Phi = 1.1 \times 10^{-11}$.

For the PQ invariant potential with $p = 2$, $\mathcal{L}_{\text{int}} \supset -\kappa_2 H_1^\dagger H_2 \Phi^2 + \text{h.c.}$, the scattering rates for $\phi\phi \rightarrow H_1^\dagger H_2$ and its hermitian conjugate, are identical,

$$\Gamma_{\phi\phi \rightarrow H_1^\dagger H_2} = \frac{1}{8\pi(1+w_\phi)\rho_\phi} \sum_{n=1}^\infty |M_n^H|^2 (E_n \beta_n^H) = \Gamma_{\phi\phi \rightarrow H_2^\dagger H_1}, \tag{98}$$

with

$$|M_n^H|^2 = 4\kappa_2^2 \phi_0^4 |\mathcal{P}_n|^2 \tag{99}$$

and $\beta_n^H = \sqrt{1 - \frac{(m_{H_1} + m_{H_2})^2}{4E_n^2}} \sqrt{1 - \frac{(m_{H_1} - m_{H_2})^2}{4E_n^2}}$. Then, after the Fourier expansion, the averaged scattering rate for $\phi\phi \rightarrow H_1^\dagger H_2$ is given by

$$\langle \Gamma_{\phi\phi \rightarrow H_1^\dagger H_2} \rangle = \frac{9\kappa_2^2 \phi_0^2 \omega}{2\pi m_\phi^2} \hat{\Sigma}_H \left\langle \left(1 - \frac{(m_{H_1} + m_{H_2})^2}{4\omega^2 n^2} \right)^{1/2} \times \left(1 - \frac{(m_{H_1} - m_{H_2})^2}{4\omega^2 n^2} \right)^{1/2} \right\rangle. \tag{100}$$

We note that the Higgs-portal coupling κ_2 is similarly bounded to $|\kappa_2| \lesssim 10^{-5}$, in order to keep the running quartic coupling λ_Φ small during inflation.

For the PQ invariant potential with $p = 1$, $\mathcal{L}_{\text{int}} \supset -\frac{1}{\sqrt{2}}\kappa_1 H_1^\dagger H_2 \Phi + \text{h.c.}$, the decay rates for $\phi \rightarrow H_1^\dagger H_2$ and its hermitian conjugate, are identical,

$$\Gamma_{\phi \rightarrow H_1^\dagger H_2} = \frac{1}{8\pi(1+w_\phi)\rho_\phi} \sum_{n=1}^\infty |M_n^{H'}|^2 (E_n \beta_n^{H'}) = \Gamma_{\phi \rightarrow H_2^\dagger H_1} \tag{101}$$

where

$$|M_n^{H'}|^2 = \kappa_1^2 \phi_0^2 |\mathcal{P}_n|^2 \tag{102}$$

and $\beta_n^{H'} = \sqrt{1 - \frac{(m_{H_1} + m_{H_2})^2}{E_n^2}} \sqrt{1 - \frac{(m_{H_1} - m_{H_2})^2}{E_n^2}}$. After the Fourier expansion, the averaged decay rate for $\phi \rightarrow H_1^\dagger H_2$

is given by

$$\langle \Gamma_{\phi \rightarrow H_1^\dagger H_2} \rangle = \frac{9\kappa_1^2 \omega}{8\pi m_\phi^2} \Sigma_H \left\langle \left(1 - \frac{(m_{H_1} + m_{H_2})^2}{\omega^2 n^2} \right)^{1/2} \times \left(1 - \frac{(m_{H_1} - m_{H_2})^2}{\omega^2 n^2} \right)^{1/2} \right\rangle, \tag{103}$$

with $\Sigma_H = \sum_{n=1}^\infty n |\mathcal{P}_n|^2$. Similarly, the κ_1 term contributes to $\phi\phi \rightarrow H_1^\dagger H_1, H_2^\dagger H_2$ too, but the corresponding scattering rates are proportional to κ_1^4 , so it turns out to be suppressed by the higher inverse powers of the Planck scale so negligible as compared to the other decay/scattering rates.

The PQ inflaton can be responsible for the generation of masses for the right-handed neutrinos through the Yukawa couplings, which give rise to the decay rate of the inflaton condensate, as follows,

$$\Gamma_{\phi \rightarrow N_R N_R} = \frac{1}{8\pi(1+w_\phi)\rho_\phi} \sum_{n=1}^\infty |M_n^N|^2 (E_n \beta_n^N) = \Gamma_{\phi \rightarrow \overline{N_R} \overline{N_R}} \tag{104}$$

where

$$|M_n^N|^2 = y_N^2 \phi_0^2 |\mathcal{P}_n|^2 E_n^2 (\beta_n^N)^2, \tag{105}$$

with $\beta_n^N = \sqrt{1 - \frac{4m_N^2}{E_n^2}}$ and $m_N = \frac{1}{\sqrt{2}} y_N \phi$ being the effective masses for the RH neutrinos. After averaging over oscillations, we get

$$\langle \Gamma_{\phi \rightarrow N_R N_R} \rangle = \frac{9y_N^2 \omega^3}{8\pi m_\phi^2} \Sigma_N \left\langle \left(1 - \frac{4m_N^2}{\omega^2 n^2} \right)^{3/2} \right\rangle, \tag{106}$$

with $\Sigma_N = \sum_{n=1}^\infty n^3 |\mathcal{P}_n|^2$. We also note that the Yukawa couplings of the RH neutrinos to the PQ field must be chosen to be small enough, namely, $y_N \lesssim 10^{-3}$, in order to make the running effects on the quartic coupling λ_Φ ignorable. We note that as $4m_N^2/\omega^2 \sim 2.79y_N^2/\lambda_\Phi$, the decay of the lowest inflaton modes into a pair of RH neutrinos are kinematically open if $|y_N| \lesssim 0.60\sqrt{\lambda_\Phi} \sim 2 \times 10^{-6}$ for $\lambda_\Phi = 1.1 \times 10^{-11}$.

Finally, the same Yukawa couplings of the PQ field to the RH neutrinos also give rise to the inflaton scattering, $\phi\phi \rightarrow N_R N_R, \overline{N_R} \overline{N_R}$, with the corresponding scattering rate,

$$\Gamma_{\phi\phi \rightarrow N_R N_R} = \frac{1}{8\pi(1+w_\phi)\rho_\phi} \sum_{n=1}^\infty |M_n^{N'}|^2 (E_n \beta_n^{N'}) = \Gamma_{\phi\phi \rightarrow \overline{N_R} \overline{N_R}} \tag{107}$$

where

$$|M_n^{N'}|^2 = \frac{32y_N^4 m_N^2 \phi_0^4 (\beta_n^{N'})^2}{E_n^2} \tag{108}$$

and $\beta_n^{N'} = \sqrt{1 - \frac{m_N^2}{E_n^2}}$. Thus, the averaged scattering rate is given by

$$\langle \Gamma_{\phi\phi \rightarrow NRNR} \rangle = \frac{36y_N^4 \phi_0^2}{\pi m_\phi^2 \omega} \hat{\Sigma}_N \left\langle m_N^2 \left(1 - \frac{m_N^2}{n^2 \omega^2} \right)^{3/2} \right\rangle, \tag{109}$$

with $\hat{\Sigma}_N = \sum_{n=1}^\infty n^{-1} |(\mathcal{P}^2)_n|^2$.

5.3 Reheating temperature

For $a_{\text{end}} \ll a \ll a_{\text{RH}}$ where a_{RH} is the scale factor at the time reheating is complete, we can ignore the inflaton decay/scattering rates and approximate the energy density for the inflaton as

$$\rho_\phi(a) \simeq \rho_{\text{end}} \left(\frac{a_{\text{end}}}{a} \right)^4. \tag{110}$$

When the reheating process is dominated by the perturbative decays and scattering processes of the inflaton, we obtain the reheating temperature during reheating, as follows,

$$T_{\text{RH}} = \left(\frac{30}{\pi^2 g_*(T_{\text{RH}})} \right)^{1/4} \left(\frac{4}{3} \sqrt{3} M_P \gamma_\phi \right). \tag{111}$$

where we took the sum of the decay and scattering rates of the inflaton by $\gamma_\phi \equiv \Gamma_\phi / \rho_\phi^{1/4}$.

We first compute the decay and scattering rates, as follows,

$$\begin{aligned} \gamma_\phi|_{\text{decay}} &\simeq \frac{3\sqrt{\pi}}{2} \lambda_\phi^{1/4} \left(\frac{\Gamma(\frac{3}{4})}{\Gamma(\frac{1}{4})} \right)^3 \\ &\times \max \left[0.5y_N^2 \Sigma_N \mathcal{R}_N^{-1/2}, \frac{\kappa_1^2}{\omega^2} \Sigma_H \mathcal{R}_H^{-1/2} \right], \end{aligned} \tag{112}$$

$$\begin{aligned} \gamma_\phi|_{\text{scattering}} &\simeq \frac{3}{\sqrt{\pi} \lambda_\phi^{3/4}} \left(\frac{\Gamma(\frac{3}{4})}{\Gamma(\frac{1}{4})} \right) \\ &\times \max \left[\lambda_{H_i\Phi}^2 \hat{\Sigma}_H \hat{\mathcal{R}}_H^{-1/2}, \kappa_2^2 \hat{\Sigma}_H \bar{\mathcal{R}}_H^{-1/2}, \right. \\ &\left. \times 8y_N^4 \frac{m_N^2}{\omega^2} \hat{\Sigma}_N \hat{\mathcal{R}}_N^{-1/2} \right]. \end{aligned} \tag{113}$$

Here, $\Sigma_N = 0.2406$, $\Sigma_H = 0.2294$, $\hat{\Sigma}_H = 0.1255$, $\hat{\Sigma}_N = 0.0310$, and we took the averaged phase space factor for $2m_N, 2m_{H_i} \gg w$ by $\mathcal{R}_N \equiv 4m_N^2/w^2$, $\hat{\mathcal{R}}_N \equiv m_N^2/w^2$, $\mathcal{R}_H \equiv (m_{H_1} + m_{H_2})^2/w^2$, $\hat{\mathcal{R}}_H \equiv m_{H_i}^2/w^2$ and $\bar{\mathcal{R}}_H \equiv (m_{H_1} + m_{H_2})^2/(4w^2)$ [24]. Thus, we can determine the reheating temperature by the inflaton decay/scattering into a pair of Higgs bosons or RH neutrinos, as follows,

$$T_{\text{RH}}|_{\text{decay}} \simeq 5.2 \times 10^5 \text{ GeV} \left(\frac{100}{g_*(T_{\text{reh}})} \right)^{1/4}$$

$$\times \left(\frac{\max[y_N, \kappa_1/\omega]}{10^{-4}} \right)^2 \left(\frac{\lambda_\phi}{10^{-11}} \right)^{1/4}, \tag{114}$$

$$\begin{aligned} T_{\text{RH}}|_{\text{scattering}} &\simeq 3.0 \times 10^{11} \text{ GeV} \left(\frac{100}{g_*(T_{\text{reh}})} \right)^{1/4} \\ &\left(\frac{\max[\lambda_{H_i\Phi}, \kappa_2, \sqrt{2}y_N^2 m_N/\omega]}{10^{-7}} \right)^2 \left(\frac{10^{-11}}{\lambda_\phi} \right)^{3/4}. \end{aligned} \tag{115}$$

Here, we note that the coefficient κ_1 of the PQ invariant term, $H_1^\dagger H_2 \Phi$, is bounded to $|\kappa_1| \lesssim 10^3 \text{ GeV} (f_a/10^8 \text{ GeV})$ for electroweak symmetry breaking as discussed in the end of Section 3.2, while $\omega \sim \sqrt{\lambda_\phi} \phi_0 \sim 10^{13} \text{ GeV}$, so $\kappa_1/\omega \lesssim 10^{-10} (f_a/10^8 \text{ GeV})$. Thus, the κ_1 term is not efficient for reheating as far as $y_N \gtrsim \kappa_1/\omega$. As a result, we find that a low reheating temperature below $T_{\text{RH}} \sim 10^5 \text{ GeV}$ can be obtained from the inflaton decay with $10^{-10} (f_a/10^8 \text{ GeV}) \lesssim y_N \lesssim 10^{-4}$, but a high reheating temperature up to $T_{\text{RH}} \sim 10^{11} \text{ GeV}$ is achievable due to the inflaton scattering with $\kappa_2 \lesssim \lambda_{H_i\Phi} \lesssim 10^{-7}$, which is consistent with a small running quartic coupling λ_ϕ during inflation.⁴

5.4 Axion dark radiation

The axions produced from the inflaton scattering can remain out of equilibrium. Then, we get the correction to the effective number of neutrino species, as follows [10,25],

$$\Delta N_{\text{eff}} = 0.02678 \left(\frac{Y_a}{Y_a^{\text{eq}}} \right) \left(\frac{106.75}{g_{*s}(T_{\text{reh}})} \right)^{4/3} \tag{116}$$

where Y_a is the axion abundance produced from the inflaton scattering, Y_a^{eq} is the axion abundance at equilibrium given by $Y_a^{\text{eq}} = \frac{45\xi(3)}{2\pi^4 g_{*s}(T_{\text{reh}})}$, and $N_{\text{eff}}^{\text{SM}} = 3.0440$ in the SM [26–28]. Then, we need $Y_a \lesssim 10Y_a^{\text{eq}}$ to be consistent with the current bounds from the Planck satellite, $N_{\text{eff}} = 2.99 \pm 0.17$ [29].

In our model, the ratio of the scattering rates into the axion pair and Higgs pair is given by

$$\frac{\Gamma_{\phi\phi \rightarrow aa}}{\Gamma_{\phi\phi \rightarrow H_i^\dagger H_i}} \simeq \frac{\lambda_\phi^2}{\lambda_{H_i\Phi}^2}. \tag{117}$$

Thus, for $\lambda_{H_i\Phi} \gtrsim \lambda_\phi$, $\phi\phi \rightarrow aa$ is subdominant for reheating, but the produced axions can contribute to the effective number of neutrinos, ΔN_{eff} . In this case, the axions produced during reheating make small contributions to ΔN_{eff} [10].

On the other hand, when the universe is reheated to a sufficiently high reheating temperature [25,30,31],

$$T_{\text{reh}} \gtrsim 1.7 \times 10^9 \text{ GeV} \left(\frac{f_a}{10^{11} \text{ GeV}} \right)^{2.246} \equiv T_{a,\text{eq}}, \tag{118}$$

⁴ Here, the condition $\kappa_2 \lesssim \lambda_{H_i\Phi}$ comes from the stability along the Higgs fields during inflation, as shown in Eq. (70).

the axions could become thermalized with the SM plasma. In this case, the contribution of the axions to the effective number of neutrino species is given just by the abundance in thermal equilibrium Y_a^{eq} , as follows,

$$\Delta N_{\text{eff}} = \frac{4}{7} \left(\frac{T_{a,0}}{T_{\nu,0}} \right)^4 = \frac{4}{7} \left(\frac{11}{4} \right)^{4/3} \left(\frac{g_{*s}(T_0)}{g_{*s}(T_{a,\text{eq}})} \right)^{4/3} \quad (119)$$

where $T_{\nu,0}, T_{a,0}$ are the neutrino and axion temperatures, respectively, at present, and $g_{*s}(T_0) = 3.91$. Thus, we get $\Delta N_{\text{eff}} = 0.02397$ for $g_{*s}(T_{a,\text{eq}}) = 116$ (adding one more Higgs doublet and three right-handed neutrinos to the SM); $\Delta N_{\text{eff}} = 0.02550$ for $g_{*s}(T_{a,\text{eq}}) = 110.75$ (adding one more Higgs doublet to the SM). We remark that future CMB experiments such as CMB-S4 [32,33] can test the excess in the effective number of neutrinos in the future.

5.5 PQ symmetry restoration after reheating

After reheating, the leading order thermal potential for the PQ field is given by

$$V_T(\Phi) = \frac{1}{24} T^2 \left(\sum_b n_b m_{b,\text{eff}}^2 + \sum_f n_f m_{f,\text{eff}}^2 \right) + \dots \\ = \beta T^2 |\Phi|^2 + \dots \quad (120)$$

with

$$\beta \equiv \frac{1}{24} (4\lambda_{H_1\Phi} + 4\lambda_{H_2\Phi} + 6y_N^2). \quad (121)$$

Thus, if $\beta T^2 + \mu_\Phi^2 > 0$, the PQ symmetry would be restored after reheating, so domain-walls and cosmic strings could be formed even after inflation. Taking $\mu_\Phi^2 \simeq -\lambda_\Phi v_\Phi^2$ in Eq. (22) and $v_\Phi \simeq f_a$, we get the upper bound on the reheating temperature in order for the PQ symmetry not to be restored after reheating, as follows,

$$T_{\text{reh}} < \sqrt{\frac{\lambda_\Phi}{\beta}} f_a \equiv T_{\text{restore}}. \quad (122)$$

Therefore, for $\lambda_{H_1\Phi}, \lambda_{H_2\Phi} \sim 10^{-10}$ and $y_N \sim 10^{-6}$, the upper bound on the reheating temperature is given by $T_{\text{restore}} \simeq 0.57 f_a$.

6 Axion kinetic misalignment

We briefly summarize the evolution of the axion velocity in the post-inflationary period and determine the axion relic density from the axion kinetic misalignment.

6.1 Post-inflationary evolution of axion velocity

After the end of inflation, the total Noether charge for the PQ symmetry is conserved approximately, so $a^3 n_\theta = a^3 \dot{\phi}^2 \dot{\theta}$ is

almost constant. As a result, the PQ Noether charge density from the axion rotation red-shifts at the end of reheating by

$$n_\theta(T_{\text{RH}}) = n_{\theta,\text{end}} \left(\frac{a_{\text{end}}}{a_{\text{RH}}} \right)^3 \quad (123)$$

where $a_{\text{end}}, a_{\text{RH}}$ are the values of the scale factor at the end of inflation and the reheating completion, respectively. Then, suppose that the reheating temperature is sufficiently high such that $\phi(a_{\text{RH}}) > 3f_a$, namely, $T_{\text{RH}} > T_{\text{RH}}^c$, with

$$T_{\text{RH}}^c \equiv \left(\frac{90\lambda_\Phi}{8\pi^2 g_*} \right)^{1/4} 2f_a \\ = \left(\frac{100}{g_*} \right)^{1/4} \left(\frac{f_a}{10^{11} \text{ GeV}} \right) (1.2 \times 10^8 \text{ GeV}). \quad (124)$$

In this case, using

$$\frac{a_{\text{end}}}{a_{\text{RH}}} = \left(\frac{\rho_{\text{RH}}}{\rho_{\text{end}}} \right)^{1/4}, \quad (125)$$

with $\rho_{\text{RH}} = \frac{\pi^2}{30} g_*(T_{\text{RH}}) T_{\text{RH}}^4$ and $\rho_{\text{end}} = \frac{3}{2} V_E(\phi_{\text{end}})$, we obtain the PQ Noether charge density at the reheating temperature, as follows,

$$n_\theta(T_{\text{RH}}) = n_{\theta,\text{end}} \left(\frac{\pi^2 g_*(T_{\text{RH}}) T_{\text{RH}}^4}{45 V_E(\phi_{\text{end}})} \right)^{3/4}. \quad (126)$$

Here, $g_*(T_{\text{RH}}), g_*(T_*)$ are the number of the effective entropy degrees of freedom at the reheating temperature and the onset of the axion oscillation, respectively. Thus, the PQ Noether charge density at $T = T_{\text{RH}}$ is independent of the reheating temperature, so is the axion abundance.

When the reheating is delayed such that $T_{\text{RH}} < T_{\text{RH}}^c$, the energy density of the inflation scales during reheating as

$$\rho_\phi = \rho_{\text{end}} \left(\frac{a_{\text{end}}}{a_c} \right)^4 \left(\frac{a_c}{a_{\text{RH}}} \right)^3 \quad (127)$$

where a_c is the scalar factor when $\phi(a_c) = 3f_a$ such that the inflation becomes matter-like for $a > a_c$. Then, using

$$\frac{a_c}{a_{\text{RH}}} = \left(\frac{\rho_{\text{RH}}}{\rho_{\phi,c}} \right)^{1/3} = \left(\frac{T_{\text{RH}}}{T_{\text{RH}}^c} \right)^{4/3}, \quad (128)$$

we get the PQ Noether charge density at the end of reheating for $T_{\text{RH}} < T_{\text{RH}}^c$ as

$$n_\theta(T_{\text{RH}}) = n_{\theta,\text{end}} \left(\frac{a_{\text{end}}}{a_c} \right)^3 \left(\frac{a_c}{a_{\text{RH}}} \right)^3 \\ = n_{\theta,\text{end}} \left(\frac{\pi^2 g_*(T_{\text{RH}}) T_{\text{RH}}^4}{45 V_E(\phi_{\text{end}})} \right)^{3/4} \left(\frac{T_{\text{RH}}}{T_{\text{RH}}^c} \right). \quad (129)$$

6.2 Axion relic density

After the axion gets massive due to the QCD instanton effects and the kinetic energy of the axion is comparable to the potential of the axion, $\frac{1}{2} f_a^2 \dot{\theta}^2(T_*) = 2 m_a^2(T_*) f_a^2$, at

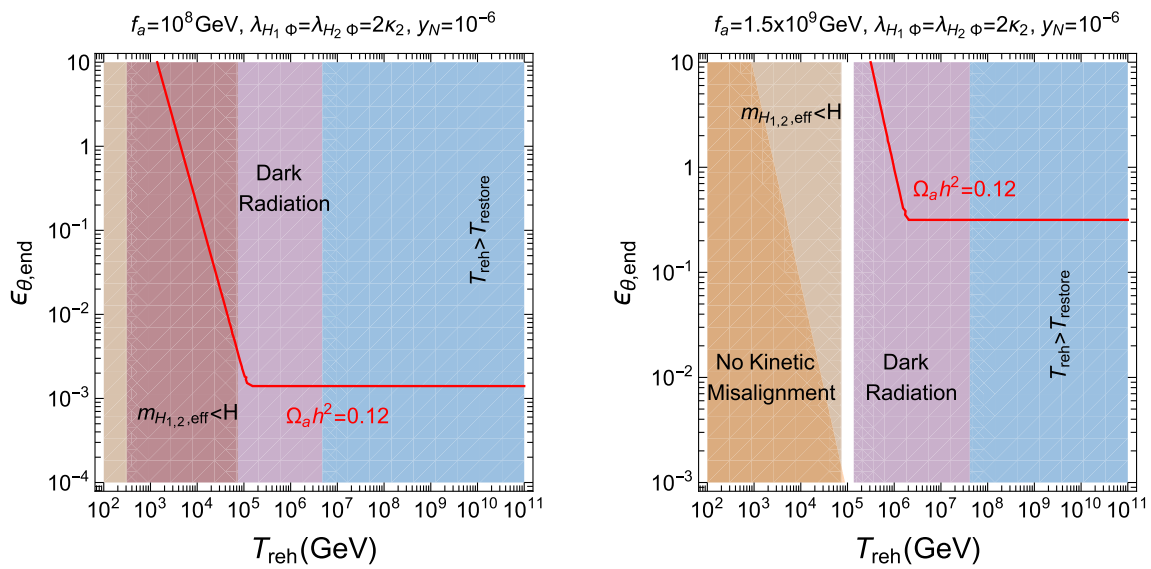


Fig. 1 Reheating temperature T_{reh} vs $\epsilon_{\theta,\text{end}}$ for axion dark matter with kinetic misalignment. The correct relic density is obtained by the axion kinetic misalignment along the red line. The kinetic misalignment is sub-dominant in orange region, the axions produced from the inflaton scattering were in thermal equilibrium in purple regions, the PQ sym-

metry is restored after reheating in cyan region, and the effective Higgs masses are smaller than the Hubble scale during inflation in brown region. We took $f_a = 10^8$ GeV, 1.5×10^9 GeV on the left and right plots, respectively. We chose $\lambda_{H_1\Phi} = \lambda_{H_2\Phi} = 2\kappa_2$ and $y_N = 10^{-6}$

the temperature $T = T_*$, the axion is confined to one of the local minima of the axion potential and it starts oscillating for $m_a(T_*) \geq 3H(T_*)$. Using $\dot{\theta}(T_*) = 2m_a(T_*)$, we get the condition for the axion kinetic misalignment as $\dot{\theta}(T_*) \geq 6H(T_{\text{osc}})$, so the onset of oscillation, namely, $T_* \leq T_{\text{osc}}$, where T_{osc} is the temperature of the standard axion oscillation determined by $m_a(T_{\text{osc}}) = 3H(T_{\text{osc}})$, with no initial axion kinetic energy.

When the axion kinetic misalignment is a dominant mechanism for determining the axion relic density, we obtain the axion relic abundance by

$$\Omega_a h^2 = 0.12 \left(\frac{10^9 \text{ GeV}}{f_a} \right) \left(\frac{Y_\theta}{40} \right) \tag{130}$$

where Y_θ is the abundance for the axion given by $Y_\theta = \frac{n_\theta(T_{\text{RH}})}{s(T_{\text{RH}})}$ with $n_\theta(T_{\text{RH}})$ and $s(T_{\text{RH}})$ being the Noether charge density and the entropy density at reheating, respectively. In comparison, the axion abundance, $Y_{a,\text{mis}} = \frac{n_a}{s}$, determined by the axion misalignment is given by $Y_{a,\text{mis}} = 0.11(f_a/10^9 \text{ GeV})^{13/6}$. So, the axion kinetic misalignment is dominant as far as $f_a < 1.5 \times 10^{11} \text{ GeV}$ [9].

In Fig. 1, we show the parameter space for the reheating temperature, T_{reh} , vs the slow-roll parameter for the axion at the end of inflation, $\epsilon_{\theta,\text{end}}$, satisfying the correct relic density from the axion in red lines. As compared to the results in Ref. [10], we have indicated the region where the PQ symmetry is restored after reheating, namely, $T_{\text{reh}} > T_{\text{restore}}$, in cyan color, and also showed the region where the effective masses for two Higgs doublets, which correspond to $m_{\pm,\text{eff}}$ in Eq. (71)

and $m_{\tilde{A},\text{eff}}$ in Eq. (48)), are smaller than the Hubble scale during inflation, namely, $m_{H_{1,2},\text{eff}} < H_I$, in brown color. In the parameter space where the relic density is explained, the axions produced from reheating can be dark radiation at a detectable level in the future CMB experiments, as shown in the purple region. We fixed the axion decay constant to $f_a = 10^8, 1.5 \times 10^9$ GeV in the left and right plots, respectively, and chose $\lambda_{H_1\Phi} = \lambda_{H_2\Phi} = 2\kappa_2$ and $y_N = 10^{-6}$ in common.

As a result, we find that the low reheating temperature below 10^5 GeV is bounded by $m_{H_{1,2},\text{eff}} > H_I$ due to small Higgs-portal couplings required for reheating, and the high reheating temperature above about 10^7 GeV is constrained by the restoration of the PQ symmetry after reheating. If the PQ symmetry is restored after reheating, cosmic strings and domain walls can be formed due to the breaking of the PQ symmetry and the discrete symmetry of the axion potential, respectively. Those cosmic strings and domain walls are not necessarily problematic if the explicit violation of the PQ symmetry is sizable.

In our model, the PQ violating potential gives rise to a nonzero pressure for domain walls formed after QCD phase transition such that they annihilate and never dominate the universe. For a bias term parametrized by $\Delta V = c \Lambda_{\text{QCD}}^4 \times 10^{-10}$ in Eq. (38) with c being a dimensionless parameter given by the PQ violating term of order l in the PQ field, as follows,

$$c = |c_{0,l,k}| \left(\frac{f_a}{\sqrt{2}q_\Phi M_P} \right)^l \left(\frac{M_P}{\Lambda_{\text{QCD}}} \right)^4 \times 10^{10}, \tag{131}$$

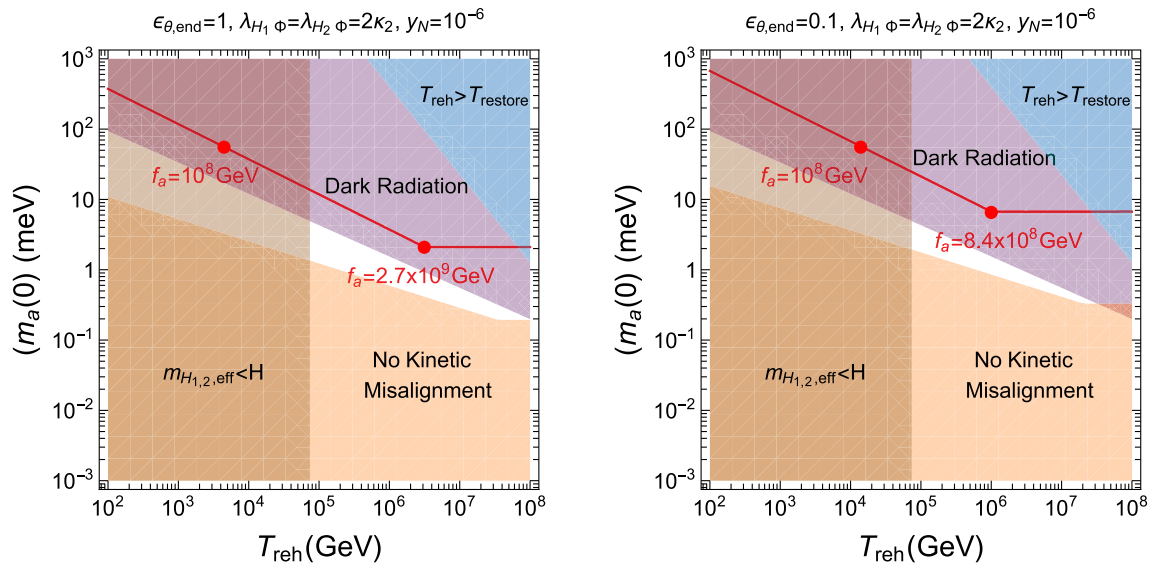


Fig. 2 Reheating temperature T_{reh} vs the axion mass $m_a(0)$ for axion dark matter with kinetic misalignment. The correct relic density is obtained by the axion kinetic misalignment along the red line. The

other color codes are the same as in Fig. 1. We took $\epsilon_{\theta,\text{end}} = 1, 0.1$ on the left and right plots, respectively. We chose $\lambda_{H_1\Phi} = \lambda_{H_2\Phi} = 2\kappa_2$ and $y_N = 10^{-6}$

the pressure for the domain walls becomes dominant over the energy density of the domain walls before Big Bang Nucleosynthesis, as far as $\Delta V \gtrsim \frac{\sigma^2}{M_P^2} (t_*/0.1 \text{ s})$ where σ is the tension of domain walls and t_* is the time at which the energy density of the domain walls dominates the radiation energy density, given by $t_* \sim \frac{M_P^2}{\sigma}$ [34]. Thus, for $\sigma \sim \Lambda_{\text{QCD}}^2 f_a$, there is no domain wall problem as far as $c \gtrsim 10^{-3} (f_a/10^9 \text{ GeV})$, which is satisfied in a consistent parameter space for l and f_a where the condition for the axion quality in Eq. (42) and the CMB bound in Eq. (85) are satisfied simultaneously. For instance, it is sufficient to take $l = 9(12)$ for $f_a = 10^9(10^{12}) \text{ GeV}$. Otherwise, in the region of the parameter space with $T_{\text{reh}} > T_{\text{restore}}$ in Fig. 1, it is relevant to consider the production of axion domain walls and their impact on the axion relic density.

As the Yukawa couplings for the RH neutrinos are increased up to 10^{-4} , the reheating temperature increases up to 10^5 GeV even if the same Higgs-portal couplings are small and the temperature for restoring the PQ symmetry gets smaller. So, for $y_N > 10^{-6}$, the brown and cyan regions are shifted to the right and left in the two plots of Fig. 1, respectively, resulting in more tight constraints for the axion kinetic misalignment.

In Fig. 2, we also depict the parameter space for the reheating temperature, T_{reh} , vs the axion mass at zero temperature, $m_a(0)$, showing the correct relic density in red lines, for $\epsilon_{\theta,\text{end}} = 1, 0.1$, in the left and right plots, respectively. The lower end, $f_a = 10^8 \text{ GeV}$, is imposed by supernova cooling constraint, whereas the upper ends, $f_a = 2.7 \times 10^9, 8.4 \times 10^8 \text{ GeV}$, in the left and right plots, are shown, because the

axion abundance becomes independent of the reheating temperature above those upper ends. We took the same parameters for $\lambda_{H_1\Phi}, \lambda_{H_2\Phi}, \kappa_2, y_N$, as in Fig. 1. Thus, we find that there is a parameter space where the axion kinetic misalignment is responsible for the correct relic density while two Higgs fields are stabilized and the PQ symmetry remains broken after reheating.

Before closing this section, we remark on the isocurvature perturbations of the axion generated during inflation [35]. The total axion relic density is composed of $Y_a = Y_{a,\text{kin}} + Y_{a,\text{mis}}$, with $sY_{a,\text{kin}} = 2m_a n_\theta$ and $sY_{a,\text{mis}} = \frac{1}{2} m_a^2 f_a^2 \theta_*^2$. Then, the power spectrum of the isocurvature perturbation at the horizon exit of the mode k_* during inflation depends only on $Y_{a,\text{mis}}$ is given by

$$P_{\text{iso}}(k_*) = \left(\frac{1}{Y_a} \frac{\partial Y_a}{\partial \theta_*} \right)^2 \langle \delta\theta_*^2 \rangle = \left[\frac{4}{\theta_*^2} \left(\frac{Y_{a,\text{mis}}}{Y_a} \right)^2 + \frac{1}{4} \left(\frac{1}{\epsilon_{\theta,*}} \frac{\partial \epsilon_{\theta,*}}{\partial \theta_*} \right)^2 \left(\frac{Y_{a,\text{kin}}}{Y_a} \right)^2 \right] \langle \delta\theta_*^2 \rangle \tag{132}$$

where the fluctuation of the initial angle is given by

$$\langle \delta\theta_*^2 \rangle = \frac{1}{f_{a,\text{eff}}^2} \left(\frac{H_I}{2\pi} \right)^2 \tag{133}$$

with $f_{a,\text{eff}} \equiv \sqrt{6} \left| \sinh \left(\frac{\phi_*}{\sqrt{6} M_P} \right) \right|$. Here, we took into account the non-canonical kinetic term for the fluctuation of θ in Eq. (63) in the Einstein frame, so the effective axion decay constant becomes very large for $\phi_* \gg M_P$ during inflation. The fluctuation of $Y_{a,\text{kin}}$ with respect to θ is nonzero, because the initial axion velocity during inflation depends on

the initial value of θ through the explicit PQ breaking term in Eq. (73), namely, the slow-roll parameter ϵ_θ for θ . Although the fluctuation of $Y_{a,\text{kin}}$ also depends on the inflaton field ϕ through the PQ charge, n_θ , in Eq. (77), it does not contribute to the isocurvature perturbations because the inflaton fluctuation is adiabatic. The bound on the isocurvature perturbations from Planck satellite [20] is given by

$$\beta_{\text{iso}} \equiv \frac{P_{\text{iso}}(k_*)}{P_\zeta(k_*) + P_{\text{iso}}(k_*)} < 0.038, \quad (134)$$

at 95% C.L., with $P_\zeta(k_*) = A_s = 2.1 \times 10^{-9}$ at $k_* = 0.05 \text{ Mpc}^{-1}$. Then, taking $Y_a \simeq Y_{a,\text{kin}}$ and using

$$\left(\frac{1}{\epsilon_{\theta,*}} \frac{\partial \epsilon_{\theta,*}}{\partial \theta} \right)^2 = 4(l-2k)^2 \cot^2 \left((l-2k)\theta_* + A_{0,l,k} \right), \quad (135)$$

we get

$$\frac{4}{\theta_*^2} \left(\frac{Y_{a,\text{mis}}}{Y_{a,\text{kin}}} \right)^2 + (l-2k)^2 \cot^2 \left((l-2k)\theta_* + A_{0,l,k} \right) \lesssim 10. \quad (136)$$

Here, we assumed a single explicit PQ breaking term at the order of f_a^l in Eq. (64) and used $\epsilon_* \simeq 2 \times 10^{-4}$ for $N = 60$ in Eq. (82), for which $\phi_* \simeq 7.1 M_P$ and the effective axion decay constant during inflation becomes $f_{a,\text{eff}} \simeq 22 M_P$. Then, the above result leads to $Y_{a,\text{mis}}/Y_{a,\text{kin}} \lesssim 5$ and $|(l-2k) \cot((l-2k)\pi + A_{0,l,k})| \lesssim 3$ for $\theta_* = \pi$, which are consistent with the dominance of the axion kinetic misalignment, as far as $|l-2k| \lesssim 3$ is taken or a mild fine-tuning between θ_* and the phase shift $A_{0,l,k}$ is made for a small value of $|\cot((l-2k)\theta_* + A_{0,l,k})|$ with a large $|l-2k|$. As a result, in our model, the isocurvature perturbations are sufficiently small due to a large value of the effective axion decay constant during inflation.

7 Conclusions

We presented a consistent framework with the $U(1)$ PQ symmetry for setting the initial axion velocity at the end of inflation. Including a PQ complex scalar field and an extra Higgs doublet conformally coupled to gravity, we obtained the PQ anomalies from the SM quarks to solve the strong CP problem by the axion and we achieved a slow-roll inflation dominantly by the PQ invariant potential. We also added three RH neutrinos for seesaw mechanism for neutrino masses. Assuming an explicit violation of the PQ symmetry at the Planck scale in the scalar potential due to quantum gravity effects, we showed that a sufficiently large initial axion velocity can be obtained at the end inflation while the axion quality problem is absent.

Focusing on the pure PQ inflation where the radial mode of the PQ field is responsible for inflation near the pole of its kinetic term, we obtained successful inflationary predictions for a small running quartic coupling for the PQ field and got a sufficiently high reheating temperature determined dominantly by the Higgs-portal couplings to the PQ field. We also showed that there is a consistent parameter space for the post-inflation era where the axion kinetic misalignment is dominant for the axion relic density. We found that the reheating temperature can be constrained by the interplay of the stability conditions for the Higgs fields during inflation and the non-restoration of the PQ symmetry after reheating. Namely, the former requires the Higgs-portal couplings or the reheating temperature to be sufficiently large while the latter favors small Higgs-portal couplings or reheating temperature.

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