

## Temperature-dependence in Seeger's liquid drop energy and the dynamical cluster-decay model

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### Introduction

Seeger's semi-empirical mass formula [1] is revisited for two of its constants (bulk constant  $\alpha(0)$  and neutron-proton asymmetry constant  $a_a$ ) readjusted to obtain the ground-state (g.s.) binding energies of nuclei within a precision of  $<1.5$  MeV and for nuclei up to  $Z=118$ . The aim is *not* to obtain a new parameter set of Seeger's liquid drop energy  $V_{LDM}$ , but to include the temperature T-dependence on experimental binding energies [2]. The T-dependence, in the constants of  $V_{LDM}$ , is introduced as per the work of Davidson *et al.* [3], where the pairing energy  $\delta(T)$  is modified as per new calculations on compound nucleus (CN) decays. The newly fitted constants of  $V_{LDM}$  at  $T=0$  are made available in a (small) tabular form for use of other workers interested in developing computer codes on nuclear dynamics of hot and rotating nuclei. The main purpose of this work is to give a procedure for calculating the fragmentation potentials of nuclei at the incident energies used in heavy ion reactions, i.e., using the T-dependent experimental binding energies, as demonstrated here for the decay of CN  $^{56}\text{Ni}^*$ .

### Methodology

The collective fragmentation potential  $V(\eta, R, T)$  that brings in the structure effects of the CN in to the dynamical cluster-decay model (DCM) of Gupta and collaborators [4, 5], is calculated according to the Strutinsky renormalization procedure ( $B = V_{LDM} + \delta U$ ), using the T-dependent liquid drop model

energy  $V_{LDM}(T)$  of Davidson *et al.* [3] and the empirical shell corrections  $\delta U$  of Myers and Swiatecki [6], for spherical nuclei, also made T-dependent to vanish exponentially with  $T_0=1.5$  MeV. It is given as

$$V(\eta, R, T) = \sum_{i=1}^2 [V_{LDM}(A_i, Z_i, T)] + \sum_{i=1}^2 [\delta U_i] \exp(-T^2/T_0^2) + V_C(Z_i, \beta_{\lambda i}, \theta_i, T) + V_P(A_i, \beta_{\lambda i}, \theta_i, T) + V_l(A_i, \beta_{\lambda i}, \theta_i, T), \quad (1)$$

where nuclear proximity  $V_P$ , Coulomb  $V_C$  and the angular momentum ( $l$ ) dependent  $V_l$  potential are for oriented nuclei and are also T-dependent. The  $V_{LDM}$  here, based on the semi-empirical mass formula of Seeger [1], is

$$V_{LDM}(A, Z, T) = \alpha(T)A + \beta(T)A^{\frac{2}{3}} + \left( \gamma(T) - \frac{\eta(T)}{A^{\frac{1}{3}}} \right) \left( \frac{I^2 + 2|I|}{A} \right) + \left( \frac{Z^2}{r_0(T)A^{\frac{1}{3}}} \right) \times \left( 1 - \frac{0.7636}{Z^{\frac{2}{3}}} - \frac{2.29}{[r_0(T)A^{\frac{1}{3}}]^2} \right) + \delta(T)f(Z, A)/A^{\frac{3}{4}}, \quad (2)$$

with  $I = a_a(Z - N)$ ,  $a_a = 1$  and, respectively, for even-even, even-odd and odd-odd nuclei,  $f(Z, A) = (-1, 0, 1)$ .

Seeger [1] fitted the constants of  $V_{LDM}$  to the ground state ( $T=0$ ) binding energies (BEs) of some 488 nuclei available at that time (in 1961). These constants certainly require modification due to the availability of large amount of data [2, 7] on ground-state BEs. The temperature dependence of the constants of  $V_{LDM}$  in Eq. (2) are given in Fig. 1 of [3].

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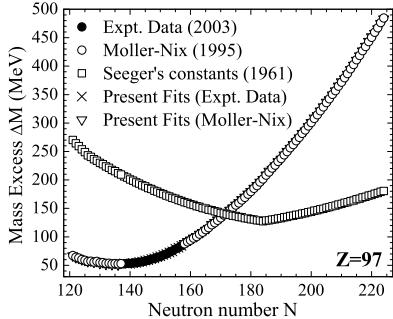


FIG. 1: The mass excess  $\Delta M (= M_A - A = NM_n + ZM_p + B(Z, N) - A)$  (in MeV) as a function of neutron number  $N$  for  $Z=97$  nuclides.

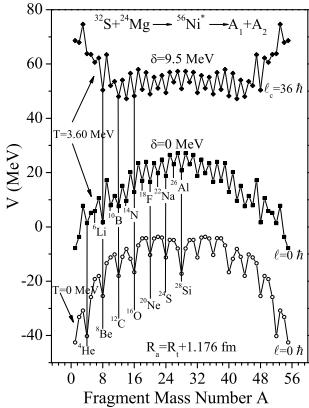


FIG. 2: Fragmentation potential [Eq. (1)] calculated for the decay of  $^{56}\text{Ni}^*$  formed in  $^{32}\text{S} + ^{24}\text{Mg}$  reaction at  $T=3.60$  MeV for  $\ell=0$  and  $36 \ h$ , and also at  $T=0$  for  $\ell=0 \ h$ .

## Calculations and Discussions

In Table 3.1 of Ref. [5], we have presented the fitted constants for the experimental [2] and theoretical [7] BEs. The fitted constant  $\alpha(0)$  is working as an overall scaling factor, and  $a_a$  controls the curvature of the experimental parabola, depicted in Fig. 1 for  $Z=97$  nuclides. Fig. 1 shows the excellent agreement between the present fits (crosses and down open triangle) corresponding to experimental data [2] (solid circles) and theoretical data [7] (open circle), respectively. The fits are obtained between 0-1.5 MeV of the available experimental and theoretical data. The calculated BEs using the 1961 Seeger's

constants are also shown in Fig. 1 (hollow square), which shows the requirement and extent of fitting required.

Next, we consider an application of the re-adjusted  $V_{LDM}$  with an idea to impress upon the need and to propose here at least a partially modified variation of the pairing constant  $\delta$  with temperature  $T$ , as compared to that of Davidson *et al.* [3]. Fig. 2 shows the fragmentation potential  $V(A)$  for the decay of  $^{56}\text{Ni}^*$  (a complete mass spectrum) into light particles (LPs) and intermediate mass fragments (IMFs) at  $T=3.60$  MeV for two different  $\ell$  values ( $\ell=0$  and  $36 \ h$ ), compared with one at  $T=0$  MeV for  $\ell=0 \ h$ . We notice that at  $T=0$  MeV for  $\ell=0 \ h$ , the pairing effects are very strong since all the even-even fragments lie at potential energy minima. On the other hand, if we include the temperature effects as per prescription of Davidson *et al.*, we find that  $\delta=0$  MeV in  $V_{LDM}$  for  $T>2$  MeV, and hence in Fig. 2 for  $T=3.60$  MeV,  $\delta=0$  MeV, the odd-odd fragments, like  $^{10}\text{B}$ ,  $^{14}\text{N}$ ,  $^{18}\text{F}$ , etc., become equally probable as the even-even fragments, since minima are now equally stronger. However, if we empirically choose  $\delta=9.5$  MeV for  $T=3.60$  MeV (for the best fit to IMFs data), the situation becomes again favorable. In other words, Fig. 2 for  $T=3.60$  MeV,  $\delta=9.5$  MeV shows once again that the even-even fragments, like  $^{12}\text{C}$ ,  $^{16}\text{O}$ , etc., are favored over odd-odd  $^{14}\text{N}$ ,  $^{18}\text{F}$ , etc. These calculations lead us to modify the variation of  $\delta$  as function of  $T$ . Apparently, many more calculations are needed.

## References

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