CLASSIFICATION OF THREE- AND FOUR-PION SYSTEMS

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(presented by Z. Koba)

With the help of the solutions of the Schrödinger equation [1)]

$$N(l_1\,,l_2\;;v)\cdot Y_{l_1}^{m_1}(\hat{r}_1)\cdot Y_{l_2}^{m_2}(\hat{r}_2)\cdot\left(\frac{r_1}{R}\right)^{l_1}\left(\frac{r_2}{R}\right)^{l_2}G_v\!\left(l_1+l_2+2\,,l_1+\frac{3}{2}\,;\frac{r_1{}^2}{R^2}\right)\frac{P^{\frac{1}{2}}}{R^2}J_{l_1+l_2+2v+2}(PR)\,,$$

and

$$N(l_1\,,l_2\,,l_3\;;v_1\,,v_2)\cdot Y_{l_1}^{m_1}(\hat{r}_1)\cdot Y_{l_2}^{m_2}(\hat{r}_2)\cdot Y_{l_3}^{m_3}(\hat{r}_3)\left(\frac{r_1}{R}\right)^{l_1}\left(\frac{r_2}{R}\right)^{l_2}\left(\frac{r_3}{R}\right)^{l_3}G_{v_1}\left(l_1+l_2+l_3+2v_2+\frac{7}{2},l_1+l_2+2v_2+3\,;\frac{r_1^2+r_2^2}{R^2}\right)$$

$$\times\left(\frac{r_1^2+r_2^2}{R^2}\right)^{v_2}G_{v_2}\!\left(l_1+l_2+2\,;l_2+\frac{3}{2}\,;\frac{r_2^2}{r_1^2+r_2^2}\right)\frac{P^{1/2}}{R^{7/2}}J_{l_1+l_2+l_3+2v_1+2v_2+7/2}(PR)\,,$$

we can construct a complete normal set of wave functions for free 3- and 4-pion systems, which possess definite values of the total angular momentum and the parity in the c.m.s. and the total isospin, the G-parity, the charge parity (when the system is neutral), and satisfy the Bose statistics. This naturally leads to a scheme of classification of n-pion states according to the increasing order of a parameter $\Lambda\equiv\sum_{i=1}^{n-1}l_i+2\sum_{j=1}^{n-2}v_j$, which expresses the degree of the centrifugal barrier effect, determines the parity, gives the upper limit of the total angular momentum, and thus plays a role similar to the relative orbital angular momentum in the two-body problem.

Lists of these functions have been presented [2, 3)]. Some of their physical implications are as follows. Table I shows the lowest values of Λ compatible with the assignment of spin, parity and symmetry property to a 3-pion system. The symmetry property should be so chosen as to conform to that of the isospin function [4)] which is shown in Table II. Values of an integral,

TABLE I

Lowest values of Λ compatible with assigned spin, parity, and symmetry for 3-pion system

Symmetry \ J^P	0^-	1^+	1^-	2^+	2^-	3^+	3^-
[3]	0	3	8	5	2	3	6
[2, 1]	2	1	4	3	2	3	4
[1, 1, 1]	6	3	2	5	4	3	4

TABLE II

Isospin states of 3-pion system with definite symmetry

Total isospin T	Symmetry	Branching ratio			
		$T_3=1$		$T_3=0$	
		$++-$	$+00$	$+-0$	000
3	[3]	1/5	4/5	3/5	2/5
2	[2, 1]	1/2	1/2	1	0
1	[3]	4/5	1/5	2/5	3/5
1	[2, 1]	1/2	1/2	1	0
0	[1, 1, 1]	—	—	1	0

$$W(P, R_0, \Lambda) \equiv \iint \exp\left(-\frac{R^2}{R_0^2}\right) \cdot |\psi(J, \Lambda; \mathbf{r}_1, \mathbf{r}_2)|^2 d^3\mathbf{r}_1 d^3\mathbf{r}_2 ,$$

$$(R_0 \approx 1/\mu) ,$$

are given in Table III for total energy 780 MeV and 550 MeV; they give a rough measure up to which values of Λ will be essential in these cases. From these tables we see for instance that the assignment $T = 0$, PS and $T = 1$, V (especially with presence of $3\pi^0$) are improbable from the point of view of the barrier effects.

Tables IV-VI give corresponding quantities for a 4-pion system. Table VII shows expected angular correlations for a system of 2 π^+ and $2\pi^-$ in the states with $\Lambda \leqq 2$. For example the state $T = 0$, $J = 2$ can have two kinds of symmetry, which could be distinguished by the charge ratio as well as by the angular correlation.

TABLE III

Barrier effect for 3-pion system. (Normalized at $\Lambda = 0$.)

Λ	0	1	2	3	4	5	6	7	8
W ($\approx$780 MeV)	1	0.6	0.3	0.1	0.04	0.01	$3 \cdot 10^{-3}$	$7 \cdot 10^{-4}$	$2 \cdot 10^{-4}$
W ($\approx$560 MeV)	1	0.2	0.03	$3 \cdot 10^{-3}$	$3 \cdot 10^{-4}$	$3 \cdot 10^{-5}$	$2 \cdot 10^{-6}$	$1 \cdot 10^{-7}$	$1 \cdot 10^{-8}$

TABLE IV

Lowest values of Λ compatible with assigned spin, parity, and symmetry for 4-pion system

Symmetry \ J^P	0^+	0^-	1^+	1^-	2^+	2^-	3^+	3^-
[4]	0	$\geqslant 9$	6	3	2	5	6	3
[2, 2]	2	5	4	3	2	3	4	5
[3, 1]	2	7	4	1	2	3	4	3
[2, 1, 1]	4	5	2	3	4	3	4	3

TABLE V

Isospin states of 4-pion system with definite symmetry

Total isospin T	Symmetry	Branching ratio						
		$T_3 = 2$		$T_3 = 1$		$T_3 = 0$		
		$++\!+-$	$++00$	$++-0$	$+000$	$++--$	$+-00$	0000
4	[4]	1/7	6/7	3/7	4/7	3/35	24/35	8/35
3	[3, 1]	1/3	2/3	11/15	4/15	1/5	4/5	0
2	[4]	6/7	1/7	4/7	3/7	8/21	1/21	12/21
2	[2, 2]	0	1	1	0	2/3	1/3	0
2	[3, 1]	2/3	1/3	2/3	1/3	0	1	0
1	[3, 1]	—	—	3/5	2/5	4/5	1/5	0
1	[2, 1, 1]	—	—	1	0	0	1	0
0	[4]	—	—	—	—	8/15	4/15	3/15
0	[2, 2]	—	—	—	—	1/3	2/3	0

TABLE VI

Barrier effect for 4-pion system. (Normalized at $\Lambda = 0$.)

Λ	0	1	2	3	4
$W(\approx 1\text{Gev})$	1	0.4	0.1	$4\cdot 10^{-2}$	$1\cdot 10^{-2}$

TABLE VII

Angular correlations for $(2\pi^+, 2\pi^-)$-system

Λ	J	Symmetry	T	Angular Correlations (*)	
				$(++)\cdot(--)$	$(+-)\cdot(+-)$
0	0	[4]	4, 2, 0	isotropic	
1	1	[3, 1]	3, 1	isotropic	$1+\frac{8}{3\pi}\cos\Omega$
2	2	[4]	4, 2, 0	$\frac{2}{5}+\frac{3}{5}\left\{\cos^2\theta_1+\cos^2\theta_2+\cos^2\Omega\right\}$	
		[3, 1]	3, 1	$\frac{13}{10}-\frac{9}{10}\cos^2\Omega$	$\frac{9}{10}+\frac{3}{20}(\cos^2\theta_1+\cos^2\theta_2)$ $-\frac{4}{5\pi}(\cos\theta_1\cos\theta_2+3\cos\Omega)$
		[2, 2]	2, 0	$\frac{13}{10}+\frac{3}{10}\cos^2\Omega$ $-\frac{3}{5}(\cos^2\theta_1+\cos^2\theta_2)$	$1+\frac{3}{10}\cos^2\Omega$ $-\frac{3}{20}(\cos^2\theta_1+\cos^2\theta_2)$ $+\frac{44}{15\pi}\cos\Omega-\frac{12}{5\pi}\cos\theta_1\cos\theta_2$
	0	[3, 1]	3, 1	isotropic	$\frac{3}{2}(\cos^2\theta_1+\cos^2\theta_2)$ $-\frac{8}{\pi}\cos\theta_1\cos\theta_2$
		[2, 2]	2, 0	isotropic	$\frac{1}{4}-\frac{4}{3\pi}\cos\Omega+\frac{9}{4}\cos^2\Omega$

(*) Denote the coordinates of four pions by $\mathbf{x}_A$, $\mathbf{x}_B$, $\mathbf{x}_C$, and $\mathbf{x}_D$, respectively, and introduce the following relative coordinates:

$$\mathbf{r}_1 = \frac{1}{\sqrt{2}}(\mathbf{x}_A-\mathbf{x}_B), \qquad \mathbf{r}_2 = \frac{1}{\sqrt{2}}(\mathbf{x}_C-\mathbf{x}_D), \qquad \mathbf{r}_3 = \frac{1}{2}(\mathbf{x}_A+\mathbf{x}_B-\mathbf{x}_C-\mathbf{x}_D).$$

Then θ_1, θ_2, and Ω represent the angles between $\mathbf{r}_1$ and $\mathbf{r}_3$, $\mathbf{r}_2$ and $\mathbf{r}_3$, $\mathbf{r}_1$ and $\mathbf{r}_2$, respectively.
$(++)$ $(--)$ means that A, B are positive, C, D are negative pions.
$(+-)$ $(+-)$ means that A, C are positive, B, D are negative pions.

LIST OF REFERENCES

1. M. Morse and H. Feshbach, Methods of Theoretical Physics, vol. 2, p. 1731, New York 1953
2. Z. Koba, Physics Letters *1*, 34 (1962); Acta Phys. Pol. (in press).
3. M. Grynberg and Z. Koba, Physics Letters, *1*, No. 4 (1962).
4. A. Pais, Annals of Physics, *9*, 548 (1960).

DISCUSSION

PEIERLS: In a given physical situation, is it clear that you will not have to use a superposition of several of your wave functions?

KOBA: In general, T does not uniquely specify the symmetry and one has to take a linear combination of functions corresponding to different symmetries and the coefficients in the superposition can change through reaction. When the interaction is factorizable into isospin and configurational spaces, each of which is invariant under permutation, those coefficients remain constant and, in particular cases, one can restrict oneself to a single function.