Proto-Neutron Star Matter

A Alp[1], D Farrell[1], F Weber[1,2], G Malfatti[3], M G Orsaria[3,4], I F Ranea-Sandoval[3,4]

[1] Department of Physics, San Diego State University, 5500 Campanile Drive, San Diego, CA 92182, USA
[2] Center for Astrophysics and Space Sciences, University of California at San Diego, La Jolla, CA 92093, USA
[3] Grupo de Gravitación, Astrofísica y Cosmología, Facultad de Ciencias Astronómicas y Geofísicas, Universidad Nacional de La Plata, Paseo del Bosque S/N, La Plata (1900), Argentina
[4] CONICET, Godoy Cruz 2290, Buenos Aires (1425), Argentina

E-mail: [1]aalp@sdsu.edu, dfarrell@sdsu.edu, [2]fweber@ucsd.edu, [3]gmalfatti@fcaglp.unlp.edu.ar, [4]morsaria@fcaglp.unlp.edu.ar, iranea@fcaglp.unlp.edu.ar

Abstract. In this paper, we explore the properties of proto-neutron star matter. The relativistic finite-temperature Green function formalism is used to derive the equations which determine the properties of such matter. The calculations are performed for the relativistic non-linear mean-filed theory, where different combinations of lepton number and entropy have been investigated. All particles of the baryon octet as well as all electrically charged states of the Δ isobar have been included in the calculations. The presence of all these particles is shown to be extremely temperature (entropy) dependent, which should have important consequences for the evolution of proto-neutron stars to neutron stars as well as the behavior of neutron stars in compact star mergers.

1. Introduction

The proto-neutron star matter studied in this paper exists deep in the cores of proto-neutron stars, which are the remnants of collapsed stars that form in the aftermath of supernova explosions [1, 2, 3, 4]. The gravitational collapse of the core goes through several stages (see figure 1) before stellar equilibrium is reached and a cold neutron star is formed [5, 6]. In the very early stages, a proto-neutron star experiences a deleptonization stage where hot and lepton-rich matter becomes lepton-poor over the course of about one minute. During this time, the entropy (s) per baryon and lepton fraction (Y_L) of the matter change quickly, from around $s = 1$ and $Y_L = 0.4$, to $s = 2$ and $Y_L = 0.2$, to $s = 2$ and $Y_{\nu_e} = 0.4$ [5, 7, 8, 9]. As neutrinos and photons continue to diffuse out over the next several minutes and temperatures cool to less than 1 MeV, a hot proto-neutron star becomes a cold neutron star. The equations of state describing these stages and the associated baryon-lepton compositions will be studied in this paper.

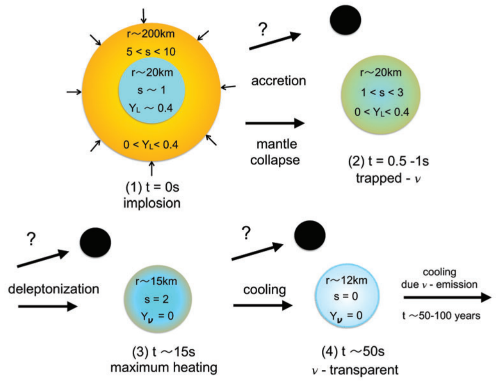

Figure 1. Schematic illustration of different temporal stages in the evolution of proto-neutron stars to neutron stars [5, 7].

2. The Non-Linear Nuclear Lagrangian

The nuclear Lagrangian of the theory is given by [7, 9, 10, 11, 12],

$$
\begin{aligned}
\mathcal{L} = \quad & \sum_B \bar{\psi}_B[\gamma_\mu(i\partial^\mu - g_{\omega B}\omega^\mu - g_{\rho B}\,\boldsymbol{\tau}\cdot\boldsymbol{\rho}^\mu) - (m_B - g_{\sigma B}\sigma)]\psi_B \\
& + \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2) - \frac{1}{3}\tilde{b}_\sigma m_N(g_{\sigma N}\sigma)^3 - \frac{1}{4}\tilde{c}_\sigma(g_{\sigma N}\sigma)^4 \\
& - \frac{1}{4}\,\omega_{\mu\nu}\,\omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu + \frac{1}{2}m_\rho^2\,\boldsymbol{\rho}_\mu\cdot\boldsymbol{\rho}^\mu - \frac{1}{4}\,\boldsymbol{\rho}_{\mu\nu}\cdot\boldsymbol{\rho}^{\mu\nu}\,,
\end{aligned} \tag{1}
$$

where m_B and m_N stand for the baryon and nucleon masses, respectively. The quantity ψ_B stands for the particle fields of the baryon octet (i.e., $\Sigma^\pm, \Sigma^0, \Lambda, \Xi^-, \Xi^-$) and the electrically charged states of the Δ isobar. The interactions among these particles are described by the exchange of σ, ω, and ρ mesons. The quantities m_σ, m_ω, m_ρ in equation (1) denote the masses of mesons and $g_{\sigma B}$, $g_{\omega B}$, and $g_{\rho B}$ are the meson-baryon coupling constants. In standard relativistic mean-field theory, the meson-baryon coupling constants are independent of the baryon density. This is different for the density-dependent relativistic mean-field theory, where the coupling constants depend on density,

$$g_{iB}(n) = g_{iB}(n_0)f_i(x)\,. \tag{2}$$

Here $i \in (\sigma, \omega, \rho)$ and $f_i(x)$ accounts for the functional form of the density dependence [12]. The density dependent coupling constants are given by [13, 14]

$$g_{iB}(n) = g_{iB}(n_0)\,a_i\left[\frac{1 + b_i(x + d_i)^2}{1 + c_i(x + d_i)^2}\right], \tag{3}$$

for $i = (\sigma, \omega)$. The quantity x is given by $x = n/n_0$, where n_0 denotes the nuclear saturation density. For the ρ meson, the expression of the meson-baryon coupling constant reads

$$g_{\rho B}(n) = g_{\rho B}(n_0)\,\exp[-a_\rho(x - 1)]\,, \tag{4}$$

with only one parameter a_ρ. The quantities $\omega^{\mu\nu}$ and $\boldsymbol{\rho}^{\mu\nu}$ in equation (1) denote field strength tensors for the vector mesons, which are given by [5, 12],

$$\omega_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu\,, \quad \boldsymbol{\rho}_{\mu\nu} = \partial_\mu\boldsymbol{\rho}_\nu - \partial_\nu\boldsymbol{\rho}_\mu\,, \tag{5}$$

where $\boldsymbol{\rho}_\mu$ denotes the ρ-meson field. Additionally, $\tilde{b}_\sigma$ and $\tilde{c}_\sigma$ represent coupling parameters associated with non-linear self-interactions among σ-meson fields. Additional scalar self-interactions are included by

$$U(\sigma) = \frac{1}{3}\tilde{b}_\sigma m_N(g_{\sigma N}\sigma)^3 + \frac{1}{4}\tilde{c}_\sigma(g_{\sigma N}\sigma)^4\,. \tag{6}$$

In this paper we use the SWL nuclear model to study proto-neutron star matter [7, 9, 12], whose parametrization reproduces the properties of symmetric as well as asymmetric nuclear matter extremely well (see section 5). The relativistic mean-field equations of motion are derived by evaluating the Euler-Lagrange equation for the fields $\mathcal{X} = \psi_B$, ψ_L, σ, ω^μ, and ρ^μ. One obtains for the baryons [7, 9, 10, 12],

$$(i\gamma^\mu \partial_\mu - m_B)\psi_B = \left(g_{\omega B}\gamma^\mu \omega_\mu + \frac{1}{2} g_{\rho B}\gamma^\mu \boldsymbol{\tau}\cdot\boldsymbol{\rho}_\mu - g_{\sigma B}\sigma \right) \psi_B \,, \tag{7}$$

and for the mesons

$$(\partial^\mu \partial_\mu + m_\sigma^2)\sigma = \sum_B g_{\sigma B}\bar{\psi}_B \psi_B - b_\sigma\, m_N g_{\sigma N}(g_{\sigma N}\sigma)^2 - c_\sigma\, g_{\sigma N}(g_{\sigma N}\sigma)^3 \,, \tag{8}$$

$$\partial^\mu \omega_{\mu\nu} + m_\omega^2 \omega_\nu = \sum_B g_{\omega B}\bar{\psi}_B \gamma_\nu \psi_B \,, \tag{9}$$

$$\partial^\mu \boldsymbol{\rho}_{\mu\nu} + m_\rho^2 \boldsymbol{\rho}_\nu = \sum_B g_{\rho B}\bar{\psi}_B \boldsymbol{\tau}\gamma_\nu \psi_B \,. \tag{10}$$

The relativistic mean-field limit of equations (8) through (10) is given by [7, 9, 10]

$$m_\sigma^2 \bar{\sigma} = \sum_B g_{\sigma B} n_B^s - \tilde{b}_\sigma\, m_N g_{\sigma N}(g_{\sigma N}\bar{\sigma})^2 - \tilde{c}_\sigma\, g_{\sigma N}(g_{\sigma N}\bar{\sigma})^3 \,, \tag{11}$$

$$m_\omega^2 \bar{\omega} = \sum_B g_{\omega B} n_B \,, \tag{12}$$

$$m_\rho^2 \bar{\rho} = \sum_B g_{\rho B} I_{3B} n_B \,, \tag{13}$$

where $\bar{\sigma} \equiv \langle \sigma \rangle$, $\bar{\omega} \equiv \langle \omega \rangle$, and $\bar{\rho} \equiv \langle \rho \rangle$ denote the meson-mean fields. The quantity I_{3B} denotes the 3-component of isospin. The scalar and particle number densities for each baryon B are denoted by n_B^s and n_B, and are given by [10, 12]

$$n_B^s = \langle\, \bar{\psi}_B \psi_B \,\rangle \,, \tag{14}$$

$$n_B = \langle\, \psi_B^\dagger \psi_B \,\rangle \,. \tag{15}$$

3. Baryonic Field Theory at Finite Density and Temperature

To solve the field equations at finite temperatures and densities we use the finite-temperature Greens function formalism. It is based on the spectral function representation of the two-point Green function [10, 15],

$$g^B(p^0, \boldsymbol{p}) = \int d\omega \frac{a^B(\omega, \boldsymbol{p})}{\omega - (p^0 - \mu_B)(1 + i\eta)} - 2i\pi\, \mathrm{sign}(p^0 - \mu_B)\, \frac{1}{\exp(|p^0 - \mu_B|/T) + 1}\, a^B(p^0 - \mu_B, \boldsymbol{p}) \,, \tag{16}$$

where μ^B denotes the chemical potential of a baryon and a^B stands for the spectral function of that particle. The effective baryon mass m_B^* and the single-baryon energy E_B^* are given by [7, 10],

$$m_B^* = m_B - g_{\sigma B}\bar{\sigma} \,, \qquad E_B^*(\boldsymbol{p}) = \sqrt{\boldsymbol{p}^2 + {m_B^*}^2} \,. \tag{17}$$

The expression for the baryon number density (15) in terms of the two-point Green function is given by [7, 10]

$$n_B = i\,\mathrm{Tr}\,\gamma^0 \int d^3\boldsymbol{x} \left(g^B(x, x^+) + g^B(x, x^-) \right) . \tag{18}$$

Transformation of equation (18) to momentum space gives

$$n_B = i\,\mathrm{Tr}\,\gamma^0 \int \frac{d^4p}{(2\pi)^4} \left(\mathrm{e}^{i\eta p^0} + \mathrm{e}^{-i\eta p^0} \right) g^B(p) . \tag{19}$$

We take note that the integration over p^0 can be carried out analytically via contour integration [10], which leads to

$$\int \frac{d^4p}{(2\pi)^4}\, \mathrm{e}^{i\eta p^0} g^B(p^0, \boldsymbol{p}) = -i \int \frac{d^3\boldsymbol{p}}{(2\pi)^3}\, a^B(\boldsymbol{p}) f_{B^-}(\boldsymbol{p}) , \tag{20}$$

$$\int \frac{d^4p}{(2\pi)^4}\, \mathrm{e}^{-i\eta p^0} g^B(p^0, \boldsymbol{p}) = i \int \frac{d^3\boldsymbol{p}}{(2\pi)^3}\, \bar{a}^B(\boldsymbol{p}) f_{B^+}(\boldsymbol{p}) . \tag{21}$$

The number density (19) can then be written as

$$n_B = \gamma_B \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} \left(f_{B^-}(\boldsymbol{p}) - f_{B^+}(\boldsymbol{p}) \right) , \tag{22}$$

where $\gamma_B \equiv (2J_B + 1)$ accounts for the spin-degeneracy of a given baryon. The Fermi-Dirac distribution functions $f_{B^\pm}$ in equations (20) to (22) are given by

$$f_{B^-}(\boldsymbol{p}) = \frac{1}{\mathrm{e}^{(E^*_B(\boldsymbol{p}) - \mu^*_B)/T} + 1} , \qquad f_{B^+}(\boldsymbol{p}) = \frac{1}{\mathrm{e}^{(-\bar{E}^*_B(\boldsymbol{p}) + \mu^*_B)/T} + 1} , \tag{23}$$

where μ^*_B denotes the effective baryon chemical potential [7]. The expression for the scalar density (14) in terms of the two-point Green function is given by [7, 10]

$$n^s_B = i\,\mathrm{Tr} \int d^3\boldsymbol{x} \left(g^B(x, x^+) + g^B(x, x^-) \right) . \tag{24}$$

Transformation of equation (24) to momentum space leads to

$$n^s_B = i\,\mathrm{Tr} \int \frac{d^4p}{(2\pi)^4} \left(\mathrm{e}^{i\eta p^0} + \mathrm{e}^{-i\eta p^0} \right) g^B(p) . \tag{25}$$

By making use of equations (20) and (21), the integration over p^0 can be carried out analytically, which leads to the final result for the scalar number density given by

$$n^s_B = \gamma_B \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} \frac{m^*_B}{E^*_B(\boldsymbol{p})} \left(f_{B^-}(\boldsymbol{p}) + f_{B^+}(\boldsymbol{p}) \right) . \tag{26}$$

The relations for the energy density ϵ and pressure P of the system follow from the energy-momentum tensor as $\epsilon = \langle T^{00} \rangle$ and $P = \frac{1}{3} \sum_i \langle T^{ii} \rangle$. For ϵ one obtains [7, 10]

$$\epsilon = i \sum_B \mathrm{Tr} \int \frac{d^4p}{(2\pi)^4} \left(\mathrm{e}^{i\eta p^0} + \mathrm{e}^{-i\eta p^0} \right) \left(p^0 \gamma^0 - \frac{1}{2} \left(g_{\sigma B} \bar{\sigma} + \gamma^0 \left(g_{\omega B} \bar{\omega} + g_{\rho B} I_{3B} \bar{\rho} \right) \right) \right) g^B(p)$$
$$- \frac{1}{6} \tilde{b}_\sigma m_N (g_{\sigma N} \sigma)^3 - \frac{1}{4} \tilde{c}_\sigma (g_{\sigma N} \sigma)^4 , \tag{27}$$

which, after some algebra, can be written as [7, 10]

$$\begin{aligned}\epsilon \;=\;& \sum_B \gamma_B \int \frac{d^3\boldsymbol{p}}{(2\pi)^3}\, E^*_B(\boldsymbol{p})\,(f_{B^-}(\boldsymbol{p}) + f_{B^+}(\boldsymbol{p})) + \frac{1}{2}m_\sigma^2\bar{\sigma}^2 + \frac{1}{2}m_\omega^2\bar{\omega}^2 \\ & + \frac{1}{2}m_\rho^2\bar{\rho}^2 + \frac{1}{3}\tilde{b}_\sigma m_N (g_{\sigma N}\sigma)^3 + \frac{1}{4}\tilde{c}_\sigma (g_{\sigma N}\sigma)^4 \,. \end{aligned} \tag{28}$$

The expression for the pressure of hot nuclear matter is given by [7, 10]

$$\begin{aligned} P \;=\;& i\sum_B \mathrm{Tr}\int \frac{d^4p}{(2\pi)^4}\left(\mathrm{e}^{i\eta p^0} + \mathrm{e}^{-i\eta p^0}\right)\left(\frac{1}{3}\boldsymbol{\gamma}\cdot\hat{\boldsymbol{p}} + \frac{1}{2}\left(-g_{\sigma B}\bar{\sigma} + \gamma^0\left(g_{\omega B}\bar{\omega} + g_{\rho B} I_{3B}\bar{\rho}\right)\right)\right) g^B(p) \\ & + \frac{1}{6}\tilde{b}_\sigma m_N (g_{\sigma N}\sigma)^3 + \frac{1}{4}\tilde{c}_\sigma (g_{\sigma N}\sigma)^4 \,. \end{aligned} \tag{29}$$

This expression takes the following form after contour integration over p^0 and some algebra [7, 10]:

$$\begin{aligned} P \;=\;& \frac{1}{3}\sum_B \gamma_B \int \frac{d^3\boldsymbol{p}}{(2\pi)^3}\,\frac{\boldsymbol{p}^2}{E^*_B(\boldsymbol{p})}\,(f_{B^-}(\boldsymbol{p}) + f_{B^+}(\boldsymbol{p})) - \frac{1}{2}m_\sigma^2\bar{\sigma}^2 + \frac{1}{2}m_\omega^2\bar{\omega}^2 \\ & + \frac{1}{2}m_\rho^2\bar{\rho}^2 - \frac{1}{3}\tilde{b}_\sigma m_N (g_{\sigma N}\sigma)^3 - \frac{1}{4}\tilde{c}_\sigma (g_{\sigma N}\sigma)^4 + \frac{\partial g_{\rho B}(n)}{\partial n} I_{3B} n_B \bar{\rho} \,. \end{aligned} \tag{30}$$

4. Chemical Equilibrium and Charge Neutrality

When determining the equation of state of proto-neutron star matter, three constraints must be taken into account. These are electric charge neutrality, baryon number conservation and chemical equilibrium. The constraint of electric charge neutrality is given by [5, 7, 9]

$$\sum_L q_L Y_L + \sum_B q_B Y_B = 0\,, \tag{31}$$

where q_L and q_B denote the lepton and baryon electric charges in units of the elementary charge, and Y_L and Y_B represent the lepton and baryon fractions, respectively. Baryon number conservation leads to

$$\sum_B n_B - n = 0\,. \tag{32}$$

Lastly, the chemical equilibrium is fulfilled if

$$\mu_B = \mu_n + q_B(\mu_e - \mu_{\nu_e})\,, \tag{33}$$

where μ_B is the baryon chemical potential, and μ_n, μ_e and μ_{ν_e} are the neutron, electron and neutrino chemical potentials, respectively. Finally we introduce the relative lepton numbers of electrons and neutrinos at a given density

$$Y_e = \frac{n_e + n_{\nu_e}}{n}\,, \qquad Y_\mu = \frac{n_\mu + n_{\nu_\mu}}{n} = 0\,, \tag{34}$$

where n_e, n_{ν_e}, n_μ and n_{ν_μ} represent the number densities of electrons, electron neutrinos, muons and muon neutrinos.

Table 1. Parameters of the SWL [16, 17] parametrization.

Parameters	Units	SWL
m_σ	GeV	0.550
m_ω	GeV	0.783
m_ρ	GeV	0.763
$g_{\sigma N}$	–	9.7744
$g_{\omega N}$	–	10.746
$g_{\rho N}$	–	7.8764
$\tilde{b}_\sigma$	–	0.003798
$\tilde{c}_\sigma$	–	−0.003197
a_ρ	–	0.3796

5. Model Parameters

The results of this paper are computed for the nuclear parametrization named SWL [16]. The parameter values of these sets are shown in table 1 and the corresponding saturation properties of symmetric nuclear matter are compiled in table 2 [9]. These are the nuclear saturation density n_0, energy per nucleon E_0, nuclear incompressibility K_0, effective nucleon mass m_N^*/m_N, asymmetry energy J, asymmetry energy slope L_0, and the value of the nucleon potential U_N at n_0. As already mentioned, the baryons considered in our calculations include all states of the spin-$\frac{1}{2}$ baryon octet comprised of the nucleons (n, p) and hyperons $(\Lambda, \Sigma^+, \Sigma^0, \Sigma^-, \Xi^0, \Xi^-)$. In addition, all states of the spin-$\frac{3}{2}$ delta isobar $\Delta(1232)$ $(\Delta^{++}, \Delta^+, \Delta^0, \Delta^-)$ are taken into account as well. A detailed discussion of the coupling constants can be found in [9, 12, 16, 18].

6. Results

Figure 2 shows the pressure as a function of the energy density of neutron star matter for temperatures from 1 MeV to 50 MeV. The temperature dependence is relatively weak since the energy density and pressure both increase at about the same rate with temperature. The situation is different for matter with constant entropy, which is shown in figure 3. The equations of state shown in this figure describe matter as it exists in the early phases (milliseconds to seconds) in the cores of proto-neutron stars. As can be seen, now temperature (i.e., entropy) has a significant effect on both the pressure and energy density, as well as on the particle composition

Table 2. Properties of nuclear matter for the SWL [16, 17] parametrization.

Saturation property	Units	SWL
n_0	fm^{-3}	0.150
E_0	MeV	−16.0
K_0	MeV	260.0
m_N^*/m_N	–	0.70
J	MeV	31.0
L_0	MeV	55.0
U_N	MeV	−64.6

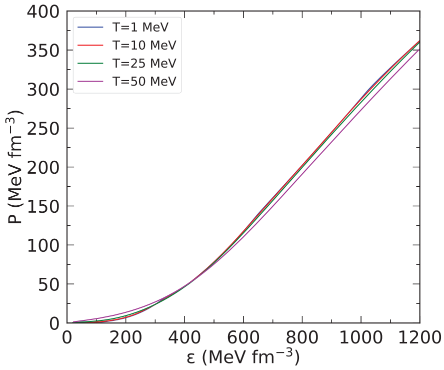

Figure 2. Equation of state at different temperatures, T.

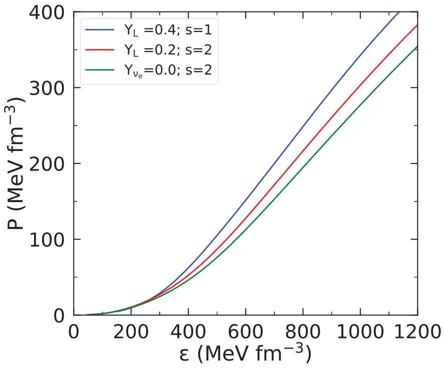

Figure 3. Equation of state at different entropies, s.

of matter as shown in figures 4 through 6. The threshold densities for the production of new particles in matter are very temperature sensitive, which is especially true for the $\Delta(1232)$) isobar states. The results of this work are not only of interest for proto-neutron stars, but also find their

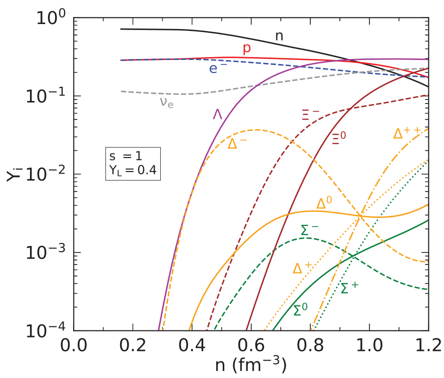

Figure 4. Particle population in matter with $Y_L = 0.4$ and $s = 1$.

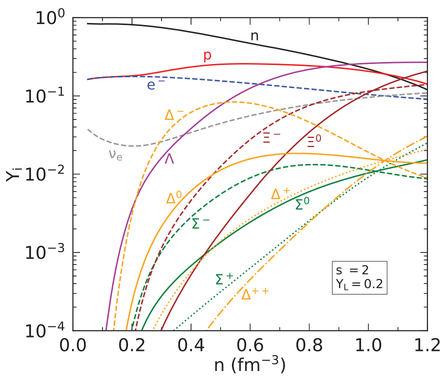

Figure 5. Particle population in matter with $Y_L = 0.2$ and $s = 2$.

application in numerical simulations of colliding neutron stars in binary systems (neutron star mergers). After contact, large shocks develop inside of such neutron stars which considerably increase their internal energy. Numerical simulations have shown that in such collisions the densities reached in matter are several times higher than the nuclear saturation density and that the temperature is on the order of 50 MeV or even higher [19, 20, 21]. The determination of a comprehensive class of modern, self-consistent models for the equation of state of hot and dense nuclear matter has therefore become a focal point of contemporary research on ultra-dense matter.

7. Summary and Conclusions

In this work, we present a consistent theoretical description of hot and dense relativistic nuclear matter based on the Green function formalism. Of particular importance is the spectral representation of the two-point Green function at finite temperature. As our studies show, the particle composition of hot and dense nuclear matter is extremely temperature dependent

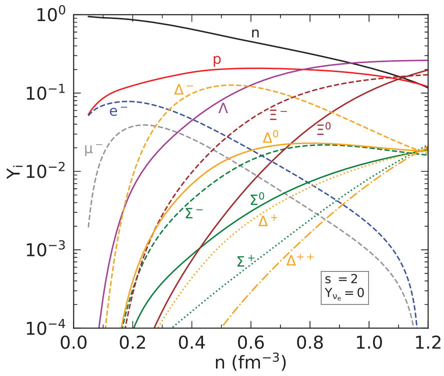

Figure 6. Particle population in matter with $Y_L = 0$ and $s = 2$.

and fundamentally different from the composition of cold nuclear matter. This has drastic implications for the equation of state as well as transport properties of proto-neutron star matter.

Acknowledgments
This research was supported by the National Science Foundation (USA) under Grant No. PHY-2012152. MO and IFR-S thank CONICET, UNLP, and MinCyT (Argentina) for financial support under grants PIP-0714, G157, G007 and PICT 2019-3662. The results of this paper contribute to the research projects of the NP3M collaboration on the Nuclear Physics of Multi-Messenger Mergers.

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