# On the super quantum information

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**Abstract**. A superqubit is a three dimensional super vector belonging to uOSP(1/2) super group. And considered as a super symmetric generalized qubit. To see how superqubits properties can be exploited in the field of the quantum computing, we propose some algorithms in order to perform the super teleportation. implementation and technologies. Using superqubit circuits will be an open field of research.

## 1. Introduction

The efficiency of quantum computation and information is coming from the weirdness of quantum mechanics, superposition, interference tunneling , duality and entanglement, and it will be more interesting when we add supersymmetry to the game and invent a new field called super quantum computing becomes more attractive [1]- [7], In the framework of the classical computation, the bit is the fundamental career of information. In the quantum computation, the carrier is the qubit. Similarly, a supersymmetric qubit can be considered and involved in the world of computation and information processing which is our central aim in this work. In fact, we introduce a super quantum computing using a superqubit leading to a new breath in the world of quantum programming by putting the fundamental piece of a super quantum algorithm. The latter can be realized by sending superqubits through a series of super gates which are investigated in this work (processing over superqubits using a series of super gates). Many models of super-algorithms are proposed such as the super-teleportation which means transmitting a superqubit from one location to another by the help of sheared a super entangled state and classical communication. Super Deutch's algorithm is also discussed and proposed for the super quantum Computation where we can mixte the features of interference and parallelism ALU (Arithmetic and Logic Unit).

## 2. Superqubits

*1.1. Single superqubits and properties*

Superqubits are the fundamental building block of the super quantum computation and information processing. It emerges from the minimal supersymmetric generalization of the conventional qubit and has some unusual properties such as crossing the Tserilson's bound in some versions of the Bell inequality (CHSH), The super qubit is a three-dimensional super vector belonging to $[uOSP(1/2)]$ This super vector contains two commuting components considered as the bosonic part of the superqubit, and one fermionic as the anti-commuting part. It can perform a parallel computation and speeds up the information processing On the other hand, the superqubit shows more non locality than any known quantum or classical system without violation of non signaling theory. This property can give it a special role in the quantum computation. The communication superqubit can be written in the following form:

$$\left|\psi\right\rangle = a_1\left|0\right\rangle + a_2\left|1\right\rangle + a_\bullet\left|\bullet\right\rangle \tag{1}$$

or

$$\left|\psi\right\rangle = (1+\frac{p^2}{2}\theta\theta^{\neq})(\alpha\left|0\right\rangle + \beta\left|1\right\rangle) + (\alpha p\theta - \beta p\theta)\left|\bullet\right\rangle \tag{2}$$

where $a_1$ , $a_2$ and $a_\bullet$ are even Grassmann numbers and $\theta$, $\theta^{\neq}$ are Grassmann variables

$$\left\langle\psi\right| = (1+\frac{p^2}{2}\theta\theta^{\neq})(\overline{\alpha}\left\langle 0\right| + \overline{\beta}\left\langle 1\right|) - p(\alpha\theta - \beta\theta^{\neq})\left\langle\bullet\right| \tag{3}$$

The computational basis for a superqubit is:

$$\left|0\right\rangle = \begin{pmatrix}1\\0\\0\end{pmatrix},\quad \left|1\right\rangle = \begin{pmatrix}0\\1\\0\end{pmatrix},\quad \left|\bullet\right\rangle = \begin{pmatrix}0\\0\\1\end{pmatrix} \tag{4}$$

leading to

$$\left\langle\psi\right\|\left.\psi\right\rangle = \delta^{A_1{}'A_1} a_{A_1} a_{A_2} - a^{\neq}{}_{A\bullet} \tag{5}$$

A superqubit transforms as a triplet under $\left[uOSP(1/2)\right]$ which is generated by the following generators:

i) *Even generators*:

$$A_1 = \frac{i}{2}\begin{pmatrix}0&1&0\\1&0&0\\0&0&0\end{pmatrix},\quad A_2 = \frac{1}{2}\begin{pmatrix}0&1&0\\-1&0&0\\0&0&0\end{pmatrix},\quad A_3 = \frac{i}{2}\begin{pmatrix}1&0&0\\0&-1&0\\0&0&0\end{pmatrix}, \tag{6}$$

ii) Odd Generators:

$$Q_1 = \frac{1}{2}\begin{pmatrix}0&0&0\\0&0&-1\\-1&0&0\end{pmatrix},\quad Q_2 = \frac{1}{2}\begin{pmatrix}0&0&-1\\0&0&0\\0&1&0\end{pmatrix} \tag{7}$$

It is important to stress out that all elements of the $\left[uOSP(1/2)\right]$ super group can be written in the following form:

$$\left[uOSP(1/2,\Lambda c)\right] = \left\{z \in ExpS \,/\, s \in uOSP(1/2)\right\}$$

$$S = \zeta_1 A_1 + \zeta_2 A_2 + \zeta_3 A_3 + \varepsilon Q_1 + \varepsilon^{\neq} Q_2 \tag{8}$$

$$S^{\neq} = -S$$
$$A_i^{\neq} = -A_i$$
’(10)

$$(\varepsilon_i^{\neq})^{\neq} = -\varepsilon_i$$

Here, the grade of the generators is:

$$|Q_i| = 1 \quad (11)$$
$$|A_i| = 0$$

*1.2. Basic Mathematics for Superqubit.*

The super space formalism may fit the description of the superqubit .In fact, the super Hilbert space is a super vector space equipped with the double dagger ‡

$$\ddagger : \mathcal{H} \rightarrow \mathcal{H}^{\ddagger}$$
$$(|\psi\rangle \rightarrow |\psi\rangle)^{\ddagger} = \langle\psi| \quad (12)$$

The inner product satisfies the following axioms

1-linearity

$$(|\psi\rangle + |\varphi\rangle)^{\ddagger} = (\langle\psi| + \langle\varphi|) \quad (13)$$

2- For any pure odd or even α and $|\psi\rangle$ where ≠ is the super start and

$$(|\psi\rangle\alpha)^{\ddagger} = (-)^{\alpha\psi}\,\alpha^{\neq}\langle\psi|$$
$$(\alpha\langle\psi|)^{\ddagger} = (-)^{\psi+\alpha\psi}\,|\psi\rangle\alpha^{\neq} \quad (14)$$
$$(|\psi\rangle)^{\ddagger\ddagger} = (-)^{\psi}\,|\psi\rangle$$

The inner product satisfies

$$(\langle\psi\|\varphi\rangle)^{\neq} = (-)^{\psi+\varphi\psi}\,\langle\varphi\|\psi\rangle$$

$$(\langle\psi\|\varphi\rangle)^{\neq\neq} = (-)^{\psi+\varphi}\,\langle\varphi\|\psi\rangle\,, \quad (15)$$

The linear super operator is required to satisfy

$$A(|\psi\rangle + |\varphi\rangle) = (A|\psi\rangle + A|\varphi\rangle)$$
$$A(|\psi\rangle\alpha) = (A|\psi\rangle)\alpha \quad (16)$$

and

$$(A+B)|\psi\rangle = (A|\psi\rangle + B|\psi\rangle)$$
$$(AB)|\psi\rangle\alpha = A(B|\psi\rangle) \quad (17)$$

*1.3. Multi- superqubit*

*2.1.1. Tensor Product*

Since a quantum computation cannot be performed using a single superqubit, a composite system can be obtained from the tensor product of two or more superqubits and a super symmetric generalization of n-qubits have been done by extending LOCC group from the group $[SU(2)]^n$ to the super group $[uOSP(1/2)]^n$ and SLOCC group $[SL(2,C)]^n$ to the super group $[uOSP(1/2)]^n$ [1]

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$$|\psi\rangle = a_{X_1X_2..........\,.....X_n}|X_1X_2..........\,.....X_n\rangle \tag{18}$$

For the composite system we can describe the two superqubits as follow:

$$\begin{gathered}|\psi\rangle = a_{AB}|AB\rangle + a_{A\bullet}|A\bullet\rangle + a_{\bullet B}|\bullet B\rangle + a_{\bullet\bullet}|\bullet\bullet\rangle \\ \langle\psi\|\phi\rangle^{\neq} = (-)^{\psi+\psi\phi}\langle\phi\|\psi\rangle \\ \langle\psi\|\phi\rangle^{\neq\neq} = (-)^{\psi+\phi}\langle\phi\|\psi\rangle\end{gathered} \tag{19}$$

and

$$A(|\psi\rangle\alpha) = (A|\psi\rangle)\alpha$$

$a_{AB}$ and $a_{\bullet\bullet}$ (respectively $a_{\bullet B}$ and $a_{A\bullet}$) are commuting (respectively anti-commuting)

$$|\psi_a\rangle\otimes|\psi_b\rangle = \begin{pmatrix} a_1b_1 \\ a_1b_2 \\ a_2b_1 \\ a_2b_2 \\ a_\bullet b_\bullet \\ a_1b_\bullet \\ a_2b_\bullet \\ a_\bullet b_1 \\ a_\bullet b_2 \end{pmatrix}, \quad |\psi_a\rangle = \begin{pmatrix} a_1 \\ a_2 \\ a_\bullet \end{pmatrix}, \quad |\psi_b\rangle = \begin{pmatrix} b_1 \\ b_2 \\ b_\bullet \end{pmatrix}, \tag{20}$$

The square of the norm is given by

$$\begin{aligned}\langle\psi\|\psi\rangle &= (-)^{X_1+X_2}\delta^{X_1X_2}\delta^{Y_1Y_2}a^{\neq}{}_{X_1Y_1}a_{X_2Y_2} \\ &= \delta^{A_1A_2}\delta^{B_1B_2}a^{\neq}_{A_1B_2}a_{A_2B_2} - \delta^{A_1A_2}a^{\neq}_{A_1\bullet}a_{A_1\bullet} \\ &\quad - \delta^{B_1B_2}a^{\neq}_{\bullet B_1}a_{\bullet B_1} + a^{\neq}_{\bullet\bullet}\,a_{\bullet\bullet}\end{aligned} \tag{21}$$

For three superqubits the corresponding state can be written as:

$$\begin{aligned}|\psi\rangle = a_{ABC}|ABC\rangle + a_{AB\bullet}|AB\bullet\rangle + a_{A\bullet C}|A\bullet C\rangle + a_{\bullet BC}|\bullet BC\rangle \\ + a_{A\bullet\bullet}|A\bullet\bullet\rangle + a_{\bullet B\bullet}|\bullet B\bullet\rangle + a_{\bullet\bullet C}|\bullet\bullet C\rangle + a_{\bullet\bullet\bullet}|\bullet\bullet\bullet\rangle\end{aligned} \tag{22}$$

$a_{ABC}, a_{A\bullet\bullet}, a_{\bullet B\bullet}, a_{\bullet\bullet C}$ (respectively $a_{A\bullet C}, a_{AB\bullet}, a_{\bullet BC}, a_{\bullet\bullet\bullet}$) are commuting (respectively anti-commuting)

*2.1.2. the super entanglement*

Two states are called super entangled when they cannot be written as a tensor product of its constituents. An example of a normalized super entangled state is:

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$$|\psi\rangle = \frac{1}{\sqrt{3}}(|00\rangle + |11\rangle + i|\bullet\bullet\rangle \quad (23)$$

## 3. Super gates

Super Gates are the building block of the quantum circuit,

*1.4. Single superqubit Gates*

A single superqubit gate is the basic operation using a super operator which is written in the form of super matrices. After this, the superqubit is changed to another state with the conservation of the norm. All transformations are represented by 3*3 super matrices (2/1)*(2/1). The following gates are designed for the super quantum computation. The transformations are vrepresented by the elements of the super group uOSP(1/2) and the superqubit transforms as a triplet or doublet

*3.1.1. Even super gates*

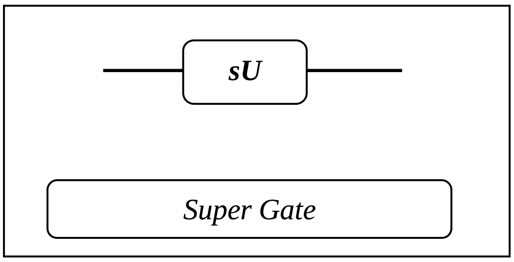


**Figure1** A super gate

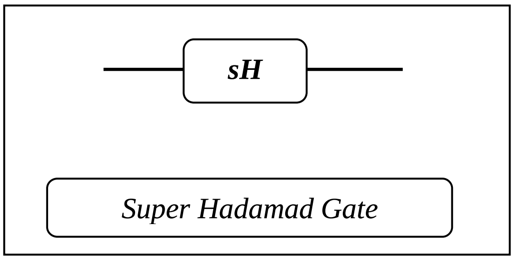


**Figure 2** A super Hadamard gate

*1.5. Multi-superqubit gates*

*3.1.2. 2*2 reversible super qubit gates*

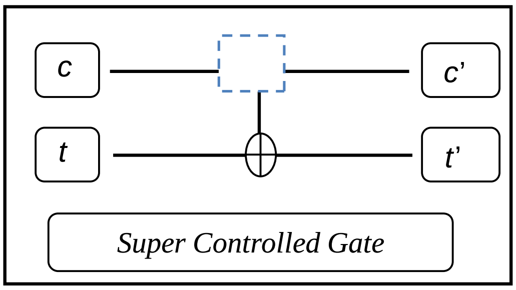


**Figure 3** A Super Controlled gate

this transformation is represented by a 9*9 super matrix as.

$$\begin{pmatrix} 1&0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0\\ 0&0&0&-1&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&0&-1\\ 0&0&0&0&0&0&0&1&0 \end{pmatrix}\begin{pmatrix} a\\ b\\ c\\ d\\ e\\ f\\ g\\ h\\ i \end{pmatrix}=\begin{pmatrix} a\\ b\\ -d\\ c\\ e\\ f\\ g\\ -i\\ h \end{pmatrix} \tag{24}$$

| *in* | | *out* | |
|---|---|---|---|
| *c* | *t* | *c'* | *t'* |
| 0 | *A* | 0 | *A* |
| 1 | *A* | 1 | *NotA* |
| • | *A* | • | *NotA* |

### *3.1.3. Super Toffoli gate*

$$\begin{pmatrix}
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
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0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1
\end{pmatrix}$$

| *in* | | | *out* | | |
|---|---|---|---|---|---|
| $c_1$ | $c_2$ | $t$ | $c_1'$ | $c_2'$ | $t'$ |
| 0 | 0 | $A$ | 0 | 0 | $A$ |
| 0 | 1 | $A$ | 0 | 1 | $A$ |
| 0 | • | $A$ | 0 | • | $A$ |
| 1 | 0 | $A$ | 1 | 0 | $A$ |
| 1 | 1 | $A$ | 1 | 1 | $NotA$ |
| 1 | • | $A$ | 1 | • | $NotA$ |
| • | 0 | $A$ | • | 0 | $A$ |
| • | 1 | $A$ | • | 1 | $NotA$ |
| • | • | $A$ | • | • | $NotA$ |

## 4. Computing with a superqubit

Performing a computation using the basic concepts of a quantum physics such as superposition and entanglement is called quantum computing where qubit plays a fundamental role. Here, we propose an extended field of computation and communication using a superqubits as fundamental career of information. In what follows, we will give an introduction to super quantum algorithms namely super teleportation and super Deutsch's algorithm, Moreover, the ALU has been also developed making parallel operations simultaneously and showing supremacy over quantum and classical computation

### *1.6. Super teleportation*

The process of transmitting an unknown qubit from one location to another without passing through space supported by a pre-shared EPR pair and classical communication between the two parts is called teleportation. It is a very interesting result of the quantum computation which was demonstrated experimentally. Teleportation will play a crucial rule in the future of quantum networks and information an make a great change over large distance communication. Similarly a quantum super teleportation is the process of transferring a superqubit from one location to another using maximally super entangled state and classical communication If Alice wants to send a superqubit to Bob,we propose the following scheme as a prototype for the quantum super teleportation :

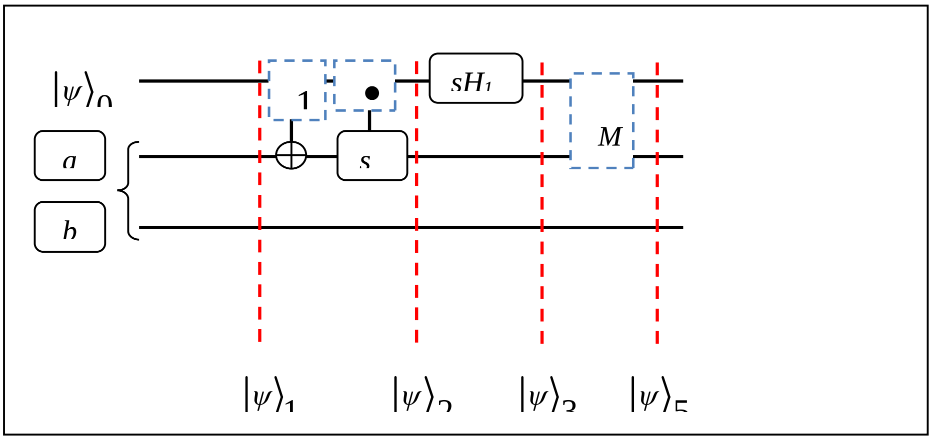


**Figure 4** prototype for the quantum super teleportation

The input state is:

$$|\psi\rangle = |\psi_0\rangle|\psi\rangle_{AB}$$

$$|\psi_5\rangle = \left[(1+\frac{p^2}{2}\theta\theta^{\neq})(|00\rangle+|10\rangle)+(p\theta-p\theta^{\neq})|\bullet\rangle|0\rangle\right]\left[a_1|0\rangle+a_2|1\rangle+a_\bullet(|0\rangle-ip\theta^{\neq}|\bullet\rangle\right]$$
$$\left[(1+\frac{p^2}{2}\theta\theta^{\neq})(|01\rangle+|11\rangle)+(p\theta-p\theta^{\neq})|\bullet\rangle|0\rangle\right]\left[a_1|1\rangle+a_2|0\rangle+a_\bullet(|1\rangle-ip\theta|\bullet\rangle\right] \quad (25)$$

The Deutch's algorithm using superqubits combines quantum parallelism with interference property of quantum mechanics

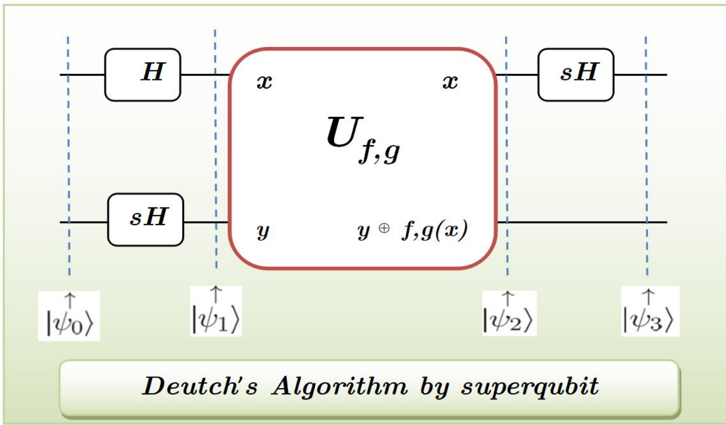


**Figure 5** Deutch's algorithm by a superqubit

$$|\psi_0\rangle = |0\rangle|1\rangle$$

After application of $U\ (f\ g\ )$

$$\left|\psi_3\right\rangle_{even} = \left\{\frac{1}{2\sqrt{2}}(1+\frac{p^2+q^2}{2}\theta\theta^{\neq})(\left|f(0)+f(1)+1\right\rangle)\right\}\left\{(\left|1\right\rangle-\left|0\right\rangle)\right\}$$
$$\left|\psi_3\right\rangle_{odd} = \left\{-(\frac{1}{\sqrt{2}}(p\theta+p\theta^{\neq})\left|g(1)+g(0)+1\right\rangle\right\}\left\{\left|\bullet+g(1)\right\rangle\right\} \quad (26)$$

## 1. Conclusion
Throughout this paper, we have exploited the superqubits properties in the field of the quantum computing. Some algorithms are proposed in order to perform the super teleportation. Implementations and technologies using superqubits circuits will be discussed in a future work.

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