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Holographic Wilson loops in anisotropic quark-gluon plasma.

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Abstract. The nonequilibrium properties of the anisotropic quark-gluon plasma are condidered from the holographic viewpoint. Lifshitz-like solution is considered as a holographic dual of anisotropic QGP. The black brane formation in such background is considered as a thermalization in dual theory. As a probe of thermalization we consider rectangular spatial Wilson loops with different orientation.

1 Introduction and holographic setup

Gauge/gravity duality is a powerful tool for strong coupling systems equilibration study. The quark-gluon plasma (QGP) produced in heavy-ion collisions (HIC) [1] and created after a very short time after the collision (about $0.2 \div 1 fm/c$) is one of such systems.

The holographic duality provides suitable background for description of this period of evolution [2, 3]. The anisotropy and anisotropy effects in the holographic description of QGP is also convenient too [4, 5].

We choose the bottom-up approach for the general AdS/CFT correspondence in background depending on some parameters. This means fitting parameters in background (assuming the dependence on the minimal number of such parameters) in such a way, that the hologprahic calculations fit the experimental data.

Holographic description [6] of the HIC modeled by shock wall collisions in the Lifshitz-like background [10, 11], parametrized by the critical exponent ν is based on background

$$ds^2 = \frac{-dt^2 + dx^2}{z^2} + \frac{dy_1^2 + dy_2^2}{z^{2/\nu}} + \frac{dz^2}{z^2}. \tag{1}$$

This background perfectly matches experimental data [6] for holographic multiplicity calculation. The experimental data is reproduced from holography for the parameter $\nu = 4$. This is the case of special interest for us. Vaidya solutions in these Lifshitz-like spacetimes have been found in [8]. The thermalization time of the 2-point correlators and the thermalization of the holographic entanglement entropy have been estimated in [7, 8].

In this proceedings, based on [9], we investigate numerically spatial Wilson loops behaviour in the Lifshitz-like background. An infalling shell of matter deforms Lifshitz-like spacetime and this corresponds to the formation of the black hole in this background. Wilson loops under consideration

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are rectangles with two infinitely long sides. Lifshitz-like spacetime depends on some parameter ν, controlling the spatial anisotropy. We study (pseudo)potentials calculated from holographic Wilson loops and their evolution in asymptotically Lifshitz-like backgrounds. We calculate the dependence of the equilibration time on spatial orientations of Wilson loops. Also we investigate the dependence on the anisotropy parameter ν.

A non-zero temperature generalization of the Lifshitz-like spacetime (1) is a black brane background with the metric [8]

$$ds^2 = \frac{-f(z)dt^2 + dx^2}{z^2} + \frac{dy_1^2 + dy_2^2}{z^{2/\nu}} + \frac{dz^2}{z^2 f(z)}, \tag{2}$$

where the function f is defined by

$$f = 1 - mz^{2+2/\nu}. \tag{3}$$

The metric of the black brane in the 5d AdS spacetime corresponds to $\nu = 1$.

The temperature of the black brane solution (2) is

$$T = \frac{1}{\pi}\frac{(\nu+1)}{2\nu}\, m^{\frac{\nu}{2\nu+2}}. \tag{4}$$

A straightforward generalization of the stationary black brane metric is the Lifshitz-Vaidya solution:

$$ds^2 = -\frac{f(v,z)dv^2 - 2dvdz + dx^2}{z^2} + \frac{dy_1^2 + dy_2^2}{z^{2/\nu}}, \tag{5}$$

with the function f taken in the following form

$$f(z,v) = 1 - \frac{M}{2}\left(1 + \tanh\frac{v}{\alpha}\right) z^{\frac{2}{\nu}+2}, \tag{6}$$

where $\alpha = 0.2$ for all calculations in this paper.

In this work we focus on rectangular spatial Wilson loops with an infinite extent. The basic formula for the computation of the expectation value of the Wilson loop operator from holography [12, 13] (specified by the contour C) reads as:

$$W[C] = \langle \mathrm{Tr}_F\, e^{i\oint_C dx_\mu A_\mu} \rangle = e^{-S_{string}[C]}, \tag{7}$$

where S is the action of the string with boundary conditions defined by C. The Nambu-Goto action [1] can be represented as

$$S_{string} = \frac{1}{2\pi\alpha'}\int d\sigma^1 d\sigma^2 \sqrt{-\det(h_{\alpha\beta})}, \tag{8}$$

with the induced metric of the world-sheet $h_{\alpha\beta}$ given by

$$h_{\alpha\beta} = g_{MN}\partial_\alpha X^M \partial_\beta X^N, \tag{9}$$

where $\alpha,\beta = 1,2$. In (9) g_{MN} is the background metric, $M,N = 1,\dots,5$, $X^M = X^M(\sigma^1,\sigma^2)$ specify the string worldsheet and σ^1, σ^2 parametrize the worldsheet.

[1] We keep α' in formulas. For all intermediate calculation we set $\alpha' = 1/2\pi$ whereas we take its appropriate value in final results to fit observable data.

Using a spatial rectangular Wilson loop with a finite extent X one can define the so called pseudopotential $\mathcal{V}(X)$ for large Y

$$W(X,Y) = \langle \mathrm{Tr} e^{i\oint_{X\times Y} dx_\mu A_\mu}\rangle = e^{-\mathcal{V}(X)Y}, \tag{10}$$

So, it is straightforward to write down the expression which relates the pseudopotential and the bulk string action:

$$\mathcal{V}(X) = \frac{S_{string}}{Y}. \tag{11}$$

2 Non-equilibrium Wilson loops

In [9] we have considered rectangular Wilson loops with different orientations to capture the anisotropy effect. Here we consider one of the orientation cases in greater details. Cases of orientation considered in [9] are

- a rectangular loop in the xy_1 (or xy_2) plane with a short side of the length ℓ

$$x \in [0, \ell < L_x], \quad y_1 \in [0, L_{y_1}]; \tag{12}$$

- a rectangular loop in the xy_1 plane with a long side of the length L_x along the longitudinal x direction,

$$x \in [0, L_x], \quad y_1 \in [0, \ell < L_{y_1}]; \tag{13}$$

- a rectangular loop in the transversal y_1y_2 plane with a long side of the length L_{y_2} along the other transversal direction y_2, namely

$$y_1 \in [0, \ell < L_{y_1}], \quad y_1 \in [0, L_{y_2}]. \tag{14}$$

2.1 Rectangular strip infinite along the y_1-direction

We consider only this case of orientation in details. For investigation of another orientations see [9].

The orientation of interest is the rectangular Wilson loop on the xy_1-plane, case (12). We choose the following worldsheet parametrization

$$\sigma^1 = x, \quad \sigma^2 = y_1, \tag{15}$$

assuming $v = v(x)$, $z = z(x)$ and $z(\pm\ell/2) = 0$.

The Nambu-Goto action in this case can be written as

$$\frac{2\pi\alpha' S_{x,y_{1(\infty)}}}{L_{y_1}} = \int \frac{dx}{z^{1+1/\nu}}\sqrt{1 - fv'^2 - 2v'z'}. \tag{16}$$

The equations of motion of dynamical system with the action (16) are

$$\begin{aligned} v'' &= \frac{1}{2}\frac{\partial f}{\partial z}v'^2 + \frac{(\nu+1)}{\nu z}(1 - fv'^2 - 2v'z'), \\ z'' &= -\frac{\nu+1}{\nu}\frac{f}{z} + \frac{\nu+1}{\nu}\frac{f^2v'^2}{z} - \frac{1}{2}\frac{\partial f}{\partial v}v'^2 \\ &- \frac{1}{2}fv'^2\frac{\partial f}{\partial z} - v'z'\frac{\partial f}{\partial z} + 2\frac{(\nu+1)}{\nu z}fv'z', \end{aligned} \tag{17}$$

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and the boundary conditions of interest are

$$z(\pm\ell) = 0, \quad v(\pm\ell) = t, \tag{18}$$

where ℓ is the length of the Wilson loop along the x-direction.

To solve these equations numerically we set the initial value problem instead in form

$$z(0) = z_* \tag{19}$$
$$v(0) = v_* \tag{20}$$
$$z'(0) = v'(0) = 0. \tag{21}$$

and then define the values of z_* and v_* that satisfy boundary conditions (18).

Let us briefly consider the structure of the boundary condition/initial value parameters space. In Fig.1 we plot the constant level lines of $z(\ell) = 0$ and ℓ beign fixed in parametric space z_* and v_* on the left plot, and the same for $v(\ell)$ on the right plot. We see, that generally for each ℓ and t one can find the unique values pair fixing the values z_* and v_* so that boundary conditions (18) are satysfied. White curve on each plot is the kind of "attractor" for constant level lines. In the parametric space the parameter values lying upper than this curve corresponds to the geodisics refracting from the shell and going to the singularity. So one should neglect these parameters values as they does not correspond to right boundary conditions.

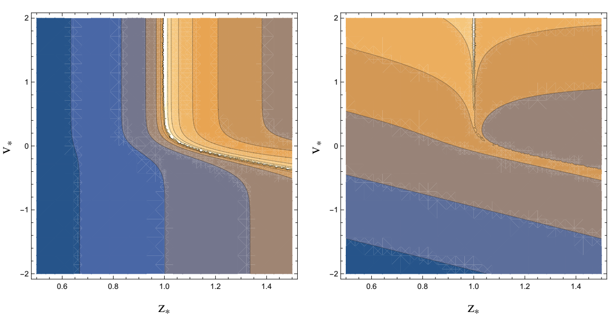

Figure 1. Left plot: parametric space for the value $z(\ell)$. Parameter $\nu = 2$ is fixed. Right plot: parametric space for the value $v(\ell)$, time t. Parameter $\nu = 2$ is fixed.

We solve equations of motion (17) numerically and show the example of solution in Fig.2.

The renormalized action (16) can be represented as

$$\frac{2\pi\alpha' S_{x,y_{1(\infty)},ren}}{L_{y_1}} = -\int_{z_0}^{z_*} \frac{[\mathfrak{b}(z) - \mathfrak{b}(z_0)]}{z^{1+1/\nu}} dz + \frac{\nu\mathfrak{b}(z_0)}{z_*^{1/\nu}},$$

where we have introduced the quantity $\mathfrak{b}$ defined by

$$\mathfrak{b}(z) = \frac{1}{z'}\left(\frac{z_*}{z}\right)^{1+1/\nu}. \tag{22}$$

and the pseudopotential is expressed as :

$$\mathcal{V}_{x,y_{1(\infty)}} = \frac{S_{x,y_{1(\infty)},ren}}{L_{y_1}}. \tag{23}$$

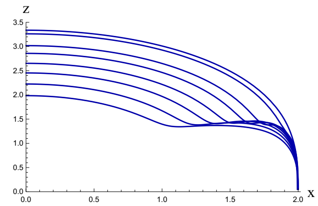

Figure 2. Profile of the string $z(x)$, $z(2) = 0$ at different moments of the boundary time for $\nu = 4$. In (6) we take $M = 1$.

It is also instructive to define and study the following quantity

$$\delta\mathcal{V}_1(x,t) = \mathcal{V}_{x,y_{1(\infty)}}(x,t) - \mathcal{V}_{x,y_{1(\infty)}}(x,t_f), \tag{24}$$

expressing the deviation of $\mathcal{V}$ from thermal equilibrium.

Evaluating (24) on numerical solutions of (17) we can see the dynamics of pseudopotential equilibration (see Fig.3 and Fig.4)

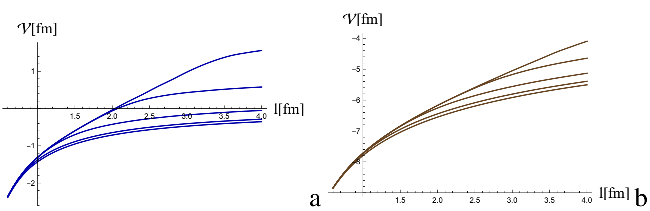

Figure 3. The pseudopotential $\mathcal{V}_{x,y_{1(\infty)}}$ as a function of ℓ at fixed values of t for $\nu = 1$ and $\nu = 4$. ((a) and (b), respectively). Different curves correspond to time $t = 0.1, 0.5, 0.9, 1.4, 2$ (from down to top, respectively). In (6) we take $M = 1$.

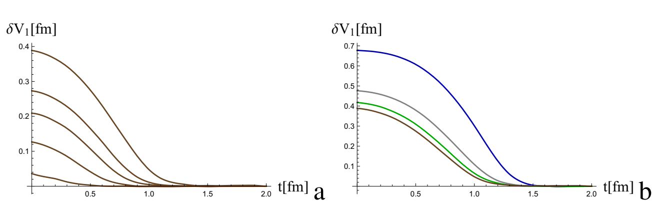

Figure 4. a. The time dependence of the quantity $-\delta\mathcal{V}_1(x,t)$, given by (24), for different values of the length ℓ, $\nu = 4$. b. The quantity $-\delta\mathcal{V}_1(x,t)$ as a function of t at $\ell = 2$ for $\nu = 1, 2, 3, 4$ (from top to down, respectively). In (6) we take $M = 1$.

3 Conclusion

The equilibration process of Wilson loop in the anisotropic quark-gluon plasma using the gauge/gravity duality is analysed. Lifshitz-like background is taken on the gravity side and the critical exponent ν controls the anisotropy. The expectation values of Wilson loops and their dynamic is considered for certain orientation of contour. Numerical issues, concerning the solution of equation of motion for the Nambu-Goto string are discussed.

The pseudopotential out of equilibrium has the universal behaviour: it tends to achievement of the saturation corresponding to large values of the boundary time. The dynamical exponent influences the thermalization process of Wilson loops: this behavior described above strengthen as the value of ν increases.

Acknowledgments

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