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Coriolis Coupling in the Continuum

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Abstract. In 1956 A. Kerman explained the observed energy levels of the lowest $K = 1/2$ and $K = 3/2$ bands in ^{183}W as resulting from rotational perturbations due to the effects of the Coriolis force. Following from that seminal paper, in this work we consider Kerman's problem when one of the Nilsson single-particle levels involved is a resonant state. This is the simplest model of Coriolis mixing that allows for analytical expressions derived in first-order perturbation theory, without losing the main physical ingredients of the continuum effects. We present results for the perturbed solution of the ground state band energies as a function of the state width, which suggest a general behavior of both increased moment of inertia and staggering.

1. Introduction

In the middle of the last century, as rotational motion in nuclei began to be studied, Kerman [1] suggested that rotating nuclei might also experience the effects of the Coriolis force, much like well known examples in classical mechanics. For an odd-A nucleus the coupling of a single nucleon to a rotating core is well described by the Particle-Rotor-Model (PRM) [2, 3]. The Hamiltonian is usually written as

$$H = e_\Omega + \frac{\hbar^2}{2\mathscr{I}} I(I+1) + H_c \tag{1}$$

e_Ω are the energies of the relevant Nilsson levels [4] with angular momentum projection on the symmetry axis Ω , $\mathscr{I}$ is the core moment-of-inertia, and H_c is the Coriolis coupling term

$$H_c = -\frac{\hbar^2}{2\mathscr{I}}(I_+ j_- + I_- j_+) \tag{2}$$

where $I_\pm$ and $j_\pm$ the ladder operators for the total and single particle angular momenta respectively. The matrix elements of H_c are:

$$\langle I, \Omega \pm 1 | H_c | I, \Omega \rangle = -\frac{\hbar^2}{2\mathscr{I}} \langle \Omega \pm 1 | j_\pm | \Omega \rangle \left[(I \mp K)(I \pm K + 1)\right]^{1/2} = -h_c[(I \mp K)(I \pm K + 1)]^{1/2} \tag{3}$$

and are particularly important for large j and increase with the total angular momentum. Given the form of the matrix element it mixes levels with $\Delta K = \pm 1$, for example Nilsson orbits with

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$K = 1/2$ and $3/2$ can be admixed, as can those with $K = 5/2$ and $7/2$. Due to symmetrization of the orbit directions in the wavefunction, a $K = 1/2$ band can also mix with itself, giving rise to a diagonal Coriolis effect (staggering) on the energies, measured by the decoupling parameter, $a = -\langle K| j_+ |\bar{K}\rangle$ [2].

Coriolis effects in rotational motion, such as backbending [5], particle alignments [6], decoupled bands [7] and re-distribution of single particle strengths [8] have been abundantly characterized across the nuclear chart [9] and to some extent these studies have dominated the high-spin physics research program for decades.

However, with the development of new rare isotope accelerator facilities worldwide, the focus in nuclear structure has shifted to exotic nuclei stretching to the drip lines. On the neutron-rich side of stability, as the neutron separation energy approaches zero, weakly bound neutrons in the single-particle levels at the Fermi surface approach the top of the nuclear potential and may move outside the core of well-bound nucleons, and possibly couple to unbound continuum states. The nature of the transition from a closed to an open quantum system [10], where binding is dominated by correlations rather than the mean field, has only just begun to be explored.

Specific to this work, are the questions of how collective motion evolves in neutron rich systems and whether continuum effects might emerge in the structure of weakly bound odd-A rotors.

A priori, one would expect that the familiar Coriolis mixing signatures can be altered in significant ways if one of the mixing states is unbound. In fact, the mixing of unbound states with finite widths is known to have unusual, intriguing, and sometimes counter-intuitive effects [11, 12, 13]. Recent theoretical works have addressed this interesting topic. Ref. [14] investigated the shell structure of one-particle resonances in deformed potentials and numerical examples applicable to the structure of ^{21}C and ^{39}Mg were discussed. In Refs. [15, 16] a nonadiabatic particle-plus-rotor model[1] formulated in the Berggren basis (explicitly containing bound states, narrow resonances, and the scattering continuum) was developed to study the collective rotation of the positive-parity deformed configurations of the one-neutron halo nucleus ^{11}Be and the structure of ^{39}Mg.

Here we take a more simplistic (and perhaps more pedagogical) approach by considering the simplest case of Coriolis coupling in an odd-A nucleus, that of Kerman's original two band-mixing problem. Our goal is to assess whether signatures of continuum effects emerge in the structure of the mixed bands, in the presence of a resonant Nilsson orbit as anticipated in Ref. [14]. In spite of its simplicity, this model captures the relevant ingredients and we will show that a renormalization of the rotational properties of the perturbed ground state band as a function of the decay width appears as a general feature of the solution.

2. Extension of Kerman's Study

Kerman's focus was on the mixing of the Nilsson orbits 1/2[510] and 3/2[512] in ^{183}W, ultimately achieving an excellent description of the perturbed energies. In this case, the PRM Hamiltonian is simply given by a 2x2 matrix

$$\begin{pmatrix} E_{3/2} & H_c \\ H_c & E_{1/2} \end{pmatrix}$$

The diagonal unperturbed energies of these bands are:

$$E_{3/2} = e_{3/2} + AI(I+1) \tag{4}$$

[1] Non-adiabatic effects have also been studied to explain proton radioactivity in proton-rich deformed rare-earth nuclei (See Ref. [17]).

and

$$E_{1/2} = e_{1/2} + A(I(I+1) + (-)^{I+1/2} a(I+1/2)) \tag{5}$$

where we have introduced the rotational constant $A = \frac{\hbar^2}{2\mathscr{I}}$ and a is the decoupling parameter.

We extend the problem by considering, for example, that the 1/2 level is a resonance with width Γ as depicted in Fig. 1. We approximate the resonance by a sequence of N discrete equidistant and equally weighted levels in the energy window spanned by Γ and centered at ϵ. Although this replaces the Breit-Wigner resonance shape with a set of sharply bounded states, it does capture the essential physics elements. In this approximation, the 2x2 Kerman's matrix becomes an $(N+1)$x$(N+1)$ matrix:

$$\left(\begin{array}{c|cccccccc} 0 & V & V & ... & V & ... & ... & V \\ \hline V & \epsilon - \frac{\Gamma}{2} & 0 & ... & ... & ... & ... & 0 \\ V & 0 & ... & ... & ... & ... & ... & 0 \\ ... & ... & ... & ... & ... & ... & ... & ... \\ V & ... & ... & ... & \epsilon & ... & ... & 0 \\ ... & ... & ... & ... & ... & ... & ... & ... \\ ... & ... & ... & ... & ... & ... & ... & ... \\ V & 0 & 0 & ... & 0 & ... & ... & \epsilon + \frac{\Gamma}{2} \end{array}\right)$$

To determine the value of the matrix element V we take the limit when $\Gamma \to 0$ and the resonance becomes a bound state. In this limit the eigenvalues of the $(N+1)$x$(N+1)$ matrix should reduce to those of the 2x2 model. In particular for the lowest state the eigenvalue is

$$\lambda = \frac{\epsilon}{2} - \sqrt{(\frac{\epsilon}{2})^2 + NV^2} \tag{6}$$

Equating this result with the 2x2 solution we obtain

$$V = \frac{H_c}{\sqrt{N}} \tag{7}$$

3. First order perturbation results

While the matrix above can be diagonalized numerically, it is interesting to consider the approach of Ref. [1] in which the results were obtained in first order perturbation theory, namely when $H_c \ll \epsilon$. In this approximation we obtain analytic expressions for the energies of the perturbed bands. It is convenient to introduce some dimensionless variables:

$$\tilde{a} = \frac{(-)^{I+1/2} a(I+1/2)}{\epsilon}, \quad \tilde{\Gamma} = \frac{\Gamma}{\epsilon}, \text{ and } \tilde{V}^2 = \frac{H_c^2}{A\epsilon} \tag{8}$$

After some mathematical manipulation (See Appendix A.) the energy of the perturbed ground state $K = 3/2$ band in units of the rotational constant is:

$$\tilde{E}_{3/2} = \frac{E_{3/2}}{A} = I(I+1) + \frac{\tilde{V}^2}{1+\tilde{a}} \left(1 + \frac{\tilde{\Gamma}}{2(1+\tilde{a})} + ...\right) \tag{9}$$

with the further approximation that $\tilde{\Gamma} \ll 1$.

The factor in front of the parenthesis in Eq. (9) corresponds to the solution of the 2x2 problem, which is recovered by taking the limit $\tilde{\Gamma} \to 0$. The factor in the parenthesis can be

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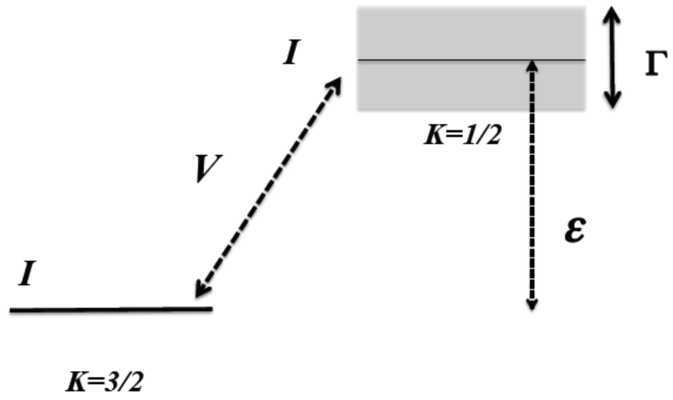

Figure 1. Schematic illustration of the mixing of two Nilsson levels, with one being a resonant state

interpreted as a renormalization of the Coriolis force introduced by the coupling to the resonance, as measured by the reduced width $\tilde{\Gamma}$.

Taking as an example the parameters used by Kerman for the case of ^{183}W (listed in Table 1) we plot in Figure 2 the renormalization term, as a function of angular momentum for some selected values of $\tilde{\Gamma}$. The observed behavior can be interpreted as an increase of both the moment of inertia and the staggering in the ground state band. Clearly, the strength of the effect increases sharply with the width and one also sees the increasing staggering with spin, reflecting the decoupling parameter.

One would be tempted to speculate about cases where these predictions could be tested. A possible region to consider is that around ^{40}Mg where first gamma-ray spectroscopy results may indicate effects of weak-binding [18]. In particular, the case of ^{39}Mg [14, 16] where the odd neutron is expected to occupy resonant Nilsson levels of f and p parentage will be interesting to study [19].

Table 1. Values of the rotational parameters used in the 2x2 band-mixing calculation in Ref. [1].

Parameter	Value
ϵ	210 keV
A	15 keV
h_c	20 keV
a	0.2

4. Summary

The evolution of shell structure and collective motion in weakly bound nuclei is a topic of much interest in nuclear structure. In this work we have extended the simplest Coriolis mixing problem

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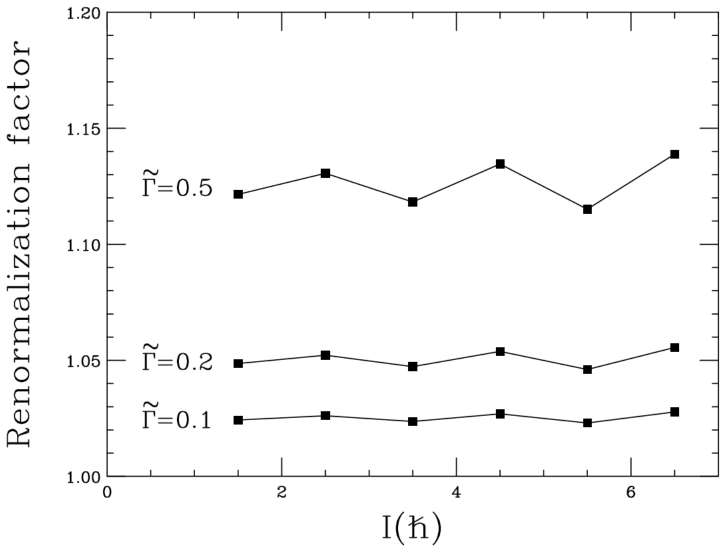

Figure 2. Renormalization of the Coriolis interaction term in Eq. (9) as a function of angular momentum for some values of the reduced width $\tilde{\Gamma}$ as indicated.

(2x2 matrix) discussed originally by Kerman to explain the band structure in ^{183}W, to include coupling to a resonant state. Our model calculation, including mixing with an unbound state, was used to explore possible consequences on the rotational properties of an odd-A system. We derived analytical expressions for the energy of the perturbed ground state band in first order perturbation theory. Qualitative effects seem to emerge when the width of the resonance becomes comparable to the intrinsic level separation energy, suggesting a renormalization of the Coriolis matrix elements which, in turn, modify the moment of inertia and band staggering. We believe that, in spite of our simplifying assumptions, the observed behavior is rather robust. Nevertheless, a numerical solution of the full Hamiltonian matrix with a more realistic Breit-Wigner weighting of the resonance and the extension to a single-j Nilsson multiplet will be the subject of a future publication.

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Appendix A.
We give here some details of the derivation of Eq. (9). In perturbation theory the perturbed energy of the ground state band is given by:

$$\tilde{E}_{3/2} = I(I+1) - \frac{H_c^2}{\epsilon A} \sum_{n=1}^{N} \frac{1}{1 + a(I)/\epsilon + (n - \frac{N}{2})\Gamma/\epsilon N} \tag{A.1}$$

where we have defined $a(I) = (-)^{I+1/2} a(I+1/2)$. In terms of the dimensionless variables in Eq. (8)

$$\tilde{E}_{3/2} = I(I+1) - \frac{\tilde{V}^2}{N} \sum_{n=1}^{N} \frac{1}{1 + \tilde{a} - \tilde{\Gamma} + n - \frac{\tilde{\Gamma}}{N}} \tag{A.2}$$

By considering an infinite number of levels we can replace the sum by an integral of the form

$$\int_0^1 \frac{dx}{b+cx} \tag{A.3}$$

giving

$$\tilde{E}_{3/2} = I(I+1) - \frac{\tilde{V}^2}{\tilde{\Gamma}} ln\left(\frac{1+\tilde{a}}{1+\tilde{a}-\tilde{\Gamma}}\right) \tag{A.4}$$

With the further assumption that $\tilde{\Gamma}$ is small we finally obtain

$$\tilde{E}_{3/2} = I(I+1) - \frac{\tilde{V}^2}{\tilde{\Gamma}}\left(-\frac{\tilde{\Gamma}}{1+\tilde{a}} - \frac{\tilde{\Gamma}^2}{(1+\tilde{a})^2} + ...\right) \tag{A.5}$$

which is Eq. (9).

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