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A Survey of Pressure Losses at Pipe Diameter Transitions and Thermal Expansion Joints, and Errata Concerning SSC Technical Note No. 90 (SSCL-N-746)

M. L. Rehak and R. P. Shutt

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A Survey of Pressure Losses at Pipe Diameter Transitions and Thermal Expansion Joints, and Errata Concerning SSC Technical Note No. 90 (SSCL-N-746)

M.L. Rehak and R.P. Shutt

In superconducting magnet systems, such as large high energy accelerators, there are many connections by means of pipes and expansion joints consisting of bellows. When the helium used for magnet cooling flows through a system, it encounters many sudden contractions (C) or enlargements (E) of adjacent pipe cross sections, and bellows, whose walls are corrugated, can therefore be expected to offer more resistance to flow than smooth-walled pipes. Resulting pressure losses are treated in the literature. Here we merely wish to consider some facts that specifically concern our work and to correct an error in our SSC Technical Note No. 90 (SSCL-N-746).

Basically a fluid is accelerated at C and decelerated at E, requiring pressure differences. We cite the well-known Bernoulli equation, based on energy conservation:

$$h + \frac{\alpha M^2}{2\rho a^2} + p = constant \tag{1}$$

in a pipe system. Fluid flow is assumed to be stationary.

Definitions:

p = pressure (atm)

M = mass flow (g/sec) (≡ a ρ v, if v = velocity)

ρ = density (usually assumed to be ≈ constant at a transition) (g/cm^3)

a = pipe cross section (cm^2)

h = "head", for instance, due to a column of liquid or equivalent.

$\alpha \approx 10^{-6}$ converts pressure dimensions to atmospheres.

At a pipe transition where $a_1 \to a_2$, omitting "head", eq. 1 results in pressure drop

$$\Delta p = p_1 - p_2 = -\frac{\alpha M^2}{2\rho}\left(\frac{1}{a_1^2} - \frac{1}{a_2^2}\right) \tag{2}$$

$\alpha\frac{M^2}{2\rho a_{1,2}^2}\left(\equiv \alpha\frac{\rho v^2}{2}\right)$ is called a velocity head. If $a_1 \gg a_2\,(C)$, $\Delta p \approx \alpha\frac{M^2}{2\rho a_2^2}$, or $p_2 \approx p_1 - \alpha\frac{M^2}{2\rho a_2^2}$. If $a_1 \ll a_2\,(E)$, $p_2 \approx p_1 + \alpha\frac{M^2}{2\rho a_1^2}$.

Bernoulli's equation is correct for idealized flow conditions. For C and E cases, a custom has been to use one or two, if some margin is desired, velocity heads for pipe end pressure drops. There are several additional effects, due to turbulence and radial flow velocity components at pipe transitions. Figure 1 gives an illustration. It is seen that, due to radial velocity components, at the pipe entrance the stream lines are expected to contract to a cross section a_c that is smaller than a_2. Between a_c and a_2, turbulence takes place. At the pipe exit the stream is expected to "jet" without expanding fully from a_2 to a_3 until flowing through lengths equal to several exit pipe diameters. Outside the jetting region, again, turbulence occurs. Pressure losses due to these effects are called "head losses" in the literature. The head losses are analyzed in the literature considering pressure differences to provide the necessary local accelerations of the flow. These head losses are then entered into eq. 1, and the following expressions are obtained: Calling

$$G_n = \alpha \frac{M^2}{2\rho_n a_n^2}, \tag{3}$$

the head loss for sudden pipe contraction (C) is (n = 21)

$$h_c = G_{21} \times 0.5 \left(1 - \frac{a_2}{\alpha_1}\right) \tag{4}$$

and for sudden pipe enlargement (E) (n = 22)

$$h_e = G_{22} \times \left(1 - \frac{a_2}{a_3}\right)^2 \tag{5}$$

Using eq. 1,

$$p_1 - p_{21} = h_c + G_{21} \left(1 - \left(\frac{a_2}{a_1}\right)^2\right) \tag{6}$$

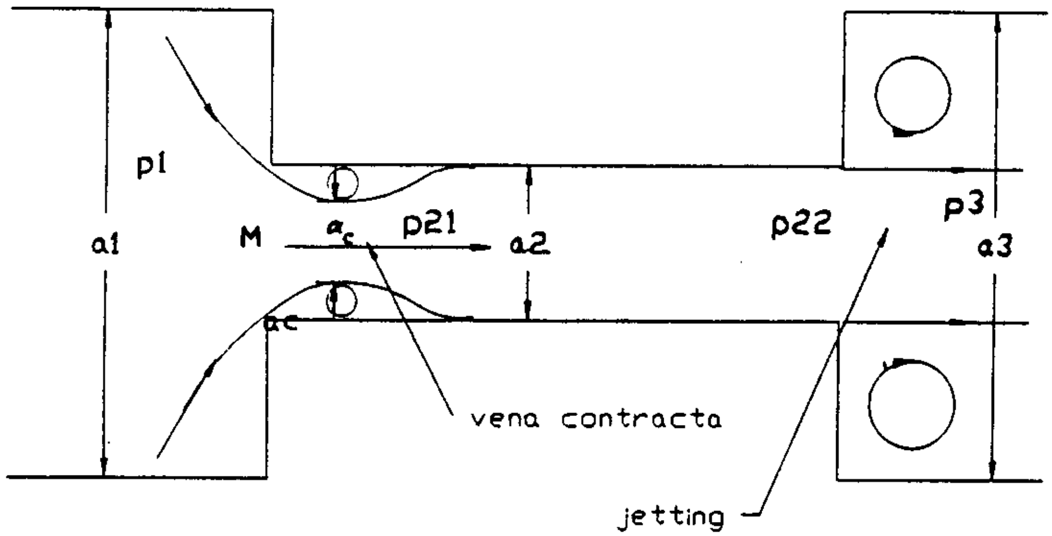
$$p_{22} - p_3 = h_e - G_{22} \left(1 - \left(\frac{a_2}{a_3}\right)^2\right) \tag{7}$$

resulting in

$$C: \quad p_1 - p_{21} = G_{21} \left(1.5 - \frac{1}{2}\left(\frac{a_2}{a_1}\right) - \left(\frac{a_2}{a_1}\right)^2\right) \tag{8}$$

$$E: \quad p_{22} - p_3 = G_{22} \times 2 \left(\left(\frac{a_2}{a_3}\right)^2 - \left(\frac{a_2}{a_3}\right)\right) \tag{9}$$

$p_{21} - p_{22}$ is the pressure difference due to friction along the pipe with cross section a_2. As mentioned above, G_{21} and G_{22} are the velocity heads at (C) and (E) if $a_2 \ll a_1$ and $a_2 \ll a_3$, respectively. According to eqs. 8 and 9, for these limits



$a_1, \ a_2, \ a_3$ = cross sections of inlet volume, pipe, and outlet volume, respectively.

$p_1, \ p_3$ = inlet and outlet volume pressures.

$p_{21}, \ p_{22}$ = pressures at or near pipe ends.

a_c = cross section of "vena contracta" (as called in literature).

M = mass flow, assumed to be constant.

Figure 1

$$p_1 - p_{21} \rightarrow 1.5 \ G_{21} \tag{10}$$

$$p_{22} - p_3 \rightarrow 0 \tag{11}$$

Thus the relevant head loss increases the pressure drop at a large contraction by 50% beyond one velocity head, and at a big pipe enlargement it cancels the velocity head. For these cases, if some margin is wanted, one can then indeed use merely 2 velocity heads at a pipe entrance and zero, or for a margin, 0.5 velocity heads at the exit.

G_n contains the fluid density. For compressible fluids ρ will change when the pressure (and temperature) change. This effect is usually neglected – as we will do here. One could certainly use some density averaging procedure, but for most cases velocity heads are small enough that this does not seem warranted.

We will now assume that pressure difference $p_{21} - p_{22}$ is zero and add eqs. 8 and 9 in order to obtain the total pressure drop across pipe C-E:

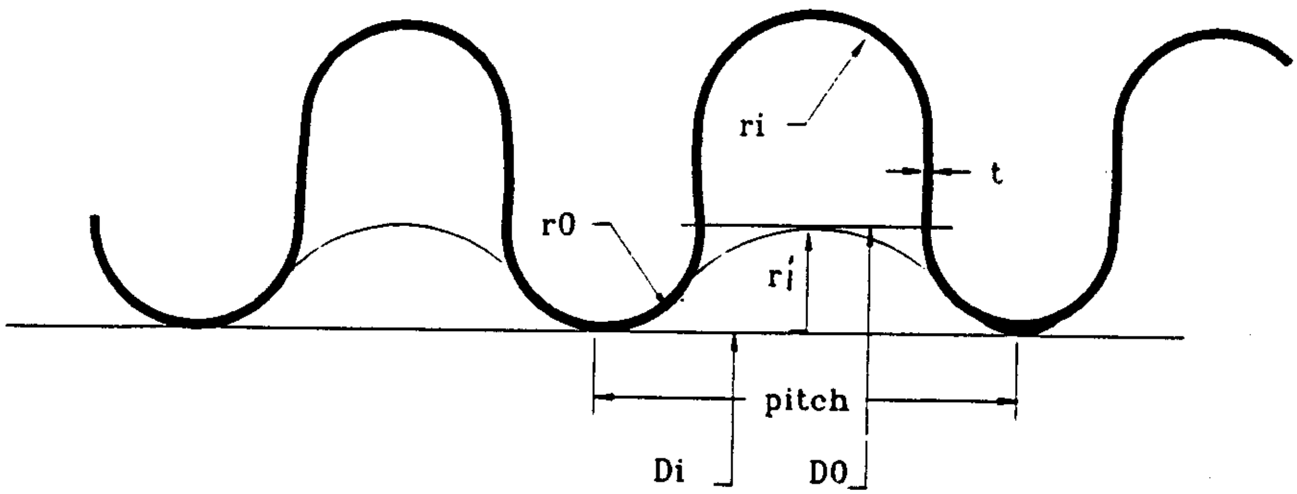
$$p_1 - p_3 = \frac{\alpha M^2}{2\rho_2 a_2^2}\left[1.5 - a_2\left(\frac{1}{2a_1} + \frac{2}{a_3}\right) - a_2^2\left(\frac{1}{a_1^2} - \frac{2}{a_3^2}\right)\right] \tag{12}$$

This expression applies to the general case, when $a_1 \neq a_3$. If $a_1 = a_3$:

$$p_1 - p_3 = \frac{\alpha M^2}{2\rho_2 a_2^2}\left[1.5 - 2.5\left(\frac{a_2}{a_1}\right) + \left(\frac{a_2}{a_1}\right)^2\right] \tag{13}$$

Having given expressions for pipe ends (eqs. 8 and 9) we will now apply eq. 13 to find an expression for pressure differences across bellows for which Fig. 2 gives an illustration, showing a convolution cross section. After entry into a bellows, stream lines could be expected to expand into the full radial width of a convolution. However most of the flow would remain nearer the inner diameter D_i because of the increase of the path length with increasing radius. A reasonable choice for a (weighted) average flow diameter inside a convolution might be $D_o = D_i + 2r_i$. If the pitch length $\Lambda = 4r_i + 2t = 2(r_o + r_i)$ (correct if convolution sides are parallel), then $D_o = D_i + 0.5\Lambda - t$. However, convolution sides are not usually parallel due to the manufacturing process but open up toward the inside of the bellows. Therefore set $\Lambda = 2(r_o + r_i) + \delta$, resulting in

$$D_o = D_i + 0.5\Lambda - t - 0.5\delta \tag{14}$$



r_i = "valley radius"

t = wall thickness

$r_o = r_i + t$

D_i = inner diameter of bellows

D_o: $\approx D_i + 2r_i$

Λ = pitch length

Figure 2

Measurements for long corrugated hose, reported in the literature have resulted in $D_o = D_i + 0.44\Lambda$. Usually for such hose t is rather small, for added flexibility, but δ, for manufacturing ease, may be larger than for a specially manufactured bellows. Thus, for our case, we can also accept that

$$D_o \approx D_i + 0.44\Lambda \tag{15}$$

Returning to eq. 13, we consider a bellows with N convolutions and set

$$a_2 = \frac{\pi D_i^2}{4}\,, \qquad a_1 = \frac{\pi D_o^2}{4} \tag{16}$$

and thus write now

$$p_1 - p_3 = \frac{8\alpha M^2}{\pi^2 \rho D_i^4}\left[1.5 - 2.5\left(\frac{D_i}{D_o}\right)^2 + \left(\frac{D_i}{D_o}\right)^4\right] \tag{17}$$

For many cases this may still be a maximum value since the "jetting" effect, when exiting from a convolution, may prevent the flow from expanding from diameter D_i to D_o because the distance between neighboring convolutions, Λ, usually is $\ll D_i$. Figure 3 illustrates the term in brackets in eq. 17,

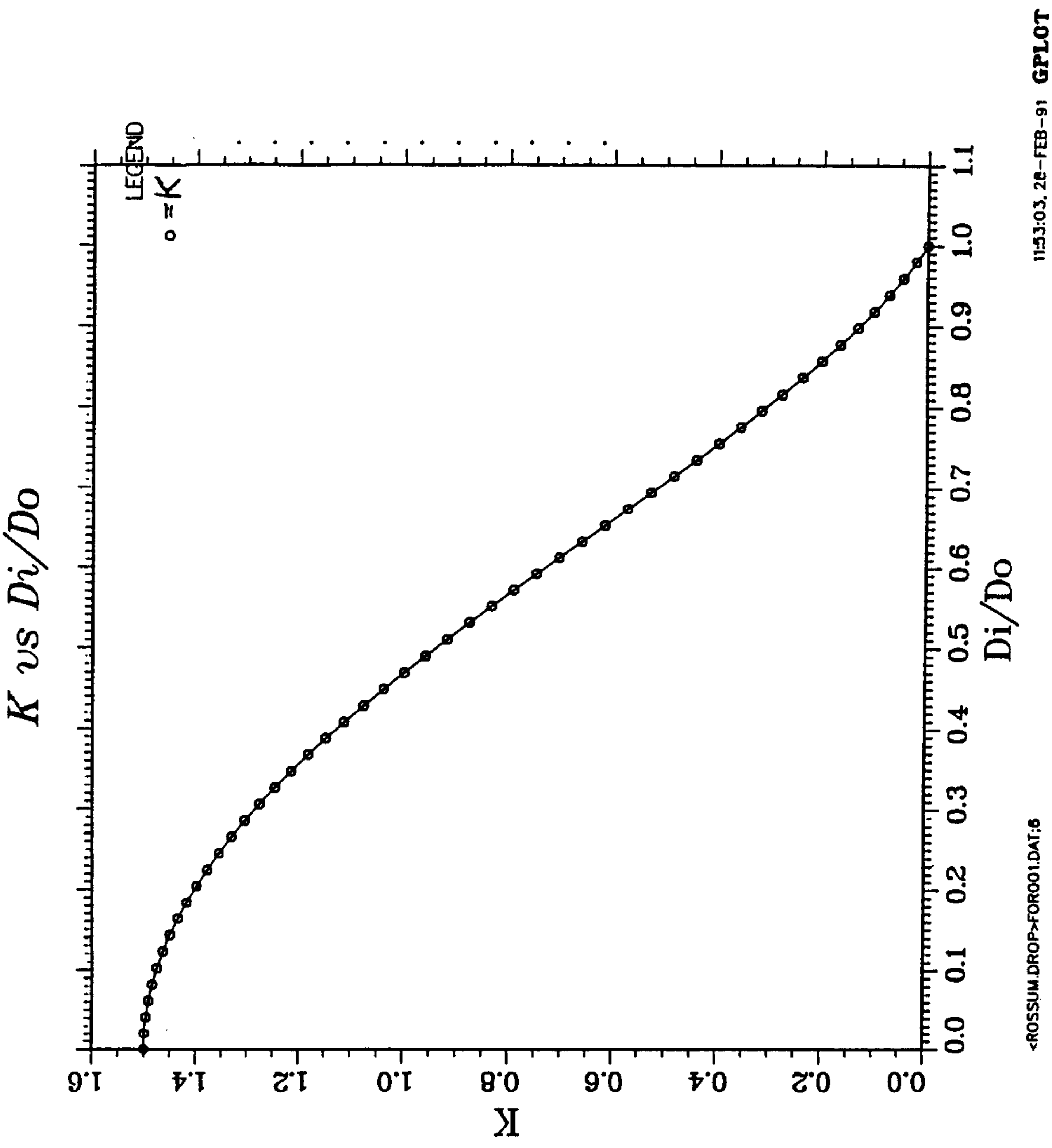
$$K = 1.5 - 2.5\left(\frac{D_i}{D_o}\right)^2 + \left(\frac{D_i}{D_o}\right)^4$$

as a function of $\frac{D_i}{D_o}$.

For a bellows with N convolutions we write, finally

$$\Delta p = \frac{2\alpha M^2}{\pi^2 \rho D_i^4} K N \tag{18}$$

For examples, we calculate pressure drops across two bellows of different sizes, considered for interconnections of SSC magnets:



K vs Di/Do
LEGEND
o = K
K
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
Di/Do
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
11:53:03, 28-FEB-91 GPLOT
<ROSSUM.DROP>FOR001.DAT;6

Figure 3

(1) $D_i = 9.842\ cm$

$\Lambda = 0.635\ cm$

$N = 23$

$M = 1000\ g/sec$

$\rho = 0.136\ g/cm^3$

$\alpha = 10^{-6}$

Here a bus bar of cross section 4.6 x 2.7 cm^2 passes through the bellows. We subtract this from the inner and outer flow cross sections (diameters $D_i,\ D_o$) and take the square root of the ratio of the results: $\frac{D_i}{D_o} = 0.967$. Then, from Fig. 3, K = 0.037 and $\Delta p = 7.7 \times 10^{-4}$ atm per bellows, using here $\Delta p = \frac{\alpha M^2}{2\rho a^2} KN$, where $a = \frac{\pi \times 9.842^2}{4} - 4.6 \times 2.7 = 63.7\ cm^2$

(2) $D_i = 6.985$

$\Lambda = 0.635$

$N = 23$

$M = 1000$

$D_i/D_o = 0.945$

$K = 0.066$

$\Delta p = 8.3 \times 10^{-3}\ atm$ per bellows.

Multiplying this by 200 which is the approximate number of dipoles (neglecting the quadrupoles) in the 17 cells across each refrigerator we would obtain a possible total bellows pressure difference of 1.7 atm, $\underline{if\ M = 1000\ g/sec}$.

In SSC Technical Note No. 90 (SSCL-N-746) we used 2 velocity heads per convolution $\left(\frac{16\alpha M^2}{\pi^2 \rho D_i^4}\left(1 - \left(\frac{D_i}{D_o}\right)^4\right)\right)$ to calculate pressure drops across bellows. As pointed out above, if $D_i \ll D_o$, this is not a bad estimate, but as $\frac{D_i}{D_o}$ increases disagreement becomes increasingly worse, amounting to an overestimate by a factor of 6 for the numerical examples given above. We thank W. Clay for pointing out this error to us.

We acknowledge valuable discussions with A.P. Meade and D.P. Brown.

Appendix: Pressure drop across interconnect

The upper part of the current interconnect design is shown schematically in fig.4. The lower part differs from the upper one only in the fact that the bus work is replaced by instrumentation of cross-section 2.3×2.7 cm. The bellows has 23 convolutions, a wall thickness of 0.04 cm, a pitch of 0.635 cm, a live length of 14.6 and installed length of 11.4 cm, an inner diameter of 7 cm. The sleeve has a length of 17.9 cm and an inner diameter of 9 cm. the pipes have a total length of 32.9 cm.

Calculations (p.11) for a pipe diameter of 7 cm indicate that the pressure drop across the interconnect is 4.3×10^{-5} atm which is about 16 times less than the pressure drop across a magnet and the two end plates. The pressure drop across the bellows and at the transition between pipes and end domes are the largest components of the total pressure drop which is quite small.

When the pipe diameter is reduced to 5 cm the pressure drop across the interconnect is half that across the dipole (with end plates) (p.12). The major contributions to this pressure drop come from head losses at pipe transitions to end domes, sleeve and bellows. Friction and bellows have negligeable effects. The area available to flow passage is reduced by more than the ratio of the diameters squared (7 cm/5 cm) because of the presence of the bus work.

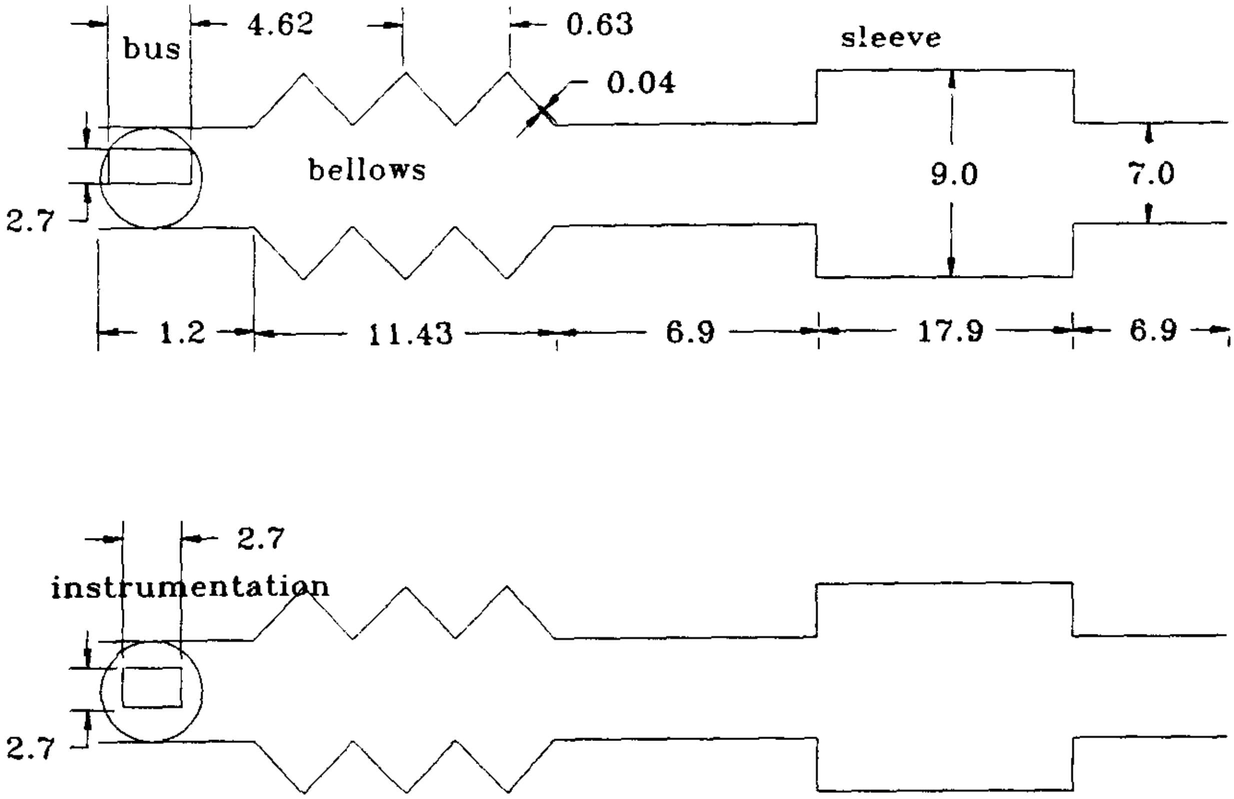


Figure 4. Upper and lower interconnections

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SSC interconnect pressure drop, mass flow g/sec: 100.0

Table 3: Δp (atm) interconnection

pressure drop	upper interconnect	lower interconnect
mass flow g/s	43.3	56.7
tank / pipe	2.027E-05	2.274E-05
pipe /bellows	0.000E+00	0.000E+00
pipe / sleeve	5.049E-06	4.781E-06
friction pipe	9.158E-07	6.638E-07
friction bellows	7.003E-07	5.032E-07
friction sleeve	1.668E-07	1.681E-07
bellows	1.540E-05	1.364E-05

Table 4: Total pressure drops (atm)

bus cm	instr cm	pipe ID cm	bellows ID cm	interconnect	dipole
4.6×2.7	2.3×2.7	7.0	7.0	4.251E-05	6.926E-04

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SSC interconnect pressure drop, mass flow g/sec: 100.0

Table 1: Δp (atm) interconnection

pressure drop	upper interconnect	lower interconnect
mass flow g/s	33.4	66.6
tank / pipe	1.604E-04	1.823E-04
pipe /bellows	7.198E-05	5.784E-05
pipe / sleeve	9.426E-05	8.906E-05
friction pipe	2.224E-05	9.549E-06
friction bellows	4.402E-07	6.726E-07
friction sleeve	1.050E-07	2.246E-07
bellows	9.149E-06	1.884E-05

Table 2: Total pressure drops (atm)

bus cm	instr cm	pipe ID cm	bellows ID cm	interconnect	dipole
4.6× 2.7	2.3× 2.7	5.0	7.0	3.586E-04	6.926E-04

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