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Study on the electromagnetic properties of the $[sc][\bar{q}\bar{b}]$ and $[sc][\bar{s}\bar{b}]$ states with $J^P = 1^+$

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Abstract A systematic study of the electromagnetic properties of exotic states is conducted to elucidate their nature, which continues to be the subject of controversy and incomplete understanding in the field. In this study, the magnetic dipole and quadrupole moments of the tetraquarks $[sc][\bar{q}\bar{b}]$ and $[sc][\bar{s}\bar{b}]$ with $J^P = 1^+$ are extracted in the context of a compact diquark-antidiquark configuration with the help of the QCD light-cone rules method. The magnetic dipole moments are given as $\mu_{[sc][\bar{u}\bar{b}]} = -2.12^{+0.74}_{-0.59}\mu_N$, $\mu_{[sc][\bar{d}\bar{b}]} = 1.66^{+0.60}_{-0.46}\mu_N$, and $\mu_{[sc][\bar{s}\bar{b}]} = 2.01^{+0.61}_{-0.50}\mu_N$. The order of magnitude of the magnetic dipole moments would suggest that these outcomes may be achievable in forthcoming experiments. The magnetic and quadrupole moments of hadrons represent another important observable, along with their mass and decay width, contributing to our understanding of the underlying quark structure and dynamics. Therefore, we hope that the results of this study will prove useful in theoretical and experimental investigations, which we anticipate will be an interesting research topic.

1 Motivation

While the idea of hadrons with more intricate structures than mesons with $q\bar{q}$ and baryons with qqq has been known for a long time, the confirmation of the existence of the exotic state designated X(3872) employing experimental evidence was achieved by the Belle Collaboration in 2003 [1]. Since then, numerous exotic hadron candidates have been discovered by various collaborations. With each new observation, the family of such exotic states is expanding and enriching. The experimental discovery of these exotic states represents a dynamic and vibrant field within hadron physics, encompassing both experimental and theoretical methodologies. The most recent developments in the field of exotic states, as well as experimental and theoretical works, are listed in Refs. [2–17]. Nevertheless, it is crucial to acknowledge that despite the extensive theoretical and experimental investigations conducted, the internal structure, decay characteristics, and quantum numbers of these exotic states remain among the pivotal questions that remain unanswered.

Most of the experimental explorations of exotic states to date have yielded either hidden-charm/bottom or doubly-charm outcomes. Theoretical investigations have also addressed the possibility of states comprising entirely different types of quarks. The exotic mesons, which are composed of four distinct quarks, have already garnered the attention of researchers. The relevant activities commenced with the announcement by the D0 collaboration regarding the resonance X(5568), which is presumed to be composed of quarks $su\overline{bd}$ [18, 19]. However, the existence of this state could not be confirmed by collaborations such as LHCb, CMS, and ATLAS. In 2021, two exotic states with minimal quark contents $[\bar{c}\bar{s}][ud]$, were reported by LHCb Collaboration utilizing full amplitude analysis $B^+ \to D^+ D^- K^+$ decays [20, 21]. The masses and widths of these states are measured as

$$\mathrm{X}_0(2900): M = 2866 \pm 7 \pm 2 \text{ MeV}, \quad \Gamma = 57 \pm 12 \pm 4 \text{ MeV},$$
$$\mathrm{X}_1(2900): M = 2904 \pm 5 \pm 1 \text{ MeV}, \quad \Gamma = 110 \pm 11 \pm 4 \text{ MeV}.$$

The LHCb Collaboration predicted that $\mathrm{X}_0(2900)$ and $\mathrm{X}_1(2900)$ have the quantum numbers $J^P = 0^+$ and 1^-, respectively. Recently, the LHCb analysis revised the findings of the initial analysis, thereby substantiating the existence of these particles. The mass and decay width values obtained from this new analysis are as follows [22]:

$$\mathrm{X}_0(2900): M = 2914 \pm 11 \pm 15 \text{ MeV}, \quad \Gamma = 128 \pm 22 \pm 23 \text{ MeV},$$
$$\mathrm{X}_1(2900): M = 2887 \pm 8 \pm 6 \text{ MeV}, \quad \Gamma = 92 \pm 16 \pm 16 \text{ MeV}.$$

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In 2022, the LHCb collaboration discovered two new states $T^{a}_{cs0}(2900)^0$ and $T^{a}_{cs0}(2900)^{++}$ in the processes $B^0 \to \overline{D}^0 D_s^+ \pi^-$ and $B^+ \to D^- D_s^+ \pi^+$ [23, 24], respectively. They were observed in the $D_s^+\pi^-$ and $D_s^+\pi^+$ mass distributions, and are states with spin-parity $J^P = 0^+$. From the analysis of the decay channels of $T^{a0/++}_{cs0}$, it becomes clear that they are fully open-flavor four-quark systems of $cd\bar{s}\bar{u}/cu\bar{s}\bar{d}$. The resonant parameters of these particles are consistent with one another, indicating that they belong to an isospin triplet. This is a significant finding, as it marks the first observation of an isospin triplet of exotic mesons comprising four distinct quark flavors. The discovery of the T^{a++}_{cs0} tetraquark represents a significant achievement, marking the first time a doubly charged exotic meson has been identified experimentally. The LHCb collaboration has recently announced the observation of a particular state in the $D^0 K^0_S$ final state of the $B^- \to D^- D^- K^0_s$ decay, the obtained mass and decay width of this state are consistent with the T^{a0}_{cs0} [25]. This result confirms the existence of the T^{a0}_{cs0} state in a new decay mode. It is worth noting that the Q, s, u, and d quarks can form several different exotic state categories, the properties of which would benefit from further examination. Before and after these experimental observations, many studies of open-flavor systems have been carried out and predictions of possible states such as mass, decay width, and production mechanisms have been made. Further details may be found in the following reviews: [2–17].

In addition to the above-mentioned states that have been identified through experimental research, there is a strong possibility that there are tetraquark states with varying structures and compositions. Another class of exotic states, tetraquarks involving heavy $c\bar{b}$ diquarks, is likely to exist. It is important to note that these states have not been discovered experimentally. However, there are theoretical studies in the literature that investigate their existence and study their properties [26–36]. One of the primary reasons for this phenomenon is that tetraquarks constructed from $c\bar{b}$ may exhibit stability against strong and electromagnetic decays. The study of these systems has revealed that, in contrast to hidden-charm and hidden-bottom tetraquark states, these tetraquark states are inherently stable, i.e., their masses lying below the corresponding meson-meson $B_c^+ K^*$ and $B_s^* D$ thresholds, displaying narrow widths. This is because they cannot annihilate into gluons in a manner distinct from other tetraquark states. As a consequence of these properties, they represent a valuable resource for the study of heavy-quark dynamics and a deeper understanding of the dynamics of QCD. Further research is required from a range of theoretical viewpoints to gain comprehensive insight into the properties of tetraquarks with $c\bar{b}$, which will help future experimental studies. In addition to the mass and decay properties of these states, it is crucial to elucidate the substructure and geometric shape of tetraquarks with $c\bar{b}$. It would be advantageous to examine their electromagnetic and radiative transition properties to gain a more profound comprehension of their intrinsic nature. The literature contains several studies on the electromagnetic properties of tetraquarks, which are designed to elucidate their internal structure [37–58].

The purpose of this study is to systematically extract magnetic dipole and quadrupole moments of the $[sc][\bar{q}\bar{b}]$ and $[sc][\bar{s}\bar{b}]$ tetraquarks. To this end, a compact diquark-antiquark interpolating current with $J^P = 1^+$ spin-parity quantum numbers is constructed. Magnetic dipole and quadrupole moments are parameters of the non-perturbative domain of QCD. Therefore, to extract these parameters a reliable method, such as the QCD light-cone sum rules (LCSR) method, is required. The LCSR method yields highly efficacious and predictable outcomes and represents a robust non-perturbative approach for investigating the dynamic and static attributes of standard and exotic hadron sectors. The LCSR approach entails calculating the correlation function in two distinct representations: one about hadrons (referred to as the "hadronic part") and the other to quark-gluon degrees of freedom (referred to as the "QCD part"). These two representations are then matched and sum rules are derived for the respective physical quantities.

This paper is organized as follows. After the Introduction, the formalism of QCD light-cone sum rules for the multipole moments of $[sc][\bar{q}\bar{b}]$ and $[sc][\bar{s}\bar{b}]$ tetraquarks with $J^P = 1^+$ are briefly introduced in Sec. 2. We present the numerical results and discussions for these states in Sec. 3. The final section is dedicated to our concluding remarks.

2 Deriving the QCD light-cone sum rules for multipole moments

The multipole moments of the $[sc][\bar{q}\bar{b}]$ and $[sc][\bar{s}\bar{b}]$ ($Z_{c\bar{b}}$ for short) tetraquarks are obtained by the LCSR technique through the construction of a correlation function expressed as follows:

$$\Pi_{\alpha\beta}(p,q) = i \int d^4x\, e^{ip\cdot x} \langle 0|\mathcal{T}\{J^i_\alpha(x) J^{i\dagger}_\beta(0)\}|0\rangle_\gamma, \tag{1}$$

where $J^i_{\alpha(\beta)}(x)$ are the interpolating currents and the subindex γ represents the external electromagnetic field. The interpolating currents for the considered multiquark systems are given by $J^i_{\alpha(\beta)}(x)$ with quantum numbers $J^P = 1^+$, which are expressed as follows:

$$J^1_\alpha(x) = \big[s^{a^T}(x) C\gamma_5 c^b(x)\big]\big[\bar{q}^a(x)\gamma_\alpha C \bar{b}^{b^T}(x) + \bar{q}^b(x)\gamma_\alpha C \bar{b}^{a^T}(x)\big], \tag{2}$$

$$J^2_\alpha(x) = \big[s^{a^T}(x) C\gamma_5 c^b(x)\big]\big[\bar{s}^a(x)\gamma_\alpha C \bar{b}^{b^T}(x) + \bar{s}^b(x)\gamma_\alpha C \bar{b}^{a^T}(x)\big], \tag{3}$$

where a and b are color indices, C is the charge conjugation operator and; $q(x)$ denotes the $u(x)$ or $d(x)$.

At the hadronic level, we plug into the correlation function a complete set of intermediate hadronic states with the same quantum numbers as the interpolating currents. Inserting the intermediate states, and isolating the ground state contributions yield the following result:

$$\Pi^{Had}_{\alpha\beta}(p,q) = \frac{\langle 0 \mid J_\alpha(x) \mid Z_{c\bar{b}}(p)\rangle}{p^2 - m^2_{Z_{c\bar{b}}}} \langle Z_{c\bar{b}}(p) \mid Z_{c\bar{b}}(p+q)\rangle_\gamma \frac{\langle Z_{c\bar{b}}(p+q) \mid J^\dagger_\beta(0) \mid 0\rangle}{(p+q)^2 - m^2_{Z_{c\bar{b}}}} + \text{higher states}, \tag{4}$$

The $\langle 0 \mid J_\alpha(x) \mid Z_{c\bar{b}}(p)\rangle$ in Eq. (4) is expressed regarding hadron characters, as indicated below

$$\langle 0 \mid J_\alpha(x) \mid Z_{c\bar{b}}(p)\rangle = \lambda_{Z_{c\bar{b}}} \varepsilon^\theta_\alpha, \tag{5}$$

with $\lambda_{Z_{c\bar{b}}}$ and $\varepsilon^\theta_\alpha$ are the pole residue and the polarization vector of multiquark systems under question, respectively. In Eq. (4), there is another matrix element, known as the radiative transition matrix element, which is expressed as [59]

$$\langle Z_{c\bar{b}}(p,\varepsilon^\theta) \mid Z_{c\bar{b}}(p+q,\varepsilon^\delta)\rangle_\gamma = -\varepsilon^\tau (\varepsilon^\theta)^\mu (\varepsilon^\delta)^\nu \Bigg[G_1(Q^2)\,(2p+q)_\tau\, g_{\mu\nu} + G_2(Q^2)\,(g_{\tau\nu}\, q_\mu - g_{\tau\mu}\, q_\nu) - \frac{1}{2m^2_{Z_{c\bar{b}}}} G_3(Q^2)\,(2p+q)_\tau q_\mu q_\nu \Bigg], \tag{6}$$

where $G_i(Q^2)$'s being form factors of the corresponding transition, with $Q^2 = -q^2$. The correlation function is derived from Eqs. (4–6) in the following manner:

$$\begin{aligned}
\Pi^{Had}_{\alpha\beta}(p,q) = {} & \frac{\varepsilon_\rho\, \lambda^2_{Z_{c\bar{b}}}}{[m^2_{Z_{c\bar{b}}} - (p+q)^2][m^2_{Z_{c\bar{b}}} - p^2]} \Bigg\{ G_1(Q^2)(2p+q)_\rho \Bigg[g_{\alpha\beta} - \frac{p_\alpha p_\beta}{m^2_{Z_{c\bar{b}}}} - \frac{(p+q)_\alpha (p+q)_\beta}{m^2_{Z_{c\bar{b}}}} + \frac{(p+q)_\alpha p_\beta}{2m^4_{Z_{c\bar{b}}}} \\
& \times (Q^2 + 2m^2_{Z_{c\bar{b}}}) \Bigg] + G_2(Q^2) \Bigg[q_\alpha g_{\rho\beta} - q_\beta g_{\rho\alpha} - \frac{p_\beta}{m^2_{Z_{c\bar{b}}}} (q_\alpha p_\rho - \frac{1}{2} Q^2 g_{\alpha\rho}) + \frac{(p+q)_\alpha}{m^2_{Z_{c\bar{b}}}} (q_\beta (p+q)_\rho + \frac{1}{2} Q^2 g_{\beta\rho}) \\
& - \frac{(p+q)_\alpha p_\beta p_\rho}{m^4_{Z_{c\bar{b}}}} Q^2 \Bigg] - \frac{G_3(Q^2)}{m^2_{Z_{c\bar{b}}}} (2p+q)_\rho \Bigg[q_\alpha q_\beta - \frac{p_\alpha q_\beta}{2m^2_{Z_{c\bar{b}}}} Q^2 + \frac{(p+q)_\alpha q_\beta}{2m^2_{Z_{c\bar{b}}}} Q^2 - \frac{(p+q)_\alpha q_\beta}{4m^4_{Z_{c\bar{b}}}} Q^4 \Bigg] \Bigg\}.
\end{aligned} \tag{7}$$

The magnetic ($F_M(Q^2)$) and quadrupole ($F_\mathcal{D}(Q^2)$) form factors can be derived in the form of the previously defined form factors, $G_i(Q^2)$, as follows:

$$\begin{aligned}
F_M(Q^2) &= G_2(Q^2), \\
F_\mathcal{D}(Q^2) &= G_1(Q^2) - G_2(Q^2) + \Big(1 + \frac{Q^2}{4m^2_{Z_{c\bar{b}}}}\Big) G_3(Q^2).
\end{aligned} \tag{8}$$

In the static limit ($Q^2 = 0$), where we are dealing with the real photon, the $F_M(0)$ and $F_\mathcal{D}(0)$ can be described as the magnetic moment (μ) and quadrupole ($\mathcal{D}$) moments by the following relationship:

$$\mu = \frac{e}{2m_{Z_{c\bar{b}}}} F_M(0), \tag{9}$$

$$\mathcal{D} = \frac{e}{m^2_{Z_{c\bar{b}}}} F_\mathcal{D}(0). \tag{10}$$

The results of the analysis at the hadron level are extracted in the form of a representation of the physical parameters under study. To obtain a description of the analysis at the quark-gluon level, it is necessary to proceed with further calculations.

At the quark-gluon level, the correlation function can be evaluated by the operator product expansion (OPE) technique and expressed in terms of photon distribution amplitudes. Upon completion of elementary mathematical operations, the following outcome for the $Z_{c\bar{b}}$ states is derived:

$$\begin{aligned}
\Pi^{\text{QCD}-J^1_\alpha}_{\alpha\beta}(p,q) = i \int d^4x e^{ip\cdot x} \langle 0 \mid \Big\{ & \text{Tr}\Big[\gamma_\alpha \widetilde{S}^{b'b}_b(-x) \gamma_\beta S^{a'a}_q(-x)\Big] \text{Tr}\Big[\gamma_5 \widetilde{S}^{aa'}_q(x) \gamma_5 S^{bb'}_c(x)\Big] \\
& + \text{Tr}\Big[\gamma_\alpha \widetilde{S}^{a'b}_b(-x) \gamma_\beta S^{b'a}_q(-x)\Big] \text{Tr}\Big[\gamma_5 \widetilde{S}^{aa'}_q(x) \gamma_5 S^{bb'}_c(x)\Big] \\
& + \text{Tr}\Big[\gamma_\alpha \widetilde{S}^{b'a}_b(-x) \gamma_\beta S^{a'b}_q(-x)\Big] \text{Tr}\Big[\gamma_5 \widetilde{S}^{aa'}_q(x) \gamma_5 S^{bb'}_c(x)\Big] \\
& + \text{Tr}\Big[\gamma_\alpha \widetilde{S}^{a'a}_b(-x) \gamma_\beta S^{b'b}_q(-x)\Big] \text{Tr}\Big[\gamma_5 \widetilde{S}^{aa'}_q(x) \gamma_5 S^{bb'}_c(x)\Big] \Big\} \mid 0\rangle_\gamma,
\end{aligned} \tag{11}$$

$$\Pi^{\mathrm{QCD}-J_\alpha^2}_{\alpha\beta}(p,q) = i\int d^4x e^{ip\cdot x}\langle 0 \mid \Big\{ \mathrm{Tr}\Big[\gamma_\alpha \widetilde{S}_b^{b'b}(-x)\gamma_\beta S_s^{a'a}(-x)\Big]\mathrm{Tr}\Big[\gamma_5 \widetilde{S}_s^{aa'}(x)\gamma_5 S_c^{bb'}(x)\Big]$$
$$+ \mathrm{Tr}\Big[\gamma_\alpha \widetilde{S}_b^{a'b}(-x)\gamma_\beta S_s^{b'a}(-x)\Big]\mathrm{Tr}\Big[\gamma_5 \widetilde{S}_s^{aa'}(x)\gamma_5 S_c^{bb'}(x)\Big]$$
$$+ \mathrm{Tr}\Big[\gamma_\alpha \widetilde{S}_b^{b'a}(-x)\gamma_\beta S_s^{a'b}(-x)\Big]\mathrm{Tr}\Big[\gamma_5 \widetilde{S}_s^{aa'}(x)\gamma_5 S_c^{bb'}(x)\Big]$$
$$+ \mathrm{Tr}\Big[\gamma_\alpha \widetilde{S}_b^{a'a}(-x)\gamma_\beta S_s^{b'b}(-x)\Big]\mathrm{Tr}\Big[\gamma_5 \widetilde{S}_s^{aa'}(x)\gamma_5 S_c^{bb'}(x)\Big]\Big\} \mid 0\rangle_\gamma, \tag{12}$$

where the $S_Q(x)$and $S_q(x)$are the propagators of heavy and light quarks with $\widetilde{S}^{ij}_{Q(q)}(x) = CS^{ij\mathrm{T}}_{Q(q)}(x)C$. The explicit forms of these propagators are defined as follows: [60, 61],

$$S_q(x) = S_q^{free}(x) - \frac{\langle\bar{q}q\rangle}{12}\Big(1 - i\frac{m_q \not{x}}{4}\Big) - \frac{\langle\bar{q}q\rangle}{192}m_0^2 x^2\Big(1 - i\frac{m_q\not{x}}{6}\Big) + \frac{i g_s\, G^{\alpha\beta}}{32\pi^2 x^2}\Big[/x\sigma_{\alpha\beta} + \sigma_{\alpha\beta}/x\Big], \tag{13}$$

$$S_Q(x) = S_Q^{free}(x) - \frac{m_Q\, g_s\, G^{\alpha\beta}}{32\pi^2}\Big[(\sigma_{\alpha\beta}\not{x} + \not{x}\sigma_{\alpha\beta})\frac{K_1(m_Q\sqrt{-x^2})}{\sqrt{-x^2}} + 2\sigma_{\alpha\beta}K_0(m_Q\sqrt{-x^2})\Big], \tag{14}$$

with

$$S_q^{free}(x) = \frac{1}{2\pi x^2}\Big(i\frac{\not{x}}{x^2} - \frac{m_q}{2}\Big), \tag{15}$$

$$S_Q^{free}(x) = \frac{m_Q^2}{4\pi^2}\Big[\frac{K_1(m_Q\sqrt{-x^2})}{\sqrt{-x^2}} + i\frac{\not{x}\, K_2(m_Q\sqrt{-x^2})}{(\sqrt{-x^2})^2}\Big], \tag{16}$$

where m_0 being the quark-gluon mixed condensate with $m_0^2 = \langle 0 \mid \bar{q}\, g_s\, \sigma_{\alpha\beta}\, G^{\alpha\beta}\, q \mid 0\rangle / \langle\bar{q}q\rangle$, $G^{\alpha\beta}$ is the gluon field strength tensor, and K_i's are the modified Bessel functions of the second type.

To perform further calculations at the quark-gluon level, it is necessary to consider two distinct contributions from the short- and long-distance interactions of the photon with quarks. To ascertain the nature of the short-distance interactions, it is necessary to proceed with the subsequent replacement as described below:

$$S^{free}(x) \longrightarrow \int d^4z\, S^{free}(x-z)\, /A(z)\, S^{free}(z)\,. \tag{17}$$

To include long-distance interactions in the analysis, it is convenient to apply the formula below:

$$S^{ab}_{\alpha\beta}(x) \longrightarrow -\frac{1}{4}\big[\bar{q}^a(x)\Gamma_i q^b(0)\big](\Gamma_i)_{\alpha\beta}, \tag{18}$$

where $\Gamma_i = \{\mathbf{1}, \gamma_5, \gamma_\alpha, i\gamma_5\gamma_\alpha, \sigma_{\alpha\beta}/2\}$. In calculating short- and long-distance contributions, we employ a single propagator in the equation, whereas the remaining propagators are treated as full propagators. Upon the inclusion of long-distance contributions in the analysis, matrix elements such as $\langle\gamma(q)\big|\bar{q}(x)\Gamma_i G_{\alpha\beta}q(0)\big|0\rangle$ and $\langle\gamma(q)|\bar{q}(x)\Gamma_i q(0)|0\rangle$ emerge. The aforementioned matrix elements, expressed in the form of photon wave functions, are crucial parameters in the calculation of long-distance interactions [62]. It is important to note that the photon distribution amplitudes (DAs) utilized in this study include contributions solely from light quarks. However, in principle, a photon can be emitted long-distance from heavy quarks. However, such long-distance photon emission from heavy quarks is highly suppressed owing to the large mass of the heavy quarks. This kind of contribution is neglected in our analysis. As explained in Eq. (17), only the short-distance photon emission from heavy quarks is considered. For this reason, DAs including heavy quarks are not considered in our analysis. The aforementioned procedures have yielded the quark-gluon correlation function, which is acquired via the use of quark-gluon parameters and photon DAs.

The above-mentioned procedures give the sum rules for the magnetic and quadrupole moments of the $[sc][\bar{q}\bar{b}]$ and $[sc][\bar{s}\bar{b}]$ tetraquarks as follows,

$$\mu_{Z_{c\bar{b}}}^{J_\alpha^1} = \frac{e^{\frac{m^2_{Z_{c\bar{b}}}}{\mathrm{M}^2}}}{\lambda^2_{Z_{c\bar{b}}}}\,\rho_1(\mathrm{M}^2,\mathrm{s}_0), \quad \mathcal{D}_{Z_{c\bar{b}}}^{J_\alpha^1} = m^2_{Z_{c\bar{b}}}\frac{e^{\frac{m^2_{Z_{c\bar{b}}}}{\mathrm{M}^2}}}{\lambda^2_{Z_{c\bar{b}}}}\,\rho_2(\mathrm{M}^2,\mathrm{s}_0), \tag{19}$$

$$\mu_{Z_{c\bar{b}}}^{J_\alpha^2} = \frac{e^{\frac{m^2_{Z_{c\bar{b}}}}{\mathrm{M}^2}}}{\lambda^2_{Z_{c\bar{b}}}}\,\rho_3(\mathrm{M}^2,\mathrm{s}_0), \quad \mathcal{D}_{Z_{c\bar{b}}}^{J_\alpha^2} = m^2_{Z_{c\bar{b}}}\frac{e^{\frac{m^2_{Z_{c\bar{b}}}}{\mathrm{M}^2}}}{\lambda^2_{Z_{c\bar{b}}}}\,\rho_4(\mathrm{M}^2,\mathrm{s}_0). \tag{20}$$

As the explicit forms of $\rho_i(\mathrm{M}^2,\, \mathrm{s}_0)$ are similar, for illustrative purposes, the result of $\rho_1(\mathrm{M}^2,\, \mathrm{s}_0)$ is provided Appendix A.

It is important to note that the derivation of Eq. (4) is contingent upon the approximation that the hadron level of the QCD light-cone sum rules can be adequately approximated by a single pole. About the tetra- or pentaquark states, it is essential to validate the aforementioned approximation through the introduction of additional arguments. This is because the hadron level of the QCD sum rules encompasses potential contributions from the two-meson intermediate states [63–66]. Thus, it may be important to consider the contribution of two-meson intermediate states when attempting to extract the parameters related to tetra- or pentaquark states. Such contributions may be deducted from the QCD sum rules or incorporated into the parameters of the pole term. The first scheme has been used in the study of pentaquarks [67–69]; while, the second scheme has been utilized to extract tetraquarks [70–77]. In the context presented herein, it is of the utmost importance that the quark propagator be modified in a manner consistent with the following equation:

$$\frac{1}{m^2 - p^2} \longrightarrow \frac{1}{m^2 - p^2 - i\sqrt{p^2}\,\Gamma(p)}, \tag{21}$$

where $\Gamma(p)$ stands for the finite width of the multiquark systems produced by the intermediate two-meson effects. Once these contributions are appropriately incorporated into the QCD sum rules, it is demonstrated that they exert an influence on the physical observables to the extent of approximately $(5-7)\%$ [70–77], and that these effects could not exceed the intrinsic errors associated with the QCD sum rule calculations. It is reasonable to posit that the results of the electromagnetic characteristics of the tetra- and pentaquarks could remain unaffected by the contributions mentioned above. It is therefore reasonable to conclude that the effects of the two-meson intermediate states at the hadron level of the correlation function can be disregarded, and the zero-width single-pole scheme should be used instead.

Analytical expressions for the multipole moments of the $[sc][\bar{q}\bar{b}]$ and $[sc][\bar{s}\bar{b}]$ tetraquarks with $J^P = 1^+$ have been derived. The subsequent section will present a numerical analysis of these quantities.

3 Numerical analysis of the multipole moments

To conduct numerical calculations of QCD sum rules for multipole moments, several input quantities are required, as given in Table 1. In numerical analysis, we set $m_u = m_d = 0$ and $m_s^2 = 0$, but consider terms proportional to m_s. Another crucial input parameter in the numerical analysis is the photon DAs and the wave functions employed therein. These expressions and the input parameters used in their explicit forms are taken from Ref. [62] and for completeness, they are listed in the Appendix B.

Besides the aforementioned input parameters, two further parameters are required for the calculations: the continuum threshold parameter $\mathrm{s_0}$ and the Borel mass $\mathrm{M^2}$. To acquire reliable results from QCD sum rules, the dependence of the multipole moments on $\mathrm{s_0}$ and $\mathrm{M^2}$ should be relatively weak, which is called working windows. The working windows of these parameters are determined from the standard procedures of the method used. These procedures are known as pole dominance (PC) and convergence of the OPE (CVG), which are described by the following formulas:

$$\mathrm{PC} = \frac{\rho_i(\mathrm{M^2}, \mathrm{s_0})}{\rho_i(M^2, \infty)}, \tag{22}$$

$$\mathrm{CVG}\,(\mathrm{M^2}, \mathrm{s_0}) = \frac{\rho_i^{\mathrm{Dim}\,7}(\mathrm{M^2}, \mathrm{s_0})}{\rho_i(\mathrm{M^2}, \mathrm{s_0})}, \tag{23}$$

where $\rho_i^{\mathrm{Dim}\,7}(\mathrm{M^2}, \mathrm{s_0})$ stands for the highest dimensional term in the OPE of $\rho_i(\mathrm{M^2}, \mathrm{s_0})$. According to the sum rules analysis, the CVG must be sufficiently small to guarantee OPE convergence; while, the PC must be sufficiently large to maximize the efficiency

Table 1 Parameters employed as inputs in the calculations [32, 33, 78–80]

Parameter	Value	Unit
m_s	$93.4^{+8.6}_{-3.4}$	MeV
m_c	1.67 ± 0.07	GeV
m_b	4.78 ± 0.06	GeV
$m_{[sc][\bar{s}\bar{b}]}$	7.30 ± 0.77	GeV
$m_{[sc][\bar{q}\bar{b}]}$	7.18 ± 0.76	GeV
m_0^2	0.8 ± 0.1	GeV^2
$f_{3\gamma}$	-0.0039	GeV^2
$\langle\bar{q}q\rangle$	$(-0.24 \pm 0.01)^3$	GeV^3
$\langle\bar{s}s\rangle$	$0.8\,\langle\bar{q}q\rangle$	GeV^3
$\langle g_s^2 G^2\rangle$	0.48 ± 0.14	GeV^4
$\lambda_{[sc][\bar{q}\bar{b}]}$	0.035 ± 0.011	GeV^5
$\lambda_{[sc][\bar{s}\bar{b}]}$	0.046 ± 0.014	GeV^5

Table 2 Predicted multipole moments of the $[sc][\bar{q}\bar{b}]$ and $[sc][\bar{s}\bar{b}]$ states with $J^P = 1^+$

Tetraquarks	μ (μ_N)	$\mathcal{D}(\times 10^{-2})$ (fm^2)	M^2 (GeV^2)	s$_0$ (GeV^2)	PC (%)	CVG (%)
$[sc][\bar{u}\bar{b}]$	$-2.12^{+0.74}_{-0.59}$	$-0.66^{+0.15}_{-0.13}$	[4.0, 4.6]	[59, 62]	[53.3, 29.4]	< 2.0
$[sc][\bar{d}\bar{b}]$	$1.66^{+0.60}_{-0.46}$	$0.33^{+0.07}_{-0.07}$	[4.0, 4.6]	[59, 62]	[52.8, 29.1]	< 2.0
$[sc][\bar{s}\bar{b}]$	$2.01^{+0.61}_{-0.50}$	$0.37^{+0.07}_{-0.07}$	[4.4, 5.0]	[61, 64]	[51.7, 28.3]	< 2.0

of the single-pole scheme. Under the aforementioned prerequisites, one can obtain the working windows of the M^2 and s$_0$, which are presented in Table 2. As a result, in the same table, the values achieved for the PC and CVG are also listed. For completeness, Figs. 1 and 2 illustrate the variations of the extracted multipole moments of these states concerning M^2 and s$_0$. As illustrated by the presented figures, the multipole moments of these states show a relatively mild dependence on the parameter M^2. Although the multipole moments of these states exhibit some dependence on s$_0$, they remain within the limits permitted by this approach and constitute the primary source of uncertainty.

Predicted multipole moments of the $[sc][\bar{q}\bar{b}]$ and $[sc][\bar{s}\bar{b}]$ states with $J^P = 1^+$, which consider the uncertainties inherent in the input parameters and the variation in the M^2 and s$_0$ working windows, are presented in Table 2. Moreover, to enhance comprehension, we have provided the results of the magnetic moments, presented in conjunction with both their central values and the combined with errors, in Fig. 3. In light of the findings presented in this study, the following key points have been identified:

- Our analysis shows that the multipole moments of these states are governed by axial-vector diquark ($[\bar{q}\bar{b}]$/$[\bar{s}\bar{b}]$) component of the interpolating currents.
- When we analyze the individual quark contributions (which can be achieved through the charge factors e_q, e_c, and e_b), it becomes evident that the magnetic dipole moments are primarily influenced by light quarks, which account for approximately 74% of the total; while, heavy quarks represent the remaining 26%. In the case of the quadrupole moments, the total contributions come from the light quarks. Upon closer examination of the quadrupole moments, it becomes evident that the missing heavy-quark contributions are due to the terms containing heavy quarks exactly canceling each other out.
- The order of the magnetic dipole moments could be used to give an idea of the experimental accessibility of the corresponding physical quantities. Their order of magnitude would suggest that these outcomes may be achievable in forthcoming experiments.
- The quadrupole moment results obtained for these states are non-zero, indicating the presence of a non-spherical charge distribution. By the predicted signs, the geometric shape of the $[sc][\bar{u}\bar{b}]$ state is oblate; whereas, the $[sc][\bar{d}\bar{b}]$ and $[sc][\bar{s}\bar{b}]$ states are prolate.
- U-symmetry violation between the $[sc][\bar{d}\bar{b}]$ and $[sc][\bar{s}\bar{b}]$ states in case of the magnetic dipole moment is about 18%, while in the case of the quadrupole moment, it is roughly 12%. A reasonable U-symmetry violation is observed.

4 Concluding remarks

A systematic study of the electromagnetic properties of exotic states is conducted to elucidate their nature, which continues to be the subject of controversy and incomplete understanding in the field. This work estimates the multipole moments of possible $[sc][\bar{q}\bar{b}]$ and $[sc][\bar{s}\bar{b}]$ states within the context of the QCD light-cone sum rules. In calculating the multipole moments of the $[sc][\bar{q}\bar{b}]$ and $[sc][\bar{s}\bar{b}]$ states, the diquark-antidiquark configuration is taken into account with $J^P = 1^+$ spin-parity quantum numbers. The order of magnitude of the magnetic dipole moment demonstrates that it may be measured in an experiment. The predictions derived in this study can be checked through the use of other theoretical models, such as lattice QCD and chiral perturbation theory. The multipole moments of the $[sc][\bar{q}\bar{b}]$ and $[sc][\bar{s}\bar{b}]$ states reveal valuable knowledge concerning the internal structure, size, and shape of the hadrons. The determination of these parameters could be a significant step in our interpretation of hadron properties concerning quark-gluon degrees of freedom and in elucidating their nature. It is similarly crucial to ascertain the branching ratios of the various decay modes/channels of these tetraquark states. It would be highly intriguing to anticipate forthcoming experimental efforts that will investigate the potential existence of $[sc][\bar{q}\bar{b}]$ and $[sc][\bar{s}\bar{b}]$ states. These findings would then be subjected to rigorous testing to validate and expand upon the present results. To gain a more comprehensive understanding of these findings, further research is recommended.

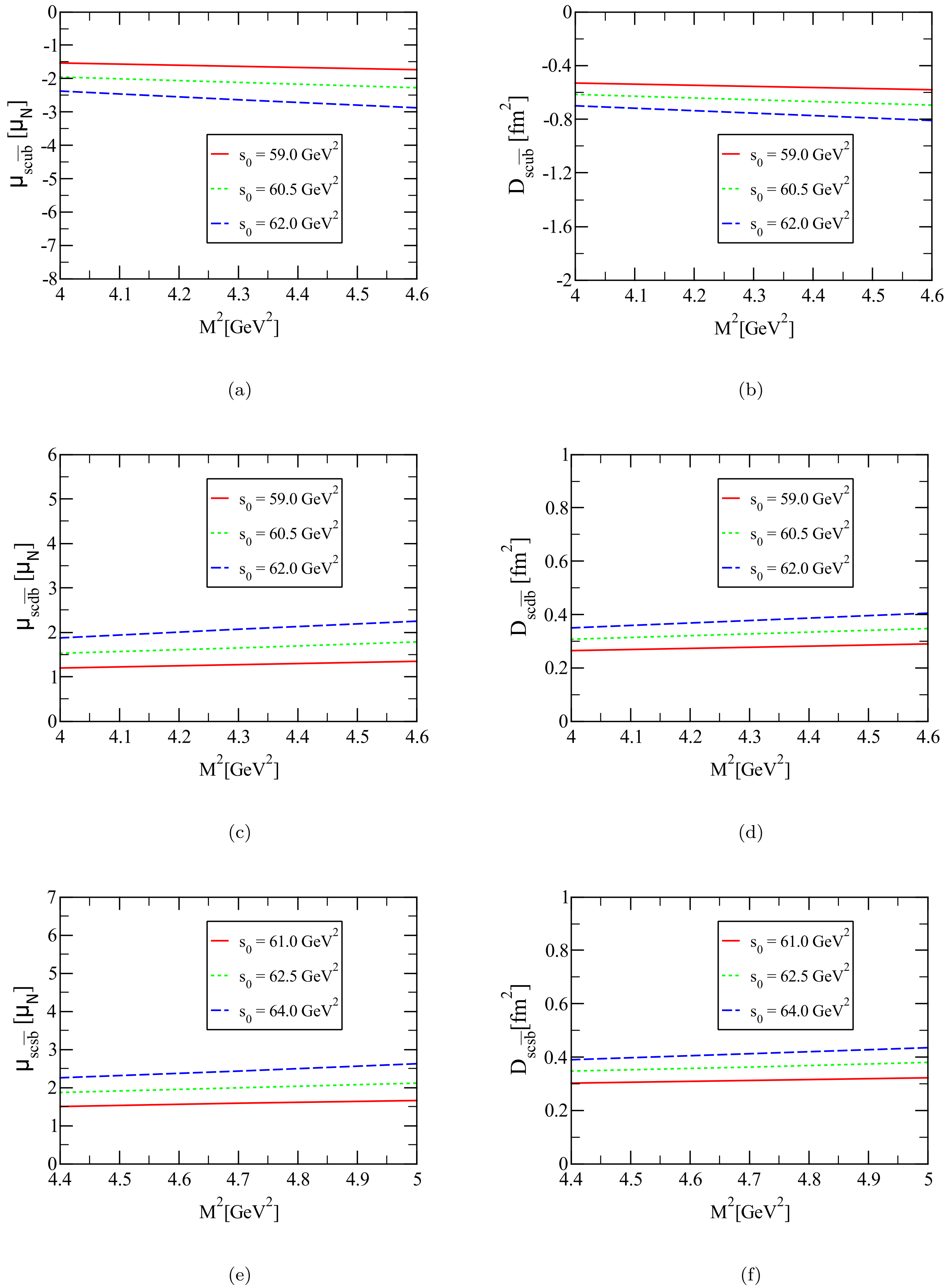
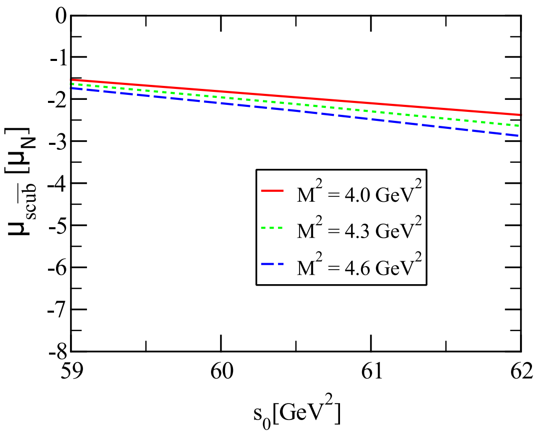
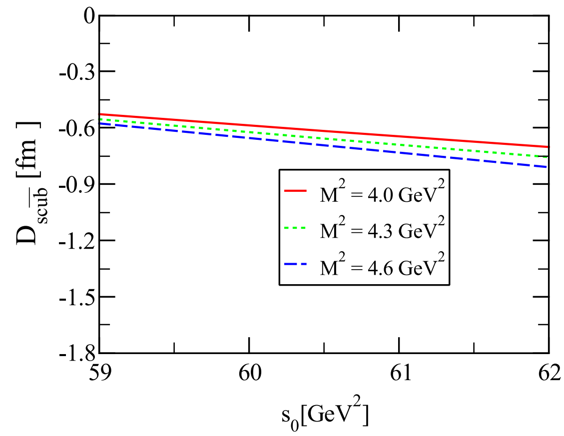


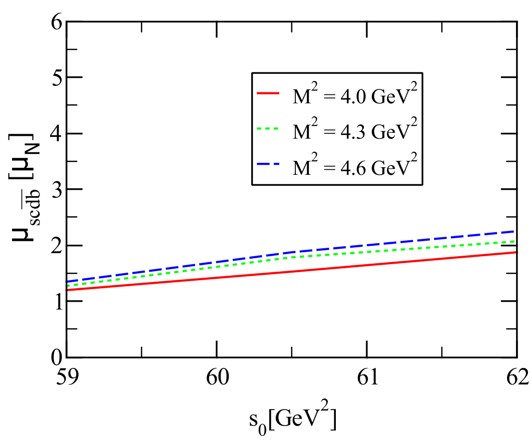
Fig. 1 Variation of multipole moments $[sc][\bar{q}\bar{b}]$ and $[sc][\bar{s}\bar{b}]$ states as a function of the M^2 at different values of s_0



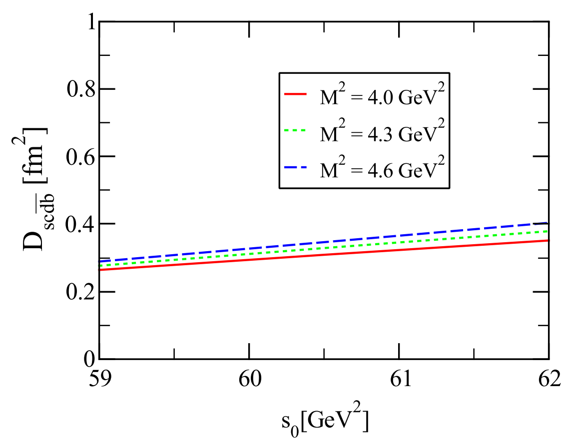
(a)



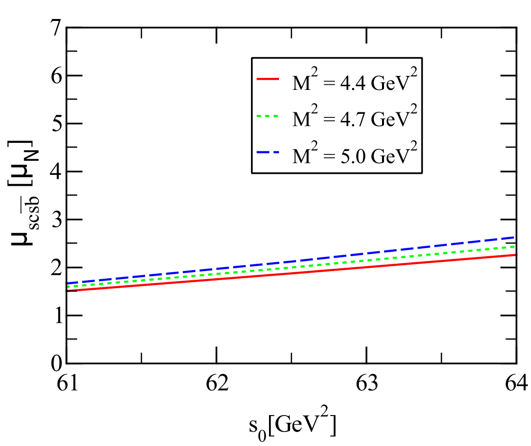
(b)



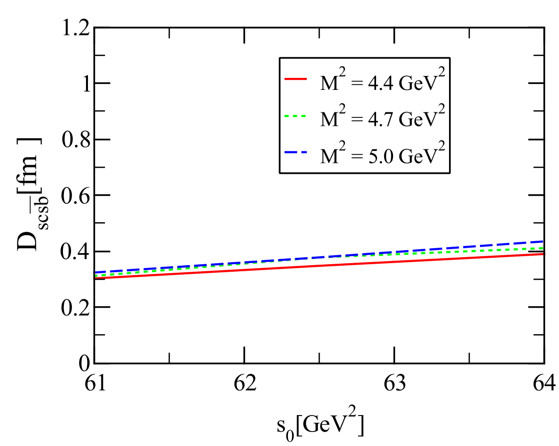
(c)



(d)



(e)



(f)

Fig. 2 Variation of multipole moments $[sc][\bar{q}\bar{b}]$ and $[sc][\bar{s}\bar{b}]$ states as a function of the s_0 at different values of M^2

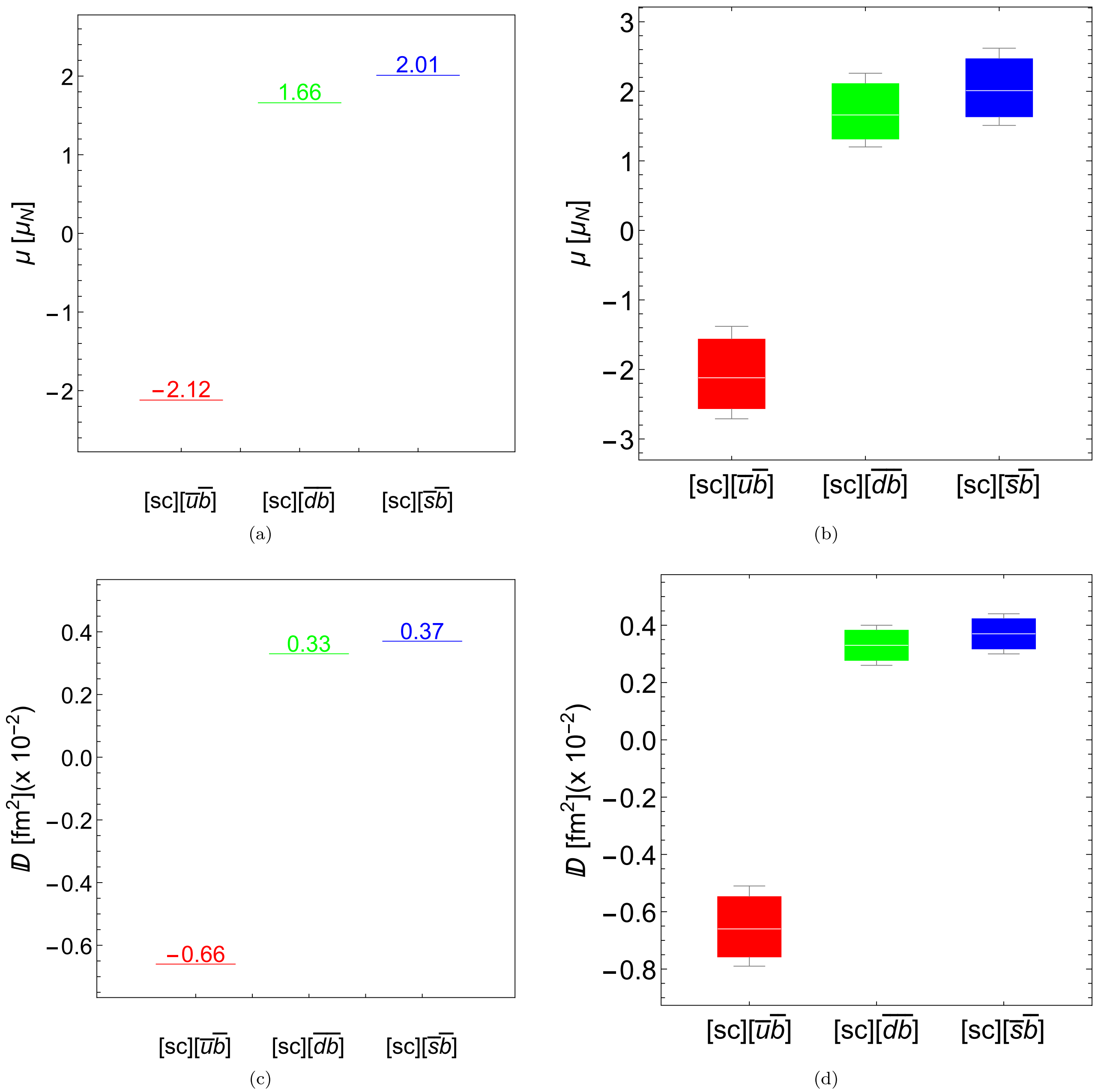


Fig. 3 The multipole moments of $Z_{c\bar{b}}$ tetraquark states: **a** and **c** for central values, and **b** and **d** for combined with errors, respectively

5 Appendix A: The explicit forms of $\rho_i(\mathrm{M}^2, \mathrm{s}_0)$ function

This appendix of the manuscript presents the explicit form of the $\rho_1(\mathrm{M}^2, \mathrm{s}_0)$ function as follows:

$$\rho_1(\mathrm{M}^2, \mathrm{s}_0) = -\frac{1}{2^{19} \times 3^2 \times 5^2 \times 7\pi^5}\Bigg[9e_q\Big(-4I[0,6] + 42m_c m_s(I[0,5] + 15I[1,4]) - 129I[1,5]\Big) + 14e_b\Big(38m_c m_s I[0,5] - 6I[0,6] + 130m_c m_s I[1,4] - 27I[1,5]\Big)\Bigg] + \frac{\langle g_s^2 G^2\rangle\langle\bar{q}q\rangle}{2^{21} \times 3^4\pi^3}\Bigg[160(e_c - e_s)m_b(I[0,2] - 2I[1,1]) - 43e_q m_c(I_1[\mathcal{S}] + I_1[\tilde{\mathcal{S}}])I[0,2]\Bigg]$$

$$
\begin{aligned}
&- \frac{\langle g_s^2 G^2\rangle}{2^{22}\times 3^5\times 5\pi^5}\Bigg[5(e_c-e_s)(21I[0,4]-172I[1,3])-162e_q(I[0,4]-4I[1,3])-12e_b(3I[0,4]+32I[1,3])\Bigg]\\
&+\frac{\langle\bar q q\rangle}{2^{16}\times 3^3\times 5\pi^3}\Bigg[48e_b m_b(20m_c m_s I[0,3]-3I[0,4])+e_c m_c\Big((60m_c m_s I[0,3]-9I[0,4])(I_4[\mathcal{S}]-I_4[\tilde{\mathcal{S}}])\\
&-4(44m_c m_s I[0,3]-9I[0,4])I_5[h_\gamma]\Big)\Bigg]\\
&\frac{\langle\bar s s\rangle}{2^{18}\times 3^3\times 5\pi^3}\Bigg[16\Big(e_b m_c(-6I[0,4]+66m_0^2 I[1,2]-64I[1,3])+27e_q m_c(I[0,4]-4I[1,3])\\
&+9e_b m_s(I[0,4]-4I[1,3])\Big)+e_s m_c(I_2[\mathcal{S}]+44I_3[\mathcal{S}])I[0,4]\Bigg]\\
&+\frac{f_{3\gamma}}{2^{22}\times 3^3\times 5^2\times 7\pi^3}\Bigg[112e_q(55m_c m_s I[0,4]-9I[0,5])I_1[\mathcal{V}]+2511e_s I_2[\mathcal{V}]I[0,5]\Bigg], \qquad \text{(A1)}
\end{aligned}
$$

where the $I[n, m]$, and $I_i[\mathcal{F}]$ functions are expressed as:

$$
\begin{aligned}
I[n, m] &= \int_{\mathcal{M}}^{s_0} ds\, e^{-s/M^2}\, s^n\,(s-\mathcal{M})^m,\\
I_1[\mathcal{F}] &= \int D_{\alpha_i}\int_0^1 dv\, \mathcal{F}(\alpha_{\bar q},\alpha_q,\alpha_g)\delta'(\alpha_q+\bar v\alpha_g-u_0),\\
I_2[\mathcal{F}] &= \int D_{\alpha_i}\int_0^1 dv\, \mathcal{F}(\alpha_{\bar q},\alpha_q,\alpha_g)\delta'(\alpha_{\bar q}+v\alpha_g-u_0),\\
I_3[\mathcal{F}] &= \int D_{\alpha_i}\int_0^1 dv\, \mathcal{F}(\alpha_{\bar q},\alpha_q,\alpha_g)\delta(\alpha_q+\bar v\alpha_g-u_0),\\
I_4[\mathcal{F}] &= \int D_{\alpha_i}\int_0^1 dv\, \mathcal{F}(\alpha_{\bar q},\alpha_q,\alpha_g)\delta(\alpha_{\bar q}+v\alpha_g-u_0),\\
I_5[\mathcal{F}] &= \int_0^1 du\, \mathcal{F}(u),
\end{aligned}
$$

where $\mathcal{M}=(m_c+m_b+m_s)^2$ for the $[sc][\bar q\bar b]$ states and $\mathcal{M}=(m_c+m_b+2m_s)^2$ for the $[sc][\bar s\bar b]$ state; and $\mathcal{F}$ denotes the corresponding photon DAs.

6 Appendix B: Distribution amplitudes of the photon

In this Appendix, the matrix elements $\langle\gamma(q)|\bar q(x)\Gamma_i q(0)|0\rangle$ and $\langle\gamma(q)\big|\bar q(x)\Gamma_i G_{\mu\nu}q(0)\big|0\rangle$ associated with the photon DAs are presented as follows [62]:

$$
\begin{aligned}
&\langle\gamma(q)|\bar q(x)\gamma_\mu q(0)|0\rangle = e_q f_{3\gamma}\left(\varepsilon_\mu - q_\mu\frac{\varepsilon x}{qx}\right)\int_0^1 du e^{i\bar u qx}\psi^v(u)\\
&\langle\gamma(q)|\bar q(x)\gamma_\mu\gamma_5 q(0)|0\rangle = -\frac{1}{4}e_q f_{3\gamma}\epsilon_{\mu\nu\alpha\beta}\varepsilon^\nu q^\alpha x^\beta\int_0^1 du e^{i\bar u qx}\psi^a(u)\\
&\langle\gamma(q)|\bar q(x)\sigma_{\mu\nu}q(0)|0\rangle = -ie_q\langle\bar q q\rangle(\varepsilon_\mu q_\nu-\varepsilon_\nu q_\mu)\int_0^1 du e^{i\bar u qx}\left(\chi\varphi_\gamma(u)+\frac{x^2}{16}\mathbb{A}(u)\right)\\
&-\frac{i}{2(qx)}e_q\bar q q\left[x_\nu\left(\varepsilon_\mu-q_\mu\frac{\varepsilon x}{qx}\right)-x_\mu\left(\varepsilon_\nu-q_\nu\frac{\varepsilon x}{qx}\right)\right]\int_0^1 du e^{i\bar u qx}h_\gamma(u)\\
&\langle\gamma(q)|\bar q(x)g_s G_{\mu\nu}(vx)q(0)|0\rangle = -ie_q\langle\bar q q\rangle\big(\varepsilon_\mu q_\nu-\varepsilon_\nu q_\mu\big)\int\mathcal{D}\alpha_i e^{i(\alpha_{\bar q}+v\alpha_g)qx}\mathcal{S}(\alpha_i)\\
&\langle\gamma(q)|\bar q(x)g_s\tilde G_{\mu\nu}(vx)i\gamma_5 q(0)|0\rangle = -ie_q\langle\bar q q\rangle\big(\varepsilon_\mu q_\nu-\varepsilon_\nu q_\mu\big)\int\mathcal{D}\alpha_i e^{i(\alpha_{\bar q}+v\alpha_g)qx}\tilde{\mathcal{S}}(\alpha_i)\\
&\langle\gamma(q)|\bar q(x)g_s\tilde G_{\mu\nu}(vx)\gamma_\alpha\gamma_5 q(0)|0\rangle = e_q f_{3\gamma}q_\alpha(\varepsilon_\mu q_\nu-\varepsilon_\nu q_\mu)\int\mathcal{D}\alpha_i e^{i(\alpha_{\bar q}+v\alpha_g)qx}\mathcal{A}(\alpha_i)
\end{aligned}
$$

$$\langle\gamma(q)|\bar{q}(x)g_sG_{\mu\nu}(vx)i\gamma_\alpha q(0)|0\rangle = e_q f_{3\gamma} q_\alpha(\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu)\int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}}+v\alpha_g)qx}\mathcal{V}(\alpha_i)$$

$$\begin{aligned}
\langle\gamma(q)|\bar{q}(x)\sigma_{\alpha\beta}g_sG_{\mu\nu}(vx)q(0)|0\rangle &= e_q\langle\bar{q}q\rangle\Bigg\{\Bigg[\left(\varepsilon_\mu - q_\mu\frac{\varepsilon x}{qx}\right)\left(g_{\alpha\nu} - \frac{1}{qx}(q_\alpha x_\nu + q_\nu x_\alpha)\right)q_\beta \\
&- \left(\varepsilon_\mu - q_\mu\frac{\varepsilon x}{qx}\right)\left(g_{\beta\nu} - \frac{1}{qx}(q_\beta x_\nu + q_\nu x_\beta)\right)q_\alpha - \left(\varepsilon_\nu - q_\nu\frac{\varepsilon x}{qx}\right)\left(g_{\alpha\mu} - \frac{1}{qx}(q_\alpha x_\mu + q_\mu x_\alpha)\right)q_\beta \\
&+ \left(\varepsilon_\nu - q_\nu\frac{\varepsilon x}{q.x}\right)\left(g_{\beta\mu} - \frac{1}{qx}(q_\beta x_\mu + q_\mu x_\beta)\right)q_\alpha\Bigg]\int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}}+v\alpha_g)qx}\mathcal{T}_1(\alpha_i) \\
&+ \Bigg[\left(\varepsilon_\alpha - q_\alpha\frac{\varepsilon x}{qx}\right)\left(g_{\mu\beta} - \frac{1}{qx}(q_\mu x_\beta + q_\beta x_\mu)\right)q_\nu \\
&- \left(\varepsilon_\alpha - q_\alpha\frac{\varepsilon x}{qx}\right)\left(g_{\nu\beta} - \frac{1}{qx}(q_\nu x_\beta + q_\beta x_\nu)\right)q_\mu \\
&- \left(\varepsilon_\beta - q_\beta\frac{\varepsilon x}{qx}\right)\left(g_{\mu\alpha} - \frac{1}{qx}(q_\mu x_\alpha + q_\alpha x_\mu)\right)q_\nu \\
&+ \left(\varepsilon_\beta - q_\beta\frac{\varepsilon x}{qx}\right)\left(g_{\nu\alpha} - \frac{1}{qx}(q_\nu x_\alpha + q_\alpha x_\nu)\right)q_\mu\Bigg]\int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}}+v\alpha_g)qx}\mathcal{T}_2(\alpha_i) \\
&+ \frac{1}{qx}(q_\mu x_\nu - q_\nu x_\mu)(\varepsilon_\alpha q_\beta - \varepsilon_\beta q_\alpha)\int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}}+v\alpha_g)qx}\mathcal{T}_3(\alpha_i) \\
&+ \frac{1}{qx}(q_\alpha x_\beta - q_\beta x_\alpha)(\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu)\int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}}+v\alpha_g)qx}\mathcal{T}_4(\alpha_i)\Bigg\},
\end{aligned}$$

where $\varphi_\gamma(u)$ is the DA of leading twist-2, $\psi^v(u)$, $\psi^a(u)$, $\mathcal{A}(\alpha_i)$ and $\mathcal{V}(\alpha_i)$, are the twist-3 amplitudes, and $h_\gamma(u)$, $\mathbb{A}(u)$, $\mathcal{S}(\alpha_i)$, $\tilde{\mathcal{S}}(\alpha_i)$, $\mathcal{T}_1(\alpha_i)$, $\mathcal{T}_2(\alpha_i)$, $\mathcal{T}_3(\alpha_i)$ and $\mathcal{T}_4(\alpha_i)$ are the twist-4 photon DAs. The measure $\mathcal{D}\alpha_i$ is defined as

$$\int \mathcal{D}\alpha_i = \int_0^1 d\alpha_{\bar{q}}\int_0^1 d\alpha_q\int_0^1 d\alpha_g\delta(1-\alpha_{\bar{q}}-\alpha_q-\alpha_g).$$

The expressions of the DAs that are entered into the matrix elements above are as follows:

$$\begin{aligned}
\varphi_\gamma(u) &= 6u\bar{u}\left(1+\varphi_2(\mu)C_2^{\frac{3}{2}}(u-\bar{u})\right), \\
\psi^v(u) &= 3\big(3(2u-1)^2-1\big)+\frac{3}{64}\left(15w_\gamma^V-5w_\gamma^A\right)\big(3-30(2u-1)^2+35(2u-1)^4\big), \\
\psi^a(u) &= \big(1-(2u-1)^2\big)\big(5(2u-1)^2-1\big)\frac{5}{2}\left(1+\frac{9}{16}w_\gamma^V-\frac{3}{16}w_\gamma^A\right), \\
h_\gamma(u) &= -10\big(1+2\kappa^+\big)C_2^{\frac{1}{2}}(u-\bar{u}), \\
\mathbb{A}(u) &= 40u^2\bar{u}^2\big(3\kappa-\kappa^++1\big)+8(\zeta_2^+-3\zeta_2)[u\bar{u}(2+13u\bar{u}) \\
&\quad + 2u^3(10-15u+6u^2)\ln(u)+2\bar{u}^3(10-15\bar{u}+6\bar{u}^2)\ln(\bar{u})\big], \\
\mathcal{A}(\alpha_i) &= 360\alpha_q\alpha_{\bar{q}}\alpha_g^2\left(1+w_\gamma^A\frac{1}{2}(7\alpha_g-3)\right), \\
\mathcal{V}(\alpha_i) &= 540w_\gamma^V(\alpha_q-\alpha_{\bar{q}})\alpha_q\alpha_{\bar{q}}\alpha_g^2, \\
\mathcal{T}_1(\alpha_i) &= -120(3\zeta_2+\zeta_2^+)(\alpha_{\bar{q}}-\alpha_q)\alpha_{\bar{q}}\alpha_q\alpha_g, \\
\mathcal{T}_2(\alpha_i) &= 30\alpha_g^2(\alpha_{\bar{q}}-\alpha_q)\big((\kappa-\kappa^+)+(\zeta_1-\zeta_1^+)(1-2\alpha_g)+\zeta_2(3-4\alpha_g)\big), \\
\mathcal{T}_3(\alpha_i) &= -120(3\zeta_2-\zeta_2^+)(\alpha_{\bar{q}}-\alpha_q)\alpha_{\bar{q}}\alpha_q\alpha_g, \\
\mathcal{T}_4(\alpha_i) &= 30\alpha_g^2(\alpha_{\bar{q}}-\alpha_q)\big((\kappa+\kappa^+)+(\zeta_1+\zeta_1^+)(1-2\alpha_g)+\zeta_2(3-4\alpha_g)\big), \\
\mathcal{S}(\alpha_i) &= 30\alpha_g^2\{(\kappa+\kappa^+)(1-\alpha_g)+(\zeta_1+\zeta_1^+)(1-\alpha_g)(1-2\alpha_g)+\zeta_2[3(\alpha_{\bar{q}}-\alpha_q)^2-\alpha_g(1-\alpha_g)]\}, \\
\tilde{\mathcal{S}}(\alpha_i) &= -30\alpha_g^2\{(\kappa-\kappa^+)(1-\alpha_g)+(\zeta_1-\zeta_1^+)(1-\alpha_g)(1-2\alpha_g)+\zeta_2[3(\alpha_{\bar{q}}-\alpha_q)^2-\alpha_g(1-\alpha_g)]\},
\end{aligned}$$

where $\varphi_2(1\ GeV)=0$, $w_\gamma^V=3.8\pm1.8$, $w_\gamma^A=-2.1\pm1.0$, $\kappa=0.2$, $\kappa^+=0$, $\zeta_1=0.4$, and $\zeta_2=0.3$.

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Data Availability Statement This manuscript has no associated data or the data will not be deposited.

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