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The Many-Worlds Interpretation of Quantum Mechanics

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Article

Set Theory and Many Worlds

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Abstract: The 2022 Tel Aviv conference on the many-worlds interpretation of quantum mechanics highlighted many differences between theorists. A very significant dichotomy is between Everettian *fission* (splitting) and Saunders–Wallace–Wilson *divergence*. For fission, an observer may have multiple futures, whereas for divergence they always have a single future. Divergence was explicitly introduced to resolve the problem of pre-measurement uncertainty for Everettian theory, which is universally believed to be absent for fission. Here I maintain that there is indeed pre-measurement uncertainty prior to fission, so long as objective probability is a property of Everettian branches. This is made possible if the universe is a set and branches are subsets with a probability measure. A universe that is a set of universes that are macroscopically isomorphic and span all possible configurations of local beäbles fulfills that role. If objective probability is a property of branches, then a successful Deutsch–Wallace decision-theoretic argument would justify the Principal Principle and be part of probability theory rather than specific to many-worlds theory. Any macroscopic object in our environment becomes a set of isomorphs with different microscopic configurations, each in an *elemental* universe (elemental in the set-theoretic sense). This is similar to the many-interacting-worlds theory, but the observer inhabits the set of worlds, not an individual world. An observer has many elemental bodies.

Keywords: Everett; many worlds; multiverse; wavefunction realism; hidden variables

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1. Many Faces of Many Worlds

If a many-worlds interpretation of quantum mechanics is ever to become generally accepted, there first has to be agreement on what *the* many-worlds interpretation is, which is very far from being the case. There is even dispute about what to call it; are we to think in terms of a single branching world or a partitioning multiplicity of worlds? Some theorists work with the Heisenberg picture and a basic ontology of operators, while some work with the Schrödinger picture and a basic ontology of wavefunctions. On both approaches, there is scope for arguing that microscopic local beäbles are needed for a satisfactory physical ontology.

Within the diversity of views, there is a fundamental dilemma that I aim to resolve here. It is between the ideas that an observer may have multiple futures or always has a single future. Everett wrote of *splitting* in quantum measurement situations and it has generally been accepted that a well-informed observer cannot be uncertain about their future prior to Everettian fission. In an attempt to introduce pre-measurement uncertainty, Simon Saunders and David Wallace developed versions of many-worlds theory that reject the concept of splitting, which is arguably Everett's key idea. There shall be more on this in the following section.

To begin with, I will address the thorny matter of understanding the relationship between probability and uncertainty. This will lead to an argument that pre-measurement uncertainty exists for a fission interpretation of branching, where an Everettian observer splits into observers seeing different outcomes. The only reason why that feels counter-intuitive is that we have inherited a folk metaphysics that interprets future probabilities as properties of alternative possibilities. It is this which stands in the way of interpreting probabilities as properties of future coexistent actualities. A thought experiment helps to sugar this pill.

Understanding uncertainty as a cognitive state of assigning partial degrees of belief to coexistent futures requires assigning objective probabilities to those futures equal to the absolute squares of their quantum amplitudes. This calls for an account of how this *branch weight* can be understood to *constitute* objective probability. I shall argue that it can do so if understood to be a subset measure. This leads to interpreting the universal wavefunction as being a set of deterministic universes that contain microscopic local beäbles. Objects in our environment become sets of objects that are macroscopically isomorphic but differ in their microscopic configurations. They are extended in *configuration space*, so to speak. The half-life of an unstable particle thereby becomes a rate of change of a subset measure.

The result is a set-theoretic metaphysics for quantum mechanics that incorporates Everettian fission and microscopic local beäbles. It opens the way to new physics if the interaction between the universes that are the set-theoretic elements of our universe is the source of phase relations. After discussing spin, separability, and locality in the context of this metaphysics, I close with further reflections on Lev Vaidman's *World Splitter* and its implications.

2. Probability and Uncertainty

For classical mechanics, all physical processes are regarded as deterministic. The idea of there being a mind-independent, i.e., *objective*, probability can only be applied to the determination of initial conditions, relegated to an inscrutable past. Probability arises, as in statistical mechanics, from the epistemic condition of ignorance on the part of observers. The lack of Laplacian omniscience as to the exact positions and momenta of particles entails that perfect prediction is impossible, and thus, epistemic probabilities are assigned to fictional *possibilities* on the basis of statistical evidence. The gathering of that evidence involves the measurement of frequencies that can be regarded as surrogate approximations of epistemic probabilities given the assumption of the law of large numbers. Uncertainty about the future is regarded as a mental state that involves the entertaining of partial degrees of belief about future observations equal to the epistemic probabilities assigned to the possibilities of those observations on the basis of measured frequencies.

In the wake of quantum mechanics came the concept of *stochastic* physical processes, which are objectively probabilistic. Continuing to employ the metaphysics of possibility, a stochastic analysis of quantum processes with multiple possible outcomes supposes that one of those outcomes will be actualized by virtue of a random selection constrained by the objective probabilities of the possibilities. Those objective probabilities are determined by the Born rule when interpreted as assigning a quantum amplitude to the fictional possibilities. As in the case of classical mechanics, stochastic theory interprets uncertainty about the future as the entertainment of partial degrees of belief about alternative possible futures but now the partial degrees of belief are equal to the supposed objective probabilities.

The idea of stochastic processes has widely been accepted as plausible by physicists. It can seem plausible that the half-life of an unstable particle is a mind-independent property of that object. However, an air of mystery surrounds the concept, often referred to as propensity. How can propensity be a property of an object? What is the ontic status of propensity?

Hugh Everett III replaced the concept of a stochastic process with that of a *dendritic* process. Consider, for example, Vaidman's *World Splitter* [1]. Connecting with the device via a smartphone, you can choose a setup that will initiate a quantum measurement process with six equal-amplitude outcomes, i.e., a quantum die. The concept of a quantum die simply having six outcomes is an idealization that I shall use for the sake of argument to begin with. Later, I shall consider the implications of abandoning that idealization.

On "rolling" the quantum die, Everett's observer fissions into six observers, each seeing one of six different outcomes, which are all actual [2] (p. 459). Where is uncertainty to be found? Presumably, Everett thought that it was nowhere to be found, which is why he first entitled his thesis *Wave Mechanics Without Probability*. Presumably, the apparent lack of uncertainty did not bother Everett; after all, what has uncertainty to do with *physics*? He was simply suggesting that the histories of quantum processes are typically not linear, they *branch*. They are partially ordered series of events, not well-ordered series. Everett's world was not many, it was one; a single branching world that Saunders once appropriately dubbed the quantum block universe [3]. However, I shall continue to use the term "Many Worlds" since it has become virtually ubiquitous and is harmless enough so long as it is qualified in ways that will become clear as we go on.

As you roll Vaidman's quantum die, believing that you will fission, can you really deny being uncertain about the future? Many theorists have thought so, including Vaidman himself [4] (Section 3). In search of pre-measurement uncertainty, others preferred to replace Everett's concept of fission with those of *overlap* and *divergence*, where the body of an observer at a time is one of a multitude of *doppelgängers* in erstwhile "parallel" worlds [5–7]. However, inasmuch as that is motivated by trying to fill a lacuna left by supposedly absent pre-measurement uncertainty for fission, it is unnecessary, as we shall see.

Content to do without pre-measurement uncertainty, Vaidman kept to the traditional path by following Everett in believing that on rolling the quantum die, you will split into six "successors", each in a different branch and each seeing a different outcome. He wrote:

The quantum world splitter lets you enjoy all the possibilities in life with no need to choose. Why choose one, when you can do it all (AT ONCE!) [1] (original emphasis).

The idea is that you decide in advance to act on each of six different enjoyable options according to which number is observed after the measurement. An obvious first objection is to ask the following: in what sense will it be "you" acting in those different futures? Each of the six successors is a different observer seeing a different number, and thus, it is logically impossible for them all to be the same observer as you. This demonstrates that the metaphysics of persistence needs to be invoked to make sense of Everettian fission even before considering uncertainty.

Vaidman's term *successors* for post-split observers has generally been used by fission theorists and simply fails to meet this objection concerning personal identity. Note that this problem is avoided for the overlap and divergence interpretations of branching because no splitting occurs. Vaidman asserts that *you* can "do it all at once", but you are *not* any of your successors.

What is required is popularly known as *stage* theory, which was introduced by Ted Sider in 1996 [8], first explicitly applied to many-worlds theory in [9], and most recently in [10] (Section 2.1). It is generally accepted that a persisting object is one and the same thing from moment to moment. That is what could be called the folk metaphysics of persistence. However, it is not necessary to think of persistence like that. One can understand the history of an object as consisting of a series of momentary temporal parts or *stages*. What Sider recognized was that an object, at any given moment, could be understood to be a stage of its history and that a persisting object can be understood to be one that has a special relationship with the stages that are called its past and future *temporal counterparts*. A persisting object *was* its past temporal counterparts and *will be* its future temporal counterparts. Contrary to folklore, a persisting object (or observer) does not have to be one and the same thing from moment to moment after all. If he were to adopt stage theory, Vaidman could say, without fear of contradiction, that you will be each of six different observers, each seeing a different number.

What is the ontic status of non-present stages on this account? That depends on one's view of the ontic status of past and future states of affairs. In the *eternalist*, block universe view, which I suggest is most appropriate for many-worlds theory, the past and future counterparts of an object will be objects that exist in the past and future of the present object. In non-eternalist views, the present persisting object will bear the temporal relations *was* and *will be* to objects that *did* and *will* exist.

I mentioned that Vaidman maintained that you cannot be uncertain about the future when you roll the quantum die. Surely you cannot be uncertain about the future when you know that all outcomes will occur! Surprisingly, that is a conviction that also arises from a folk metaphysics from which we can profitably free ourselves.

2.1. The Logic of Uncertainty

For stochastic theory, a quantum die involves an objectively probabilistic process with six possible outcomes. One of those outcomes will be actualized randomly and each possibility can be assigned an objective, mind-independent probability. To put it another way, the quantum die has a propensity. The propensity is such that each of the six possible outcomes has an equal probability of being actualized. There has long been a sense of mystery about propensity, which I hope to dispel.

For stochastic theory, an observer rolling a quantum die is uncertain about the future for the following reason. As with classical mechanics, uncertainty is understood to be a mental state involving the assignment of subjective probabilities and degrees of belief to alternative possible futures. Stochastic theorists derive the values for the degrees of belief by appealing to what has become known as the Principal Principle, which is basically the idea that an observer should assign subjective probabilities to possible outcomes equal to what they believe the objective probabilities of those outcomes to be [11] (Section 2.2). For stochastic theory, the degrees of belief are guided by what are taken to be objective probabilities, whereas, for classical mechanics, the degrees of belief are guided by estimated epistemic probabilities arising from the ignorance of microstates. According to stochastic theory, an observer is uncertain about the future prior to rolling the quantum die because they assign degrees of belief of 1/6 to each of the possible outcomes, whose objective probabilities are 1/6.

Vaidman followed Everett in understanding the process involved in rolling the quantum die to be dendritic rather than stochastic. All six outcomes actually occur, each in a different branch of physical reality. Each branch is assigned the same quantum amplitude as is assigned to the *possible* outcomes of stochastic theory, and since the branches actually exist, quantum amplitude must be a physical property that they possess. The absolute square of quantum amplitude is the quantity that stochastic theorists identify with objective probability and that seems acceptable when amplitudes are assigned to alternative possibilities, but can it be acceptable when amplitudes are assigned to coexistent actualities? Can objective probability be a property of branches?

It is certainly *logically* possible, for if the objective probability of all the outcomes occurring together is 1, then that entails that each of the outcomes will occur but that does not give reason to believe that the objective probabilities of each of those individual outcomes should also be 1. The objective probability of the occurrence of each outcome can be 1/6, contrary to the common belief that if an event will occur, then the objective probability of its future occurrence must be 1. That may seem to involve a contradiction because an observer must be certain that any particular outcome will occur whilst assigning it an objective probability of 1/6. However, the observer is not required to apply the principal principle here, where the future *occurrence* of the outcomes is concerned.

There is as yet no agreed justification for the Principal Principle; it is used by stochastic theorists simply because it seems self-evident. If you believe that a process has six possible outcomes whose objective probabilities are 1/6, then what else can you do but assign a degree of belief of 1/6 to the future occurrence of any particular outcome? However, stochastic theorists are in the habit of applying this idea in the context of multiple futures thought of as *alternatives*, whereas in the context of the dendritic quantum die, the futures are thought of as coexistent. In this context, the application of the Principal Principle is overruled by logical consequence because, again, if the objective probability of all outcomes occurring together is 1, then, necessarily, each outcome will occur, whatever its individual objective probability of occurrence. The observer can assign a subjective probability of 1 to the occurrence of all the outcomes because the objective probability of their combined occurrence is 1. This entails that the observer is certain each outcome will occur, *despite the objective probability of the occurrence of each outcome being 1/6.*

I should mention in passing that this brings an alternative perspective to the Deutsch–Wallace decision theory argument that observers should assign degrees of belief to future measurement outcomes in accordance with the Born rule [12] (pp. 160–189). If, as I am arguing, objective probability can be understood to be a property of future branches, then the decision-theoretic argument, if good, constitutes a justification of the Principal Principle, and thus, belongs to the philosophy of probability rather than specifically to many-worlds theory.

In what sense, then, can an observer be uncertain about the future prior to rolling the dendritic quantum die? They can be uncertain in the sense of assigning a subjective probability of 1/6 to each of the *future observations.* The observer will be each of six observers seeing different outcomes whose objective probabilities are 1/6. Applying the Principal Principle, the observer assigns a degree of belief of 1/6 to the future observation of each outcome. That is exactly what the stochastic theorist does when uncertain about what will be observed. Whether the futures are understood as alternative possibilities or coexistent actualities is beside the point, uncertainty is the very same thing in both cases. The thrall of a folk metaphysics of alternative possibilities can make this hard to grasp.

Should doubt remain, a thought experiment demonstrates that an observer can believe that they are assigning subjective probabilities to alternative possible outcomes whilst they are *in fact* assigning them to coexistent actual outcomes. This involves a set-theoretic metaphysics for physical objects that leads directly to an explanation of how objective probability can be a physical property of Everettian branches.

2.2. Many Worlds without Everett

What cosmologists call the observable universe is a finite region of space that is currently estimated to have a radius of about 46 billion lightyears. Since there is as yet no evidence that space is finite, there may be a countably infinite number of regions that are observationally identical.

Consider an observer who inhabits one of an infinite set of observationally identical universes where quantum dice are, hypothetically, stochastic. On rolling a die, an infinite number of *doppelgängers* in the set of erstwhile "parallel" universes move in concert and an infinite number of quantum dice are rolled. The set of universes subsequently partitions into six subsets whose measures are *necessarily* 1/6. The reason being that what it *means* in stochastic theory for an outcome of a particular type of process to have an objective probability of 1/6 is that the subset measure for that outcome tends to 1/6 as the sample tends to infinity. That is how the *probability measure* on an infinite set gets its name.

Now, drop the ubiquitous assumption of folk metaphysics that there is a one-to-one relation between observers and *doppelgängers*. This requires an exercise in what Donald Davidson has called *radical interpretation* [13]. The idea is that truth values must be preserved for relevant utterances by an observer on the original interpretation and the alternative. On the original interpretation, a single utterance by an observer is tokened by a single noise emitted by a single *doppelgänger*, but on the alternative, a single utterance is tokenized by the infinite number of isomorphic noises emitted by each of the *doppelgängers*. Likewise, for intensional acts; on the original interpretation the act of rolling a die is tokened by the movements of a single *doppelgänger*, whereas on the alternative interpretation, the act of rolling a die is tokened by the parallel movements of all the *doppelgängers.* In the alternative interpretation, a single die is rolled, which is constituted by all the parallel dice. This is the *unitary interpretation of mind* [14] (Section 2).

A novel use of set theory is required [10] (Section 4). Following Willard Van Orman Quine, physical objects in each observable universe are to be construed as self-membered singletons that are each identified with their hierarchy of unit sets [15] (p. 31). Quine spent much energy trying to find a way for mathematics to be understood without an ontic commitment to sets but failed. Having become resigned to the necessity of sets, he noticed that non-sets could be brought into the set-theoretic fold in a way that is harmless in the sense that it does not impair the use of set theory in mathematics. He introduced what are called *Quine atoms* by logicians, though the phrase is sometimes simply used to denote sets that are their own sole element. Quine's definition went further. Take any individual, an apple, say. What it is is the set that contains the apple as its only element, plus the unit set of that set, the unit set of that set, and so on. The singleton set containing the apple is identified with that hierarchy of unit sets as being one thing: the apple. I shall refer to such a thing as a *Quineian individual*.

Thus the body of an observer in the conventional interpretation of the mind–body relation is a single *doppelgänger* that is a Quineian individual. For the alternative unitary interpretation of mind, Quine's idea is extended so that any set of Quineian individuals is also defined as an individual, likewise identified with its unit set, that set's unit set, and so on.

If the observer's body is to be a set of *doppelgängers* in this way, it follows that a set of Quineian individuals must have the properties that its elements share, with some logically necessary exceptions, such as the number of elements and value-definiteness. Therefore, in the conventional interpretation of the setup involving an infinite set of observable universes, each observer has a body of mass M, which is a Quineian individual. In the alternative interpretation, the *single* observer has a body of mass M, which is an infinite set of Quineian individuals. In the alternative interpretation, the single observer inhabits a *single* observable universe, which is an infinite set of *elemental* universes (elemental in the set-theoretic sense). The observer's spatial location is a set of corresponding *elemental* locations. I say more about what correspondence involves below.

Now suppose that in the original interpretation of the setup, each observer believes that they inhabit an observable universe that is a Quineian individual and where quantum dice are stochastic. In this case, they believe that when they roll a quantum die, there will be a single outcome, which is one of six possible outcomes, each of which has an objective probability of 1/6. Switching to the unitary interpretation of mind, the single observer necessarily believes likewise but now they are mistaken because the single observer, unbeknown to them, inhabits an observable universe that is an infinite set of universes, which are Quineian individuals.

When the single observer rolls the quantum die, each of the *doppelgängers* that are elements of the observer's body moves isomorphically so that the parallel quantum dice are caused to roll. In each elemental universe, the outcome gives rise to sensory input to a *doppelgänger* so that as the set of elemental universes partitions into six subsets with different outcomes, the set of *doppelgängers* partitions into subsets with different sensory input. Differences in sensory input give rise to different observations so the single observer fissions into six observers making different observations. The bodies of the six downstream observers are each an infinite set of *doppelgängers* whose subset measures relative to the body of the upstream observer are 1/6, i.e., the probability measure.

For the foregoing non-Everettian cosmological setup, the single quantum die of the unitary interpretation of mind is not stochastic, it is dendritic. The conclusion must be that an observer can be mistaken when believing that their uncertainty prior to rolling a quantum die derives from there being six alternative possible outcomes that all have an objective probability of 1/6. Their uncertainty can derive from there being six coexistent actual outcomes that all have an objective probability of 1/6.

3. A Metaphysics for Everettian Fission

According to Everett, the quantum die splits into six dice, each showing a different number, and the observer splits along with it. As he saw it, *of course*, there can be no probability since there is no uncertainty, thus, his pursuit of a back door to probability via typicality.

Everett's key idea was that the concept of a stochastic process could be replaced by that of a dendritic process. To make it fully intelligible, there has to be an account of how a well-informed observer can be uncertain about future observations in a quantum measurement situation, i.e., observations they will make, together with other nearby observers who have split, along with the measuring device and the laboratory. We now have an account:

Uncertainty without alternatives:

> *Uncertainty about the future is the cognitive state of assigning partial degrees of belief to multiple futures; whether those futures are thought of as alternative possibilities or coexistent actualities is an arbitrary choice because the occurrence of a future does not entail that the probability of its occurrence is 1.*

If it is useful to our understanding of physics to employ the concept of fission rather than that of stochasticity, then we are free to do so. To be certain that all outcomes will occur entails that each will occur. Therefore, we can be certain that any particular outcome will occur whilst believing that the objective probability of that outcome is 1/6. Assuming the Principal Principle, the observer assigns a degree of belief of 1/6 to the future observation of that outcome, by observers who they and their laboratory colleagues will be.

How can the real-world quantum die split in such a way that the objective probability of each of its immediate future temporal counterparts is 1/6? By being an infinite set that partitions into subsets with a probability measure. The cosmological thought experiment provides the framework for a metaphysics for quantum fission that incorporates a modification of Quine's definition of individuals as being self-membered singletons identified with their hierarchies of unit sets:

Concrete sets:

> *Any physical object is a set of Quineian individuals, which is identified with its hierarchy of unit sets. It has all the properties that its elements share, other than those logically excluded, such as the number of elements and value-definiteness.*

3.1. From Metaphysics to Physics

The cosmological thought experiment invokes an infinite set of elemental parallel stochastic universes populated by Quineian individuals. However, the whole point of Everett's idea was to *replace* stochasticity with fission. For Everettian physics, the elemental universes must have deterministic, linear histories with branches emerging as the set partitions. Pilot wave theory provides possible candidate elemental universes [10]. Interacting worlds theory also provides candidate universes with a purely particle ontology [16–18], though it may be replaceable by a field ontology [19]. However, both the pilot wave and interacting worlds theories are restricted to non-relativistic quantum mechanics and involve nonlocality in the sense that there can be causal connections between spacelike-separated events.

An often-vaunted advantage of many-worlds theory is that it does not face those problems. When conceived of, following Everett, as a *pure wave* theory, all of the physics used by physicists can be recovered, so the story goes. In defense of many-worlds theory as a pure wave theory, Wallace has recommended a *mathematics-first* approach to the ontology of quantum mechanics, which excludes microscopic local beäbles as objects bearing properties [20]. The project of ontic structural realism, which he supports, is an interesting one, but I suggest that it is better suited to a pre-spacetime ontology than to that of quantum mechanics, where stuff happens *in* spacetime.

As Louis de Broglie once remarked, a Schrödinger wave is supposedly in configuration space but lacks configurations [21] (p. 381). There are currently other attempts to fix that by introducing local beäbles to many-worlds theory [22,23]. What I have been describing is a metaphysical framework that is independent of whatever physics may actually be involved. Assuming a particle ontology, just for the sake of illustration, this framework has it that any macroscopic object in our environment is a set of objects that are macroscopically isomorphic but which differ in their microscopic particle configurations. There is a sense in which we inhabit configuration space. Objects in our environment have a spatial extension, and they are extended in configuration space too, as are our bodies. In effect, the unitary interpretation of mind is a *consequence* of assuming that objects in our environment are extended in configuration space.

As explained in Section 2.2, recall that the unitary interpretation of mind is the idea that multiple *doppelgängers* instance a single observer, not multiple observers in qualitatively identical mental states. If your body is understood to be extended in configuration space, in the sense of being a set of bodies that are only anisomorphic at the level of microscopic configurations, then your mental state, now, is instanced by a multiplicity of *doppelgängers*. You are legion, to adapt a biblical phrase.

In light of this, think about Vaidman's quantum die again. It is an apparatus in a quantum optics lab that is a set of labs including all possible configurations of particles consistent with the Born rule. That is the reason why the set partitions in the same way as a set of stochastic dice would. However, is it an infinite set? Earlier, I argued that the subset measures of branches could be identified with objective probabilities since the hypothetical set of stochastic dice in the cosmological setup was presumed to be infinite. Is the set of all possible particle configurations infinite? That would depend on whether space is continuous. Can the branch subset measures still be identified with objective probabilities if the set of quantum die is finite? Perhaps not, or perhaps an effective law of large numbers is good enough for very large samples. In any case, given the cosmological setup, if there is a finite number of configurations, there can be a countably infinite set of each configuration until such time as we have evidence that space is finite.

According to this framework, an unstable particle in our environment would be a set of particles constantly partitioning into a decay subset of increasing measure. An observer with a detector would be constantly splitting into an observer not seeing decay and observers seeing decays at later and later times. The probability of observing decay within a given period would depend on the rate of change of the decay subset measure for that type of particle, i.e., its propensity to decay. We are thus free to hypothesize that the quantum die is a very large or infinite set of isomorphic dice that will partition in the same way as a corresponding set of stochastic dice would. Therefore, the subset measures of the downstream dice will be $1/6$ relative to the upstream die.

For another illustration of the idea that objects in our environment are extended in configuration space, consider a free electron at any given moment. It is a set of elemental electrons that are in different corresponding positions and have different corresponding momenta in the elemental universes. The term *elemental* here is strictly set-theoretic. Again, our universe is being construed as a set of universes and any object in an observer's environment is a set of objects. A free electron in our universe is a set of elemental electrons that are on different trajectories in the universes that are elements of our universe. That is why the electron has an indefinite position and momentum in our non-elemental universe, where objects have a definite position and momentum only if their elements have corresponding positions and momenta in the elemental universes.

The introduction of particles as local beäbles in the way I have just described, as being the set-theoretic elements of particles in our environment, effectively preserves the full structure of the wavefunction and avoids the drawbacks of the pilot wave and interacting worlds theories, as I shall now explain.

3.2. The World as a Wavefunction

Consider the wavefunction of a free electron understood in terms of set-theoretic metaphysics. For the pure wave theory, any region of space is assigned a quantum amplitude and the absolute square is taken to give the probability of finding the electron there if a position measurement is made. There is no account of how an electron can be "spread out" in this way, hence Wallace's appeal to a thingless ontology. However, in the set-theoretic metaphysics, the absolute square of amplitude for a spatial region yields a subset measure for the single free electron, which is a set of elemental electrons. Each elemental electron in that subset is at an elemental location that is an element of a location within the given spatial region. There is thus a *fully concrete* interpretation of the electron's wavefunction within the given region. It is not in any sense counterfactual. Every location in that region is a set of elemental locations where elemental electrons may be *actually located*.

It is often said that the paradox of superposition is dealt with by the many-worlds theory by understanding superpositions as being composed of definite states on different branches. Thus, Schrödinger′s cat is dead on some branches and alive on others (sometimes put as dead in some worlds and alive in others). However, Everettian theory has only ever given an account of *macroscopic* superpositions in this way. Mystery still surrounds the concept of microscopic superpositions; hence, again, the motivation for defending a pure wave theory in terms of an ontology that does not involve objects. The set-theoretic metaphysics resolves this problem by construing microscopic superpositions as also being constituted by multiple definite states. Again, the free electron becomes an extended object, extended in configuration space. It does so by being a set of electrons, each of which is on a different trajectory in a universe that is a set-theoretic element of the observer′s universe.

However, that only provides metaphysics for a momentary snapshot of the electron′s wavefunction. There needs to be the dynamics of unitary evolution too; where is that to come from? It strikes me that the most plausible option here is to adopt the interacting worlds theory. The individual elemental universes that contain the set-theoretic elements of the observer′s electron interact in such a way as to generate the unitary dynamics. Here there is scope for new physics in order to understand how universes separated in configuration space interact. The possibility of such new physics has already been suggested by interacting worlds theorists, but what must be stressed here is the radically different perspective that the set-theoretic metaphysics brings to the interacting worlds theory.

All the difference is in how the observer is situated. For extant interacting worlds theory, the observer is situated within an individual world, which corresponds to what I have been calling an elemental universe. For the set-theoretic metaphysics, the observer is situated in the *set* of interacting universes; objects in the observer′s environment, including their body, are sets. The observer′s universe becomes a set of interacting universes.

In a sense, the observer spans the set of interacting universes. They span the universes in the sense that the mental states of an observer are instanced by a multitude of brains in a multitude of *doppelgängers*. Each of those brains is a set-theoretic element of the brain to which the observer indexically refers by a tap to the skull. The observer′s mental states are instanced by a multitude of brains rather in the way that a single novel is instanced by a multitude of books.

Extant interacting worlds theory involves causal nonlocality because particle trajectories in the observer′s world are mutually interactive at spacelike separation by virtue of the interactions between worlds. By construing our universe as a set of interacting universes rather than an element of the set, this problem is avoided. The long-recognized causal locality of the many-worlds theory is preserved, as we shall see.

4. Being Indefinite

Consider an observer who rolls a quantum die blindfolded. According to Vaidman, the observer will fission into six successors, each on an Everettian branch where the outcomes are different. According to the set-theoretic metaphysics, the body of the observer will partition into six subsets and each subset will have elements that are *doppelgängers* in the presence of elements of one of the six outcomes. The partitioning of the observer′s body will be caused by slight physical effects propagating from the six different post-roll dice, even if those effects are very slight indeed, such as gravitational differences. However, the observer themself will not fission because the *doppelgängers* are not different enough to instance distinct perceptual states. The observer does not fission because their perceptual mechanism is screened by the blindfold. Post-measurement and pre-observation, there will be a single successor whose body is the set of all the *doppelgängers* in the six subsets. The environment of that single successor will contain a die with subsets that are six dice displaying different numbers. In other words, the die in the vicinity of the post-measurement, pre-observation successor will be in a *macroscopically indefinite* state.

Now consider a terrestrial observer watching the roll of a quantum die on Mars through a powerful telescope. Post-roll, there will be no causal influence on the observer's body for several minutes, and thus there will be no consequent partitioning of the observer's body. When light from the roll of the die reaches the observer's eyes, their body will partition into six subsets and then, after retinal states have been processed, there will be six sets of *doppelgängers* instancing elements of different perceptual states and the observer will have fissioned. During the intervening few minutes, the quantum die will have been in a macroscopically indefinite state relative to the terrestrial observer, but not, of course, relative to a Martian observer.

Given the set-theoretic metaphysics, an observer cannot fission into observers seeing different outcomes until the observer's body partitions into subsets which are bodies instancing different cognitive states. Note that this has nothing to do with *consciousness*, it has to do with *mental content*. It is well established that we can perceive states of the world around us whilst not being conscious of those perceptions. Two distinct observers may be in identical conscious states and yet act differently because of different unconscious perceptions.

Therefore, necessarily, quantum measurements with multiple outcomes that occur at spacelike separation from an observer are in macroscopically indefinite states relative to that observer. As has generally been recognized for the many-worlds theory, this is enough to scotch the idea that the observation of correlations between spacelike-separated measurements on entangled particles entails nonlocal causation. That conclusion only follows if measurement outcomes necessarily have single definite outcomes.

However, the set-theoretic metaphysics construes the observer's universe as a set of elemental universes and within the elemental universes there seems to be nonlocality because hypothetical spacelike-separated measurements would always have single definite outcomes. So, is nonlocality involved in the set-theoretic metaphysics after all?

No, because the apparent nonlocality at the elemental level is not really nonlocality at all. It would be if observers inhabited the individual elemental universes but the whole idea is that they do not. Observers inhabit sets of elemental universes and, at that level, nonlocality is absent for the reason I have just given. Elemental nonlocality is not nonlocality because elemental locations are not locations. For the set-theoretic metaphysics, there is no reason to suppose that there is causal influence between events at spacelike-separated *locations*, which are locations in the observer's spacetime, which is a set of elemental spacetimes. This will become clearer with an analysis of EPR–Bell experiments, and what is needed by way of preparation for that is a set-theoretic characterization of spin and entanglement.

4.1. Spin

Spin poses a further challenge to set-theoretic metaphysics. We have to take a step back. The universe is being construed as a set of elemental universes. An electron only has a location if all its elemental electrons are at corresponding elemental locations. For the sake of argument, consider an electron to be a point particle. In this case, it is at a spatial point only if all its elements are at corresponding elemental points.

The correspondence can be thought of in the following way. For an observer at a certain time, the universe exhibits a definite distribution of objects in space on the surface of the past light cone. The observer's universe at a time is to be construed as the set of universes containing all possible configurations of particles consistent with that definite distribution of objects. A particle only has a position in the observer's universe if its elements are all at the same position relative to isomorphic distributions of macroscopic objects in each elemental universe.

The set-theoretic metaphysics interprets objects with indefinite properties as sets of objects with definite properties; therefore, when it comes to spin, *elemental* electrons cannot have indefinite spin relative to any axis. An *elemental* electron must have a definite spin, i.e., *up* or *down*, relative to some axis, period. Just as an *environmental* free electron has an indefinite position and momentum whilst the electrons that are its *elements* follow trajectories, likewise, an *environmental* electron has indefinite spin relative to all axes but one whilst the electrons that are its *elements* simply have definite spin relative to a single axis. I shall continue to italicize these terms to avoid confusion. The spin of an *elemental* electron cannot be *measured*. Measurement is something we do in our universe, but not in the *elemental* universes that are its set-theoretic elements. Bearing that in mind, here is an attempt to provide a set-theoretic metaphysics for spin.

In the spirit of string theory, let an *elemental* point be baton-like, having an orientation. In this case, an *environmental* point in the observer's universe will have an orientation too, following the concrete sets rule, since all its elements have orientations. Let an observer's *environmental* point be a set of *elemental* points with all possible orientations. In this case, the *environmental* point will have an *indefinite* orientation. We are in the habit of thinking of spatial points in our environment as lacking orientation but that is to be replaced by the idea that a spatial point has an indefinite orientation because it is a set of *elemental* points with different orientations. This is like the earlier idea that a free electron in our environment does not lack a trajectory but rather has an indefinite trajectory since the electrons that are its elements are not on corresponding trajectories in each *elemental* universe.

As a point particle, an *elemental* electron can be supposed to have an orientation too. Let any *elemental* electron have an orientation that is exclusively either parallel or orthogonal to the orientation of the *elemental* point that it occupies. We can adopt the convention that an *elemental* electron that is parallel is spin-up and an *elemental* electron that is orthogonal is spin-down. An *environmental* electron that is x-spin-up can then be construed in the following way. All its *elemental* electrons that are at *elemental* points oriented parallel to the x-axis are spin-up. A little formalism may help.

Let $\mathrm{e_E}$ be an *environmental* electron and $\mathrm{e_e}$ an *elemental* electron. Likewise, let $\mathrm{p_E}$ be an *environmental*, spatial point and $\mathrm{p_e}$ an *elemental* point. Every *elemental* point has an orientation; therefore, for an *elemental* point oriented parallel to the x-axis, we can write $\mathrm{xp_e}$. Every *elemental* electron is at an *elemental* point ($\mathrm{e_e@p_e}$) and is either oriented parallel or orthogonal relative to that point, with parallel being spin-up and orthogonal being spin-down. Therefore, we can write $\mathrm{e_e@_{up}xp_e}$ for an *elemental* x-spin-up electron and $\mathrm{e_e@_{down}xp_e}$ for an *elemental* x-spin-down electron. An x-spin-up *environmental* electron is defined thus:

$$\mathrm{e_E}\ (\text{x-spin-up})\ \text{iff}\ \forall \mathrm{e_e}\ [(\mathrm{e_e} \in \mathrm{e_E})\&(\mathrm{e_e@xp_e})] \rightarrow [\mathrm{e_e@_{up}xp_e}]$$

An x-spin-up *environmental* electron measured on the z-axis has equal probabilities for being measured spin-up and spin-down. Given the earlier analysis of objective probability in terms of subset measure, this implies that the *environmental* x-spin-up electron has a subset of *elemental* electrons that are at *elemental* points parallel to the z-axis and, of that subset, the spin-up and spin-down elemental electrons are of equal measure. In other words, the measures of $\{\mathrm{e_e@_{up}zp_e}\}$ and $\{\mathrm{e_e@_{down}zp_e}\}$ on $\{\mathrm{e_e@zp_e}\}$ are equal.

As a consequence, an observer measuring an x-spin-up electron on the z-axis will fission into observers whose bodies are of equal measure, one observing an *environmental* electron that is z-spin-up and the other observing an *environmental* electron that is z-spin-down. For the post-measurement z-spin-up *environmental* electron, all its *elemental* electrons that are at *elemental* points parallel to the z-axis are spin-up; correspondingly for the z-spin-down electron in the other post-measurement environment.

What does it mean for the post-measurement observers to have bodies of equal measure? Recall the cosmological thought experiment with an infinite set of hypothetically stochastic universes. Now think in terms of an equal-chance measurement being made in each universe, i.e., a quantum coin flip. The set of universes will partition into two subsets of equal probability measure where different outcomes occur. For this setup, if the unitary interpretation of mind is adopted, there is a single observer at the outset whose body is a set of bodies (*doppelgängers*) that partitions into two subsets of equal measure, which are the bodies of the post-coin-flip observers. Recall also that in Section 3.1, the set of hypothetical stochastic universes was replaced by a set of pilot wave or interacting worlds universes, which would partition in the same way as a set of stochastic universes would, i.e., the branch subset measures would take the same values. The reason for this would be that the set of hidden-variable universes would include all possible configurations consistent with the Born rule (corresponding to the assumption of particular initial conditions in the pilot wave theory). To put it another way, the universal wavefunction is being interpreted as a set of hidden-variable universes that include all possible configurations, and thus, Everettian branching is construed as the partitioning of a set where the subset measures just *are* the outcome probabilities. Again, the perspective being taken is that of the unitary interpretation of mind, where the fissioning of the observer arises because of the partitioning of the observer's body.

To return to spin, let an *environmental* x-spin-up electron have subsets of *elemental* electrons at *elemental* points parallel to all possible orientations. For any orientation ô, the subset of *elemental* electrons at *elemental* points parallel to ô has two subsets, namely, $\{e_e@_{up}\hat{o}p_e\}$ and $\{e_e@_{down}\hat{o}p_e\}$, which are *non-elemental* spin-up and spin-down electrons, since any set of *elemental* electrons is an electron. They become the post-measurement *environmental* electrons if a spin measurement is made on the ô orientation. The measures of those subset electrons relative to $\{e_e@\hat{o}p_e\}$ are the probabilities for observing spin-up and spin-down at that orientation.

This provides a characterization of spin for the set-theoretic metaphysics. However, before we can apply it to the analysis of EPR–Bell experiments we need a set-theoretic characterization of entanglement.

4.2. Entanglement

A pair of electrons in a singlet state has zero net spin because they have opposite spins. Emitted from a source and collimated, the wavefunction propagates as a sphere with peaked amplitudes in opposite directions. The wave propagates in configuration space but the set-theoretic metaphysics provides, at any given moment, a characterization of the wave as a distribution in 3D space. Both the *environmental* electrons are sets of *elemental* electrons. At any region of *environmental* space at a certain time (the space in the environment of an observer), there will be subsets of the *elemental* electrons of each of the two *environmental* electrons, which are situated at *elemental* points that are set-theoretic elements of the *environmental* points in the given *environmental* region. This is what I meant earlier when I set that the set-theoretic metaphysics completely recovers the structure of a wavefunction. Here we see an instantaneous reconstruction. The dynamics, which provide the phase aspect of the wave, might be recovered via a many-interacting-worlds theory or by replacing the hypothetical set of stochastic universes with an appropriate set of pilot wave universes with a dual particle–wave ontology.

Consider congruent local spacetime regions of measurement, namely, A and B, that are equidistant from the source and spacelike-separated. Both *environmental* regions are sets of *elemental* regions containing electrons that are elements of each of the two entangled electrons. For both A and B, some *elemental* points will be the location of one of the elements of one of the two entangled electrons, assuming that no two *elemental* electrons can be located at the same *elemental* point. In each *environmental* region, for every *elemental* point that is the location of an element of one of the entangled electrons, there will be another *elemental* point that is the location of an element of the other electron. Since electrons lack haecceity, there is no sense in which elemental electrons can be permutated.

Furthermore, for each of the two *environmental* regions, there will be *elemental* points of all orientations, which are the locations of electrons that are elements of the two entangled electrons. Also, for every orientation, there will be two *non-elemental* electrons, which are subsets of *elemental* electrons of equal measure. One of those *non-elemental* electrons will be spin-up and the other will be spin-down. Both the entangled electrons will be equally present in both regions, so to speak, where the presence of an electron in a region is construed as it having subsets of elements that are located at points that are elements of points in that region. Therefore, the two entangled *environmental* electrons are separable because they are two distinct objects. They are two distinct sets of *elemental* electrons, with no elements in common. This analysis is contrary to what was claimed in [10] (Section 3).

Being entangled, the two *environmental* electrons are causally linked. If one of the electrons is measured spin-up in region A the other must be measured spin-down in region B and vice versa. To see why this does not violate causal locality, we now need to think about the EPR–Bell setup.

4.3. EPR–Bell

We are to consider Alice and Bob who inhabit regions A and B. When Alice makes her spin measurement on the x-axis, she fissions into Alice$_{\mathrm{UP}}$ and Alice$_{\mathrm{DOWN}}$, whose bodies occupy the local regions A_{UP} and $\mathrm{A}_{\mathrm{DOWN}}$.. The set of the points that are elements of the points in A is the fusion of the two distinct subsets that are the elements of points in A_{UP} and $\mathrm{A}_{\mathrm{DOWN.}}$ The fissioning of Alice's body involves the fissioning of spacetime itself. Prior to the measurement, Alice inhabited an *environmental* region that was a set of *elemental* regions, each in an *elemental* universe. Post-measurement Alice$_{\mathrm{UP}}$'s and Alice$_{\mathrm{DOWN}}$'s bodies inhabited two distinct *environmental* regions that contained *elemental* points in two distinct subsets.

What distinguishes A_{UP} and $\mathrm{A}_{\mathrm{DOWN}}$ is that they contain two different *environmental* electrons and different elements of the macroscopic superposition, which is a future temporal counterpart of Alice's body. A_{UP} contains all the elements of Alice$_{\mathrm{UP}}$'s body and none of the elements of Alice$_{\mathrm{DOWN}}$'s and vice versa. In Bob's absolute elsewhere, Alice's body is in a superposition and Alice$_{\mathrm{UP}}$ and Alice$_{\mathrm{DOWN}}$ occupy distinct branches, i.e., distinct subsets of region A.

Note that Alice's measurement need not change anything in the structure of region B. Keeping things simple to begin with, let Bob make his measurement on the x-axis too. He fissions into Bob$_{\mathrm{UP}}$ and Bob$_{\mathrm{DOWN}}$ in regions B_{UP} and $\mathrm{B}_{\mathrm{DOWN}}$. The key point here is that, because of the entanglement, these two subsets of region B differ from A_{UP} and $\mathrm{A}_{\mathrm{DOWN}}$ whilst regions A and B are isomorphic. *Necessarily*, Alice$_{\mathrm{UP}}$ cannot have measured the *same* electron as Bob$_{\mathrm{UP}}$, and Alice$_{\mathrm{DOWN}}$ cannot have measured the same electron as Bob$_{\mathrm{DOWN}}$. This is a consequence of the two environmental electrons having been in causal contact at their origin.

Now Bob's successor (immediate future temporal counterpart) is in superposition relative to Alice$_{\mathrm{UP}}$ and to Alice$_{\mathrm{DOWN}}$ and Alice's successor is in superposition relative to B_{UP} and to $\mathrm{B}_{\mathrm{DOWN}}$. These four observers' results cannot come into causal contact sooner than half the light-time between regions A and B. To see why, consider Clotilde, halfway along a light path between regions A and B and watching Alice and Bob. When Clotilde sees the results of Alice's and Bob's measurements, she fissions into Clotilde$_{\mathrm{AliceUP+BobDOWN}}$ and Clotilde$_{\mathrm{AliceDOWN+BobUP}}$. As Cai Waegell and Kelvin McQueen put it, "A world containing a Bob and an Alice is only created when the wavefront from Alice's measurement meets the wavefront from Bob's measurement" [24] (Section 6). However, it is unclear why they use the term "wavefront"; it is rather a matter of the past lightcones of Alice's and Bob's future temporal counterparts coming to overlap.

Things get a bit more complicated if Bob makes his measurement on a different axis from Alice. Alice measuring spin-up on the x-axis entails that Clotilde$_{\mathrm{AliceUP}}$ must see Bob$_{\mathrm{DOWN}}$ *if Bob measures on the x-axis*. However, as we saw in Section 4.1, the structure of the region where Bob's successor would measure x-spin-down is such that if the measurement had been made on a different axis, the results spin-up and spin-down would have probabilities determined by the subset measures of elements of the electron not measured by Alice$_{\mathrm{UP.}}$ Those *elemental* electrons would be the ones located at *elemental* points oriented parallel to the axis chosen by Bob. Therefore, a series of measurements would have to be made on a succession of singlet states for Clotilde's future temporal counterparts to gather statistical evidence confirming the predicted probabilities.

5. Beyond Idealization

With the set-theoretic metaphysics in place, consider a non-idealized version of Vaidman's quantum die. Apart from the six equiprobable outcomes, there will be a plethora of extremely low-amplitude outcomes. Outcomes where "quantum accidents" occur, such as your smartphone transforming into a simulacrum of a salamander rather than displaying one of six numbers. These sorts of future events were also conceivable in classical physics, as the result of highly improbable particle trajectories consistent with current observations. However, in the context of the fission interpretation of the many worlds theory, all such bizarre events exist in the multiple futures of an observer. Vaidman does not take them into account because such events have, as he would put it, a very low *measure of existence* [25]. I have effectively argued that Vaidman's measure of existence can be strictly identified with objective probability. Therefore, bizarre futures should be left out of the account when rolling a quantum die because they have ridiculously low probabilities. There is nothing new in this idea.

However, the idea that all these bizarre futures *actually exist* is not necessarily anodyne. Pause for thought is called for in view of scenarios such as Huw Price's *Legless at Bondi* [26] (p. 382). More briefly, suppose that you are ill and are offered treatment that involves quantum processes with multiple outcomes. There is a high probability that you will be cured but a low probability that you will end up much worse off. In a conventional context, you take the risk, even if a little anxiously. In the fission context, you can be sure that the cured person will know that someone else is suffering because of the decision you took. Is it consolation enough to know that the suffering person will also have been the person who took the decision? It is not obvious that a fission interpretation of many worlds is free from moral conundrums, but then why should we expect such a profound change of worldview not to have consequences for the ways we choose to act?

6. Parting Lines

I have argued that Everett's key idea was to replace the concept of a stochastic process with that of a dendritic process, which is the idea that quantum phenomena induce the splitting of observers and their environments. This ostensibly raises problems that cannot be resolved by physics alone because assumptions rooted in folk metaphysics stand in the way. Observers cannot make predictions and test them unless they persist, but how can an observer persist through fissioning into multiple observers? Sider's stage theory solves this problem, but it did not become available until 1996 and remains neglected in the philosophical literature on persistence.

How can an observer be uncertain about future observations whilst believing that all outcomes occur? The folk metaphysics of possibility and actuality stands in the way but logic does not. That the objective probability of all outcomes occurring is 1 does *not* entail that the objective probability of each outcome's occurrence is 1. In that case, uncertainty can be understood as assigning partial degrees of belief to multiple futures without those futures needing to be alternative possibilities, as has always been thought.

How can objective probability be a property of multiple actual outcomes? The proposal that is described and further developed here involves the hypothesis that individual objects in an observer's environment can be construed as sets with many elements that are macroscopically isomorphic and microscopically anisomorphic because they are constituted by different configurations of local beäbles. Quantum processes induce the partitioning of those sets into macroscopically distinct subsets whose measures are the objective probabilities of outcomes. As a consequence, a single observer's body is a set of *doppelgängers*, so the idea that there can be multiple copies of *observers*, which is widely held amongst many-worlds theorists, must be rejected. A future-looking account of objective probability is provided by the idea that a single observer, whose body is a set of *doppelgängers* fissions into multiple observers whose bodies are subsets of *doppelgängers* with probability measures. According to stage theory, the pre-measurement observer bears the relation *will be* to each of the post-measurement observers and is uncertain about the future because the observer assigns degrees of belief to future observations equal to their probability measures. There is no question as to *which* post-measurement observer the pre-measurement observer will be; they will be *each* of them.

This set-theoretic metaphysics provides a framework for a version of the many-worlds interpretation that involves locality, separability, and Everettian fission, rather than divergence. It provides an account of probability that does not appeal to self-location uncertainty and an account of microscopic reality that includes local beäbles. It leaves work to be done on the physics of those beäbles and how they participate in branching processes.

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