INFRARED DIVERGENCE PHENOMENA, HIGH-ENERGY PROCESSES, AND REGGE POLES

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1. INTRODUCTION

Infrared divergence phenomena are already well known from semiclassical arguments. For example, suppose an electron in motion is deflected due to its interaction with a potential. The Lorentz-contracted proper field of the electron will be altered by the collision, and the change in the proper field will be emitted as electromagnetic radiation. For sufficiently long wavelengths ($kR << 1$, where k is the wave number and R is a dimension of the scattering region), the radiation can be calculated without knowledge of the details of the trajectory in the scattering region. It depends only on the initial and final momenta of the electron and the direction in which the radiation is observed (assuming the electron suffers no time delay in the scattering region). As is well known, the energy emitted per unit frequency is constant in this limit. Making the transcription to the photon description, it is clear that the number of photons emitted per unit frequency range is inversely proportional to the frequency; i.e., the photon spectrum is of the form dk/k, which diverges as $k \rightarrow 0$. This is the infrared divergence for real photons.

The angular distribution can also be understood by the semiclassical argument. In the extreme relativistic limit, the proper fields will be Lorentz-contracted in a small region near the plane perpendicular to the direction of motion of the charge and moving along with the charge. This leads to a strong peaking of the radiation parallel either to the incident or final direction of motion.

The essential idea for understanding the problems posed by infrared divergences was introduced by BLOCK and NORDSIECK [1] in 1937. They pointed out that in any practical experiment involving charged particles it is impossible to specify completely the final state of the system. Because individual photons can be emitted with arbitrarily small energies, it is always possible that some photons will escape detection. In fact, they showed that the probability that only a finite number of photons will escape detection is precisely zero; this is due to the infrared divergence associated with soft virtual photons. On the other hand, when the cross-section is summed over all final states compatible with the detection arrangement, including all possible undetected photons, a nonvanishing result is obtained. In fact, the observed cross-section is very nearly the cross-section that would be obtained if all radiative corrections were ignored. This is the well known cancellation between the real and virtual infrared divergences.

2. SEMICLASSICAL PHENOMENA, HIGH-ENERGY PROCESSES AND REGGE POLES

Now I want to begin by describing how these semiclassical phenomena emerge from quantum electrodynamics. A well-defined separation into infrared terms and "shorter wavelength" terms can be made, and the infrared terms calculated explicitly to all orders. This infrared terms turn out to have special significance in high energy processes, and among other things they contain a Regge - like behaviour. Up to the discussion of the Regge behaviour, I shall follow the paper of YENNIE, FRAUTSCHI and SUURA [2], where references to some of the alternative treatments of the subject [3] can be found.

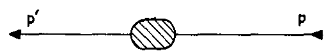

Fig. 1

A representation of the matrix element M_0.

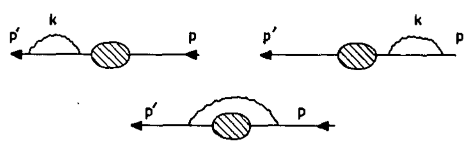

Fig. 2

Some of the ways in which an additional virtual photon can be inserted in Fig. 1

Consider a process in which an electron scatters from a state of 4-momentum p into one of p′. Let M_0 be the matrix element corresponding to any set of Feynman diagrams (Fig. 1.). Add one virtual photon to M_0, in the manner indicated in Fig. 2. That part of the contribution which diverges at small k can be represented by $M_0\ \alpha\ B$ where B is the gauge-invariant expression

$$B = \frac{i}{(2\pi)^3} \int \frac{d^4k}{k^2 - \lambda^2} \left(\frac{2p'_\mu - k_\mu}{2p'.k - k^2} - \frac{2p_\mu - k_\mu}{2p.k - k^2} \right)^2 \tag{1}$$

and λ is a fictitious photon mass. At small k the integral has the form dk/k, characteristic of the infrared divergence.

Now add a second virtual photon in the same way - inserting it only into the outside lines of the previous diagrams. Symmetrize the two virtual photons, introducing a factor $\frac{1}{2}!$ to prevent double counting. One obtains $M_0 (\alpha B)^2/2!$ plus other terms. Some of these other terms also have an infrared divergence, but a careful check reveals that they cancel when <u>all</u> other ways of introducing two virtual photons into M_0 are considered [2]. The same property is found in higher orders, so the addition of arbitrary numbers of virtual photons to M_0 yields the series

$$M_0 \sum_{n=0}^{\infty} (\alpha B)^n / (n\,!) = M_0 \exp(\alpha B) \tag{2}$$

plus terms M′ which have no infrared divergence. We can proceed to treat M′ in the same way as M_0, and eventually we find that the entire matrix element has a common factor exp (α B).

Since infrared effects are "long range", it is not surprising that the corresponding Feynman diagrams involve virtual photons emitted from external electron lines (Fig. 3). This provides electron propagators which are nearly real at small k and can spread far out into space. The exponential form (2) is also reasonable. Emission and absorption of low energy, low momentum photons do not appreciably disturb the motion of the electron; this means that such photons are emitted and absorbed independently, resulting in a Poisson distribution $(\alpha\ B)^n/(n!)$.

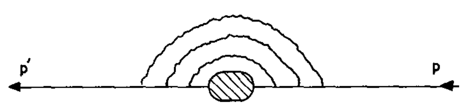

Fig. 3

A typical infrared diagram involving several virtual photons.

Equation (2) leads to a cross-section $d\sigma_0$, proportional to exp (2 α B). We must add to this cross-section the cross-section for emission of an undetected real photon, with energy bounded by K_m in order to escape detection. The infrared terms are associated with diagrams in which the real photon

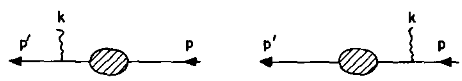

Fig. 4

An additional real photon, inserted into M_0 in these ways, gives an infrared-divergent contribution.

is emitted from the external lines (Fig. 4), and one obtains essentially $2\alpha\tilde{B}\, d\sigma_0$ where $2\alpha\tilde{B}$ is given by a product of phase space and squared-matrix element:

$$\tilde{B} = -\frac{1}{8\pi^2} \int_0^{K_m} \frac{d^3 k}{(k^2+\lambda^2)^{1/2}} \left(\frac{p'_\mu}{p'.k} - \frac{p_\mu}{p.k}\right)^2 \tag{3}$$

(for correction factors arising from a more careful treatment of energy conservation, see Ref. [2]).

At small k the integral again has the form dk/k. At high energy, where $|\vec{p}| \sim E$, etc, we see that the photon tends to emerge along the initial or final electron direction, as expected from the semiclassical argument.

The cross-section for emission of two undetected real photons must also be added. As in the case of virtual photons, this gives $(2\alpha\tilde{B})^2\,(2\,!)^{-1}$

$d\sigma_0$, plus terms which have no infrared divergence. The sum over all undetected real photons gives

$$d\sigma_0 \sum_{n=0}^{\infty} (2\,\alpha\,\tilde{B})^n/(n!) = d\sigma_0 \exp(2\,\alpha\,\tilde{B}) \tag{4}$$

and the observed cross-section is proportional to $\exp 2\,\alpha\,(B + \tilde{B})$.

It is well known, and can easily be verified explicitly from Eqs (1) and (3), that the infrared divergence in the lowest order real and virtual photon radiative corrections $2\,\alpha\,(B + \tilde{B})$ cancel, leaving a finite result. Since the higher order infrared terms simply raise $2\,\alpha\,(B + \tilde{B})$ to an exponential, the cancellation holds in all orders. One also finds that B approaches $-\infty$ as $\lambda \to 0$, ensuring that the cross-section for emission of no undetected real photons vanishes.

An attractive feature of these results is that the factor $\exp 2\,\alpha\,(B + \tilde{B})$ is known in general, independent of the details of short-range interactions in the matrix element. In the case of electron scattering from a nuclear target, for example, $\exp 2\,\alpha\,(B + \tilde{B})$ has the same form whether the target is left in its ground state or an excited state.

Another interesting aspect of the infrared factor can be seen from its form at large electron energies and momentum transfers ($E >> m, E' >> m$, $p.p' >> m^2$). If E/K_m is large the leading (double logarithm) term is

$$\exp\left[-(\alpha/\pi) \ln (2\,p.p'/m^2) \ln (EE'/K^2 m)\right]. \tag{5}$$

Here the small denominators which allow the virtual particles in infrared terms to travel far out from the target have been integrated over to give large logarithmic factors. Shorter wavelength photons are associated with at most one small denominator, and give at most single logarithms. Thus the infrared terms tend to provide the dominant radiative correction at high energy.

This result means that while the power of α provides a good index to the size of a radiative correction, the "range" of the correction should also receive some attention; long-range effects should be treated with special care.

Consider, for example, corrections of order α to electron-electron scattering. TSAI [4] has considered a clashing beam arrangement in which the two scattered electrons are detected in coincidence with good angular resolution $\Delta\theta$ but virtually no energy resolution ($\Delta E \sim E$). It is clear that if a photon is emitted parallel to either final electron, K_m is then of order E. However, if it is emitted perpendicular to the direction of the final electrons, K_m is much smaller and is determined by the angular resolution ($\Delta E \sim E\Delta\theta$). Thus $K_m(\theta)$ has a very strong angular dependence. An incorrect treatment of this angular dependence would change the double log term (5) by several per cent; the experimental energy resolution has to be treated carefully before it becomes worth while to calculate shorter range corrections of the same order in α but with no logarithms.

A numerical example will illustrate the related point that $\alpha^2 \ln^4$ terms can be at least as important as shorter range terms of order α. If E is 500 MeV, K' = 5 MeV, E' $\simeq$ 500 MeV, $p.p' = E^2$, then the $\alpha^2 \ln^4$ term obtained by expanding (5) contributes + 3%.

Another aspect of the infrared terms can be seen by recasting them as functions of the 4 - momentum transfer $t = (p-p')^2 = 2m^2 - 2p.p'$. The virtual photon factor $\exp(2\alpha B)$ behaves like a form factor for electron scattering and depends only on t. But the real photon factor $\exp(2\alpha\widetilde{B})$ introduces energy dependence as well (we let E = E′ now for simplicity), and (5) has the form:

$$\exp\left[-\frac{2\alpha}{\pi}\ln\left(\frac{2m^2-t}{m^2}\right)\ln\frac{E}{K_m}\right]=\left(\frac{E}{K_m}\right)^{-\frac{2\alpha}{\pi}\ln\left(\frac{2m^2-t}{m^2}\right)}. \tag{5.a}$$

Since the differential cross-section is proportional to this exponential,(5. a) resembles the formula

$$d\sigma/dt \sim E^{2J(t)-2} \tag{6}$$

expressing the exchange of a Regge pole with spin J(t) at high energies [5].

Fig. 5

Fig. 3 as seen in the t channel.

Do we expect to find a state of spin J(t) in the t channel? Consider Fig. 5, a typical infrared diagram as seen in the t channel where the incoming particles are an electron with 4 - momentum and a positron with - p′. The Fig. is obviously a ladder diagram with multiple exchange of photons. It would not be surprising if such diagrams gave some hint of the Bohr or positronium states which cluster just below threshold at $t = 4m^2$. The angular momenta of the positronium states (ignoring decay into photons) increase through the integers from J = 0 to J = ∞ as threshold is approached, according to the Bohr formula

$$g_t^2/2\mu = (-\mu c^2\alpha^2/2(n+J+1)^2 \tag{7}$$

where μ is the reduced mass and g_t the momentum of the electron in the centre-of-mass. It is known [6] that a Regge pole with spin J(t) interpolates smoothly between the integer J, still following Eq. (7), for each n. The Regge pole with highest J at each energy level corresponds to n = 0, and with the specializations $\mu = m/2$, $c = 1$, $4q_t^2 = t - 4m^2$, appropriate to our case, one finds from (7):

$$J(t) = -1 + m\alpha/\sqrt{4m^2 - t}. \tag{8}$$

Before the infrared factor is compared with this result it must be recalculated, since the form(5.a)is valid only at large $|t|$. (Note that even the threshold for the t channel is wrong in (5.a). One finds that near $t = 4m^2$ the leading term in the infrared factor is

$$\exp\left[\frac{2\alpha m}{\sqrt{4m^2 - t}} \ln \frac{E}{K_m}\right] = \left(\frac{E}{K_m}\right)^{\frac{2m\alpha}{\sqrt{4m^2 - t}}} . \quad (9)$$

The other factors in the cross-section give much smaller powers of energy, which are nearly constant in the region of the Bohr levels. Thus when a cross-section containing the infrared factor (9) is interpreted according to the Regge form $d\sigma/dt \sim E^{2J(t)-2}$, it gives essentially the Bohr trajectory (8) at $t \sim 4m^2$ to within a constant of order one.

3. CONCLUSION

In conclusion I should mention several peculiar features:

(1) Although the infrared factor essentially contains the power associated with the Bohr trajectory, it does not contain the poles associated with the Bohr levels. After all, we have only taken the lowest order radiative correction, which has no bound state poles, and raised it to an exponential, which does not introduce further singularities. A fuller treatment of the scattering would be required to obtain the poles [7].

(2) Eq. (5b) can alternatively be written in a form appropriate for large $|t|$,

$$\exp\left[-\frac{2\alpha}{\pi} \ln\left(\frac{2m^2 - t}{m^2}\right) \ln \frac{E}{K_m}\right] = \left(\frac{2m^2 - t}{m^2}\right)^{-\frac{2\alpha}{\pi} \ln \frac{E}{K_m}}, \quad (5.b)$$

giving a power whose rate of variation depends on the experimental energy resolution.

(3) The power of lab energy in (9) can be identified with $2J(t)-2$, where $J(t)$ refers to the Bohr trajectory, only if K_m is energy-independent in the laboratory.

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